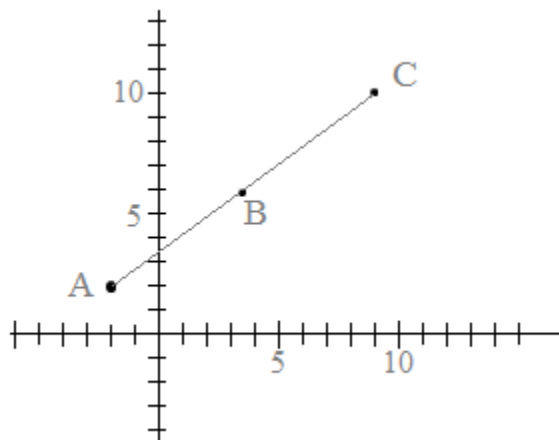


Geometry

Midpoint and Distance

Notes, Applications, and Practice Quiz (& Solutions)



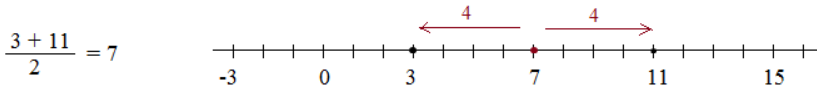
Topics include number lines, cartesian plane, formulas, triangles, circles, and more.

Midpoint and Distance: Notes, Examples, and Formulas

Midpoint

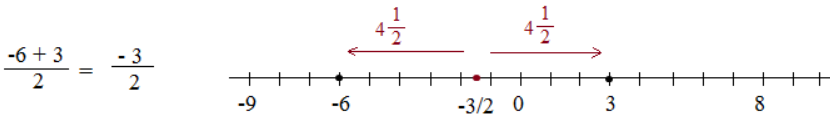
What is it? The "half-way point between two locations".
It is equidistant to each point.

Number line: The midpoint between 3 and 11 is 7.
7 is four units from both 3 and 11.



$$\frac{3 + 11}{2} = 7$$

The midpoint between -6 and 3 is $-\frac{3}{2}$



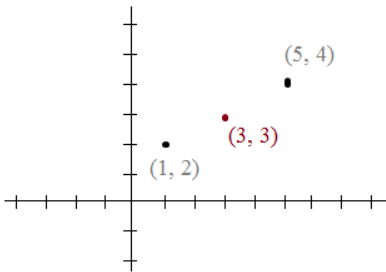
$$\frac{-6 + 3}{2} = -\frac{3}{2}$$

The midpoint is similar to the "average"

$$\frac{P_1 + P_2}{2} = \text{Midpoint}$$

The midpoint extends to the Cartesian Plane:

Simply find the midpoint of the X values. And, the midpoint of the Y values.



The midpoint of the X Values:

$$\frac{1 + 5}{2} = 3$$

The midpoint of the Y Values:

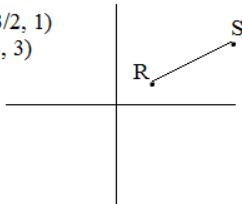
$$\frac{2 + 4}{2} = 3$$

$$\left(\frac{X_1 + X_2}{2}, \frac{Y_1 + Y_2}{2} \right)$$

Midpoint Formula

Where does the perpendicular bisector pass through \overline{RS} ?

R = (3/2, 1)
S = (4, 3)

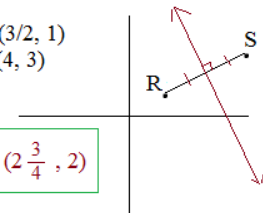


Find the midpoint of \overline{RS} :

$$\text{X coordinate: } \frac{3/2 + 4}{2} = \frac{11/2}{2} = \frac{11}{4}$$

$$\text{Y coordinate: } \frac{1 + 3}{2} = 2$$

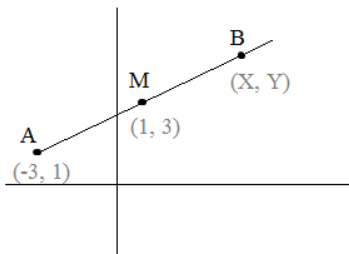
R = (3/2, 1)
S = (4, 3)



$$M = \left(2 \frac{3}{4}, 2 \right)$$

Given AB with midpoint M:

A = (-3, 1) M = (1, 3) What is B?



