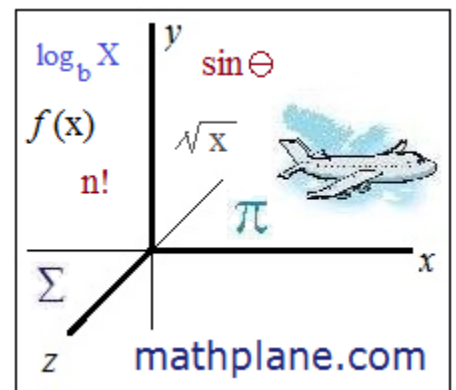


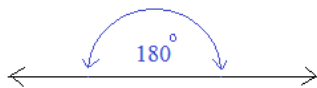
Angle Properties

Notes and Quick Quiz (& Solutions)

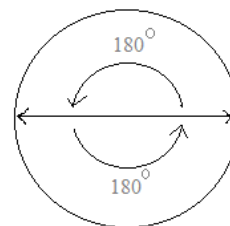


Angle properties: Notes, proofs, and examples

I. A *Straight Angle* is 180°



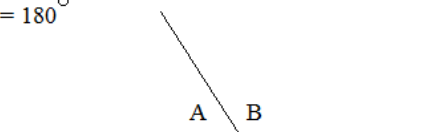
A circle has 360°



It follows that the semi-circle is 180 degrees.

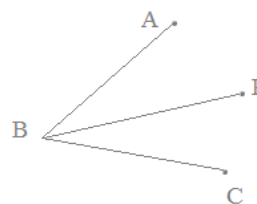
II. *Supplementary Angles* add up to 180°

$$m\angle A + m\angle B = 180^\circ$$

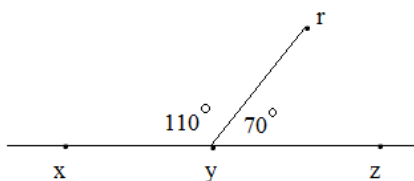


Angle Addition Postulate: If point P lies in the interior of $\angle ABC$, then

$$m\angle ABP + m\angle CBP = m\angle ABC$$



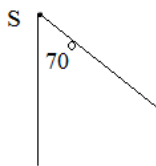
Example:



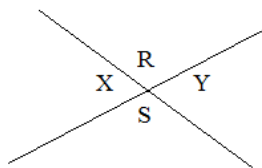
($\angle ABP$ is *adjacent* to $\angle CBP$ because they share a common vertex and side)

$\angle xyr$ and $\angle yrz$ are supplementary angles.

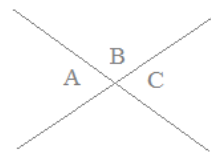
And, although they are not adjacent, $\angle S$ and $\angle xyr$ are supplementary as well.



III. *Vertical Angles* are congruent



$$\angle R \cong \angle S \quad \angle X \cong \angle Y$$



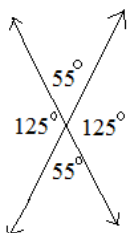
Informal proof: $\angle A = \angle C$

$$A + B = 180 \text{ degrees (supplementary angles)}$$

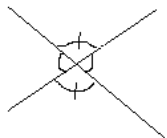
$$B + C = 180 \text{ degrees (supplementary angles)}$$

$$A = C \quad \text{(substitution)}$$

Examples:

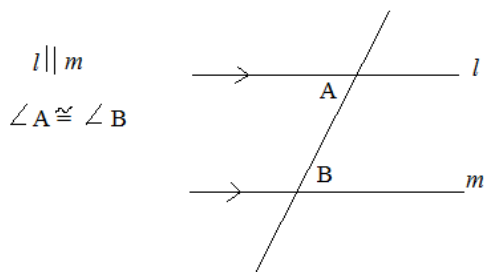


(sample notation for congruent angles)

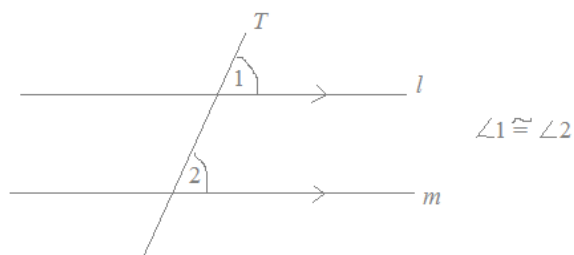


Angle properties: Notes, proofs, and examples

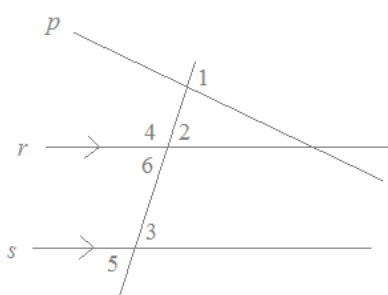
IV. If parallel lines are cut by a transversal, the *alternate interior angles* are congruent



Parallel Line Postulate: If 2 parallel lines are cut by a transversal, then their corresponding angles are congruent.