"Formula" Method

$$\frac{X_A + X_B}{2} = X_M \quad \frac{Y_A + Y_B}{2} = Y_M$$

$$\frac{-3 + X_B}{2} = 1 \quad \frac{1 + Y_B}{2} = 3$$

$$X_B = 5 \quad Y_B = 5$$

$$(5, 5)$$

"Travel" Method

Start at the endpoint. Determine how far you "travel" to the midpoint. Then, add the same amount.

$$\begin{matrix} A & & M \\ (-3, 1) & \longrightarrow & (1, 3) \end{matrix}$$

X value increased 4 units..
Y value increased 2 units..

$$\begin{matrix} M & & B \\ (1, 3) & \longrightarrow & (1 + 4, 3 + 2) \end{matrix}$$

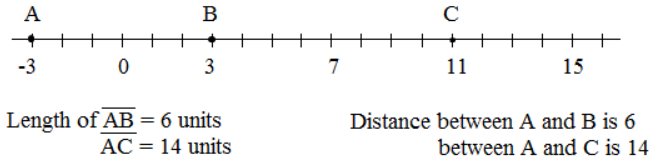
$$(5, 5)$$

Midpoint and Distance: Notes, Examples, and Formulas

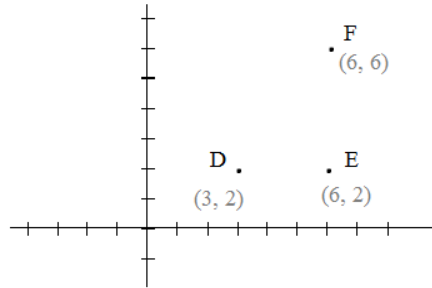
Distance

What is it? The space between 2 points.
The length of the line segment connecting two points.

Number Line:



Cartesian Plane:



The distance between D and E is 3 units...

(3, 2), (4, 2), (5, 2), and (6, 2)

And, the distance between E and F is 4 units...

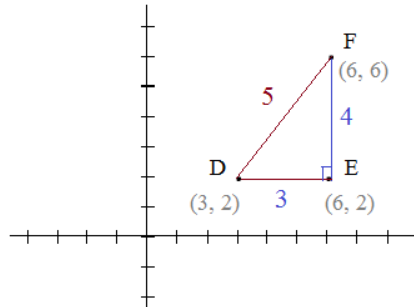
(6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

So, what is the distance between D and F?

(And, it is not 7!!)

Pythagorean Theorem

$$a^2 + b^2 = c^2$$



Notice, in this case, that the points can be vertices of a right triangle..

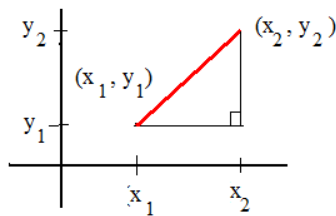
$$\text{So, } \overline{DE}^2 + \overline{EF}^2 = \overline{DF}^2$$

$$9 + 16 = 25$$

Therefore, the length of \overline{DF}
(i.e. distance between D and F)
= 5

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Distance Formula



Find the distance between (-2, 5) and (4, 7).

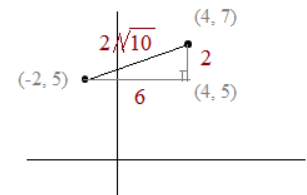
Using Distance Formula:

$$d = \sqrt{(-2 - 4)^2 + (5 - 7)^2}$$

$$= \sqrt{(-2 - 4)^2 + (5 - 7)^2}$$

$$= \sqrt{36 + 4} = 2\sqrt{10}$$

Using Pythagorean Theorem:



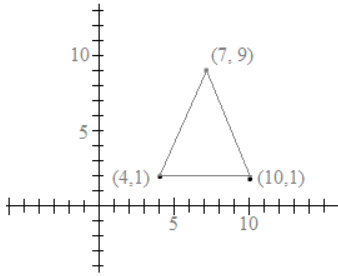
A vertical line drawn from (4, 7) intersects a horizontal line from (-2, 5) at (4, 5).. These form a right triangle!

Then, using the pythagorean theorem, the hypotenuse is $2\sqrt{10}$

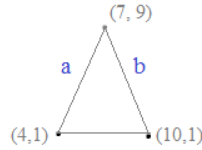
Distance and Midpoint Applications

I. Verify the following

1) The triangle is isosceles



Def. of isosceles: triangle with 2 congruent sides.



$$a = \sqrt{(7-4)^2 + (9-1)^2}$$

$$= \sqrt{9+64} = \sqrt{73}$$

$$b = \sqrt{(7-10)^2 + (9-1)^2}$$

$$= \sqrt{9+64} = \sqrt{73}$$

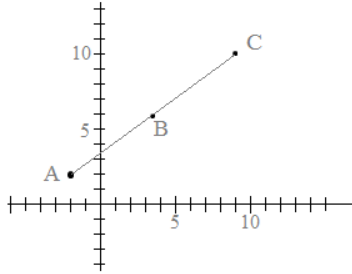
$a = b$, therefore the triangle is isosceles...

2) The length of AB equals the length of BC

$$A = (-2, 2)$$

$$B = (3.5, 6)$$

$$C = (9, 10)$$



Midpoint:

Midpoint of \overline{AC}

$$\left(\frac{-2+9}{2}, \frac{2+10}{2} \right)$$

$$(3.5, 6)$$

since B is the midpoint of \overline{AC} , $\overline{AB} = \overline{BC}$

Distance:

$$d_{\overline{AB}} = \sqrt{(-2-3.5)^2 + (2-6)^2}$$

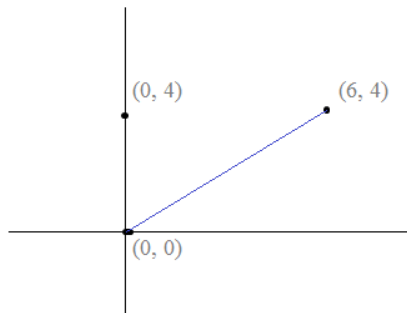
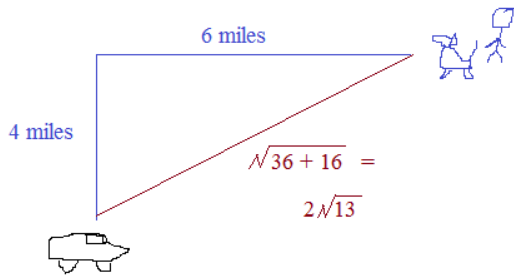
$$= \sqrt{30.25 + 16} = 6.80$$

$$d_{\overline{BC}} = \sqrt{(3.5-9)^2 + (6-10)^2}$$

$$= \sqrt{30.25 + 16} = 6.80$$

$$d_{\overline{AB}} = d_{\overline{BC}}$$

II. My dog and I go for a hike in a field. We leave the car and walk due north 4 miles. Then, we turn 90° to the right and continue 6 miles due east. We get hungry and decide to go straight back to the car. How far must we go?



distance of \overline{AB}

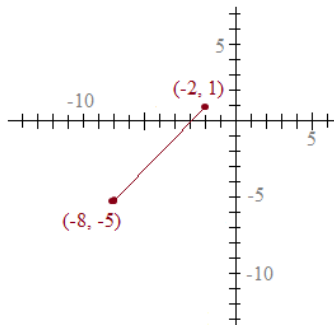
$$\sqrt{(6-0)^2 + (4-0)^2}$$

$$= \sqrt{52} = 2\sqrt{13}$$

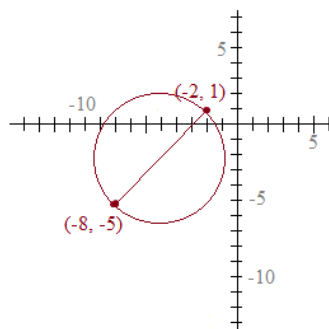
Distance and Midpoint Applications

III. Write the standard form of a circle with endpoints $(-2, 1)$ and $(-8, -5)$

Step 1: Sketch the figure



Step 2: Establish the strategy



The standard form of a circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

where r is the radius and (h, k) is the center.

We need the center: *midpoint* of the diameter..