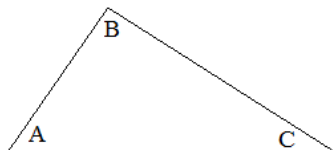
Examples:



- If $\angle 2 = 70^\circ$ and r is parallel to s ,
- $4 = 110^\circ$ (2 and 4 are supplementary)
- $3 = 70^\circ$ (3 and 2 are corresponding)
- $5 = 70^\circ$ (3 and 5 are vertical angles)
- $6 = 70^\circ$ (3 and 6 are alt. interior angles)
- $1 = ?$ (p is not parallel to r or s)

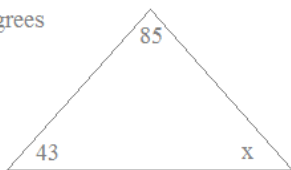
V. The sum of the *interior angles of a triangle* is 180°

$m\angle A + m\angle B + m\angle C = 180^\circ$

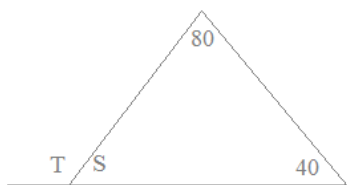


Examples:

$x + 43 + 85 = 180$ degrees
 $x = 52$ degrees



$S + 40 + 80 = 180$
 $S = 60$ degrees



$T + S = 180$ degrees
 $T + 60 = 180$
 So, $T = 120$ degrees

** Illustrates the *triangle (remote) exterior angle theorem*: the measure of an exterior angle equals the sum of the 2 non-adjacent interior angles.

Informal Proof: $1 + 2 + 3 = 180^\circ$

Add parallel line to one of the sides

$A + 1 + B = 180$ degrees (straight angle and addition postulate)

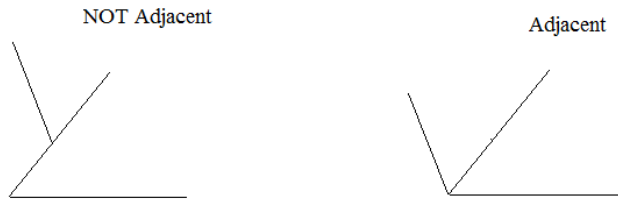
$A = 2$ and $B = 3$ (parallel lines cut by transversal, then alt. interior angles are congruent)

$2 + 1 + 3 = 180$ degrees (substitution)