And, radius: *distance* between center and endpoint (or, $1/2$ distance of diameter)

Step 3: Solve

Center: Find the midpoint of endpoints $(-2, 1)$ and $(-8, -5)$

$$\left(\frac{-2 + (-8)}{2}, \frac{1 + (-5)}{2} \right) = (-5, -2)$$

Diameter: Distance between endpoints $(-2, 1)$ and $(-8, -5)$

$$\text{distance} = \sqrt{(-2 - (-8))^2 + (1 - (-5))^2} = \sqrt{36 + 36} = 6\sqrt{2}$$

Or, Radius: Distance between center $(-5, -2)$ and endpoint $(-8, -5)$

$$\text{distance} = \sqrt{(-5 - (-8))^2 + (-2 - (-5))^2} = \sqrt{9 + 9} = 3\sqrt{2}$$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - (-5))^2 + (y - (-2))^2 = (3\sqrt{2})^2$$

$$(x + 5)^2 + (y + 2)^2 = 18$$



Eventually, Noah realizes that this assignment was NOT a geometry construction

Practice Review Quiz

Distance and Midpoint Review Quiz

Answer the following Questions. (Suggestion: Plot points and use graphs to verify solutions)

I. Midpoint

1) Find the midpoint between:

A) (0, 1) and (8, 3)

B) (11, -4) and (-6, -4)

C) (-17, -7) and (-7, -6)

2) Answer the following:

A) The midpoint of AB is (3, -3). If point A = (-2, -4), what is point B?

B) The endpoint of a segment is (5, -5). The midpoint of the segment is (9, -5). What is the other endpoint?

II. Distance

1) What is the distance between:

A) (3, 6) and (7, 9)

B) (7, -1) and (7, 7)

C) (-4, 5) and (1, 12)

2) The distance d between two points is given. Find the value(s) of b :

A) (0, b) and (3, 1); $d = 5$

B) (b , -7) and (-5, 1); $d = 10$

C) (-9, -2) and (b , 5); $d = 7$

Distance and Midpoint Review Quiz (continued)

III. Geometry application

A) Using the distance formula, determine whether the following are vertices of a right triangle (i.e. Distances and converse of Pythagorean Theorem)

1) (5, 8) (5, 2) and (0, 2)

2) (3, -1) (1, 4) and (-3, 0)

3) (-1, 1) (2, 4) and (3, -3)

B) Find the perpendicular bisectors of the following line segments:
(express your answer in point slope form)

1) Line segment \overline{AB} , where $A = (4, 7)$ and $B = (11, 6)$

2) Line segment \overline{CD} , where $C = (3, -9)$ and $D = (-6, -9)$

Distance and Midpoint Review Quiz

SOLUTIONS

Answer the following Questions. (Suggestion: Plot points and use graphs to verify solutions)

I. Midpoint

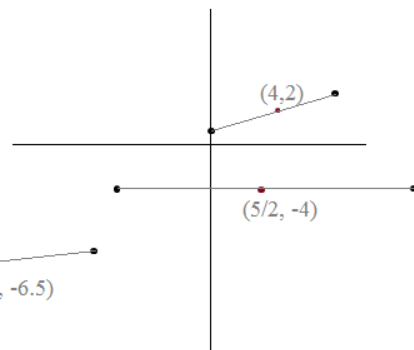
1) Find the midpoint between:

A) (0, 1) and (8, 3) $\left(\frac{0+8}{2}, \frac{1+3}{2}\right) = (4, 2)$

B) (11, -4) and (-6, -4) average of x terms: $(11 + (-6))/2 = \frac{5}{2}$ $(5/2, -4)$

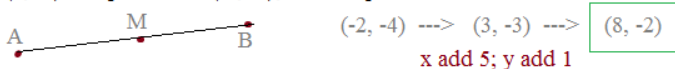
y terms are the same (no vertical change)

C) (-17, -7) and (-7, -6) $(-12, -6.5)$



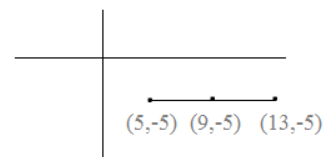
2) Answer the following:

A) The midpoint of AB is (3, -3). If point A = (-2, -4), what is point B?



B) The endpoint of a segment is (5, -5). The midpoint of the segment is (9, -5). What is the other endpoint?

$(9, -5) = \left(\frac{5+x}{2}, \frac{-5+y}{2}\right)$ $x = 13$ $y = -5$ $(13, -5)$



II. Distance

1) What is the distance between:

A) (3, 6) and (7, 9) $d = \sqrt{(7-3)^2 + (9-6)^2} = \sqrt{16+9} = 5$

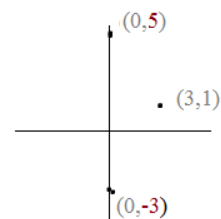
B) (7, -1) and (7, 7) Vertical line connecting both points: 8 units from -1 to 7

C) (-4, 5) and (1, 12) $d = \sqrt{(-4-1)^2 + (5-12)^2} = \sqrt{25+49} = \sqrt{74}$

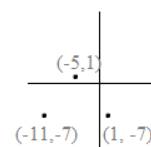
$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

2) The distance d between two points is given. Find the value(s) of b :

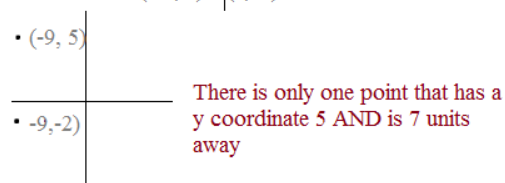
A) (0, b) and (3, 1); $d = 5$ $25 = 9 + (b-1)^2$
 $5 = \sqrt{(0-3)^2 + (b-1)^2}$ (square both sides and solve) $16 = b^2 - 2b + 1$ $(b-5)(b+3) = 0$
 $b^2 - 2b - 15 = 0$ $b = -3$ and 5



B) (b , -7) and (-5, 1); $d = 10$ $100 = b^2 + 10b + 25 + 64$
 $10 = \sqrt{(b+5)^2 + (-7-1)^2}$ $b^2 + 10b - 11 = 0$ $b = -11$ and 1
 $(b+11)(b-1) = 0$



C) (-9, -2) and (b , 5); $d = 7$ $49 = 81 + 18b + b^2 + 49$
 $7 = \sqrt{(-9-b)^2 + (-2-5)^2}$ $0 = 81 + 18b + b^2$ $b = -9$
 $(b+9)(b+9) = 0$



III. Geometry application

A) Using the distance formula, determine whether the following are vertices of a right triangle (i.e. Distances and converse of Pythagorean Theorem)

1) (5, 8) (5, 2) and (0, 2)
A B C

$$d_{AB} = 6$$

$$d_{BC} = 5$$

$$\begin{aligned} d_{AC} &= \sqrt{(5-0)^2 + (8-2)^2} \\ &= \sqrt{25 + 36} = \sqrt{61} \end{aligned}$$

$$\begin{aligned} AB &= 6 \\ BC &= 5 \\ AC &= \sqrt{61} \end{aligned}$$

$$AB^2 + BC^2 = AC^2$$

$$36 + 25 = 61$$

$$61 = 61 \checkmark$$

vertices of right triangle

2) (3, -1) (1, 4) and (-3, 0)
E F G

$$\begin{aligned} d_{EF} &= \sqrt{(3-1)^2 + (-1-4)^2} \\ &= \sqrt{29} \end{aligned}$$

$$\begin{aligned} d_{FG} &= \sqrt{(1+3)^2 + (4-0)^2} \\ &= \sqrt{32} \end{aligned}$$

$$\begin{aligned} d_{EG} &= \sqrt{(3+3)^2 + (-1-0)^2} \\ &= \sqrt{37} \end{aligned}$$

$$EF^2 + FG^2 = EG^2$$

$$29 + 32 = 37$$

$$61 \neq 37 \times$$

Vertices are not a right triangle

3) (-1, 1) (2, 4) and (3, -3)
M N P

$$d_{MN} = \sqrt{18}$$

$$d_{NP} = \sqrt{50}$$

$$d_{MP} = \sqrt{32}$$

$$MN^2 + MP^2 = NP^2$$

$$18 + 32 = 50$$

$$50 = 50 \checkmark$$

Yes! Vertices of a right triangle

B) Find the perpendicular bisectors of the following line segments: (express your answer in point slope form)

1) Line segment \overline{AB} , where A = (4, 7) and B = (11, 6)

Find midpoint of \overline{AB} :

$$\left(\frac{4+11}{2}, \frac{7+6}{2} \right) = (15/2, 13/2)$$

To find perpendicular line, find slope of \overline{AB} :

$$m = \frac{7-6}{4-11} = \frac{-1}{7}$$

slope of perpendicular line is 7.

(opposite reciprocal)

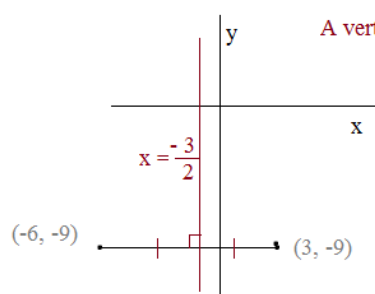
$$y - \frac{13}{2} = 7(x - \frac{15}{2})$$

2) Line segment \overline{CD} , where C = (3, -9) and D = (-6, -9)

Midpoint of \overline{CD} is (-3/2, -9)

Segment \overline{CD} is horizontal!

Therefore, the perpendicular bisector will be vertical...



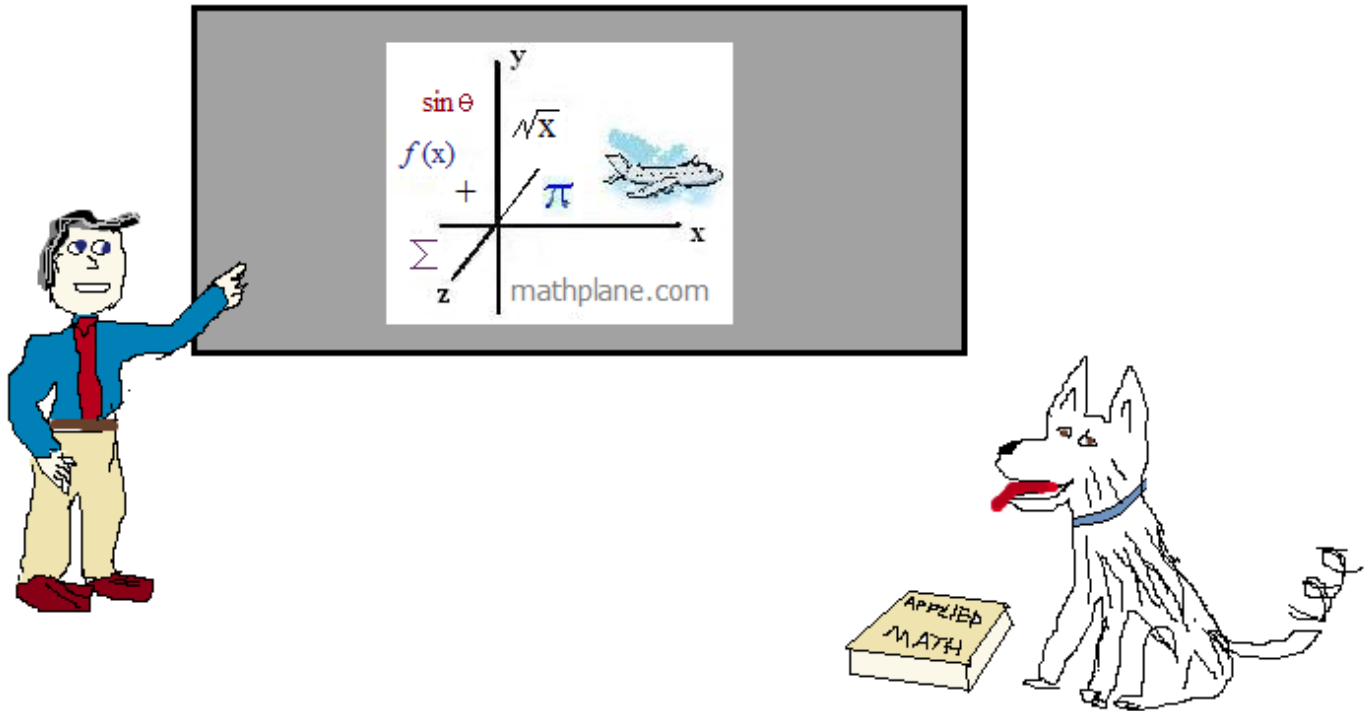
A vertical line going through $(-\frac{3}{2}, -9)$

$$x = -\frac{3}{2}$$

Thanks for visiting the site. (Hope it helped!)

If you have questions, suggestions, or requests, let us know!

Cheers.



One more question

The distance between A and B is 10 units.

If A is (3, 11) and B is (x, 5), then what is x?

Answer-→

The distance between A and B is 10.

If A is (3, 11) and B is (x, 5), what is x?

$$\text{distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$10 = \sqrt{(3 - x)^2 + (11 - 5)^2}$$

$$100 = (3 - x)^2 + 36$$

$$64 = (3 - x)^2$$

$$\pm 8 = 3 - x$$

$$x = -5 \text{ or } 11$$