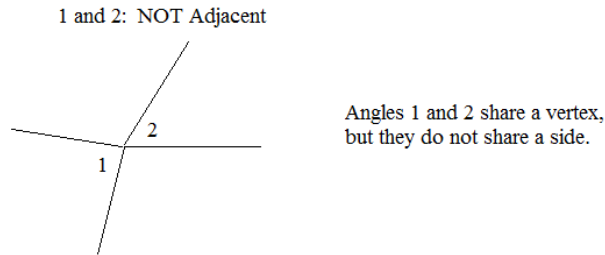
VI. Properties of *Adjacent Angles*

Adjacent Angles

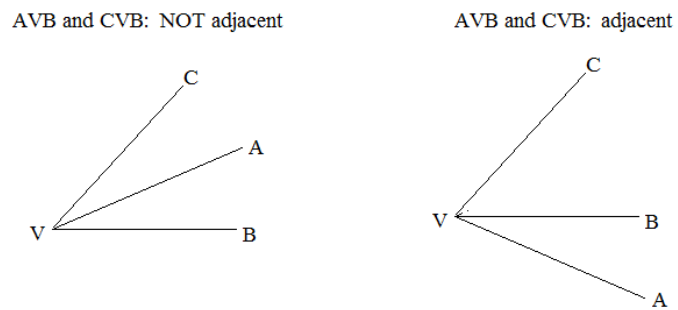
1) Angles share a common vertex



2) Angles share a common side

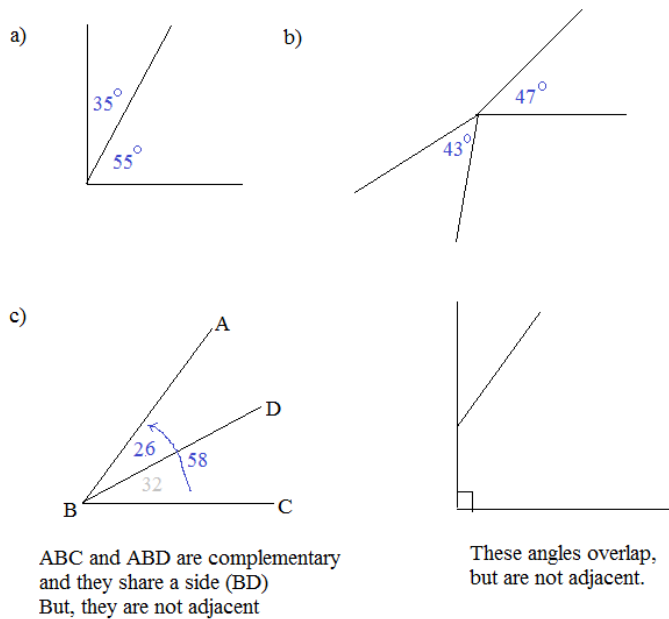


3) No side lies within the other angle



Example: Draw two complementary angles that are

- a) adjacent
- b) share a vertex, but are NOT adjacent
- c) share a side, but are NOT adjacent



Angle Properties and Algebra

Example: $\overline{AB} \perp \overline{BC}$ Find the measure of $\angle ABD$

Since AB is perpendicular to BC,
 $\angle ABC$ is a right angle.

Therefore,

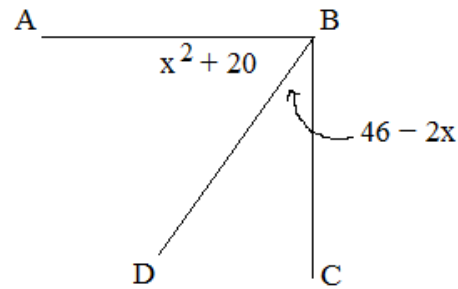
$$\angle ABD + \angle CBD = 90 \text{ degrees}$$

$$(x^2 + 20) + (46 - 2x) = 90$$

$$x^2 - 2x + 66 - 90 = 0$$

$$(x + 4)(x - 6) = 0$$

$$x = -4 \text{ or } 6$$



If $x = -4$: $\angle ABD = 36$

$\angle CBD = 54$

If $x = 6$: $\angle ABD = 56$

$\angle CBD = 34$

The measure of angle
 ABD is either 36° or 56°

Example: Find the measures of the angles in the figure.

Since there are 2 unknown variables, we need to
 find 2 equations.

$$2y + 4 = x + 6 \quad (\text{vertical angles are congruent})$$

$$(2x + 3y) + (x + 6) = 180 \quad (\text{supplementary angles})$$

$$\rightarrow x - 2y = -2$$

$$\rightarrow 3x + 3y = 174$$

Using Combination/Elimination method:

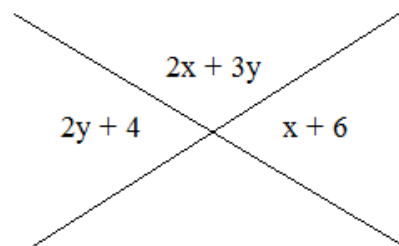
$$\begin{aligned} -3x + 6y &= 6 \\ 3x + 3y &= 174 \end{aligned}$$

$$\begin{aligned} 9y &= 180 \\ y &= 20 \end{aligned}$$

Substitute $y = 20$ into

$$2y + 4 = x + 6$$

$$x = 38$$



Since $x = 38$ and $y = 20$,

The measure of the acute angles
 is 44.

And, the measure of the obtuse
 angles is 136.

Angle Word Problems

Examples: — One of 2 complementary angles is 6 less than twice the other.

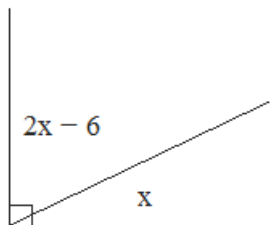
What is the measure of the larger angle?

— The measure of a supplement of an angle is 5 times that of the angle's complement.

Find the complement.

1) One of 2 complementary angles is 6 less than twice the other.

What is the measure of the larger angle?



Let $x =$ "the other angle"

then,

$(2x - 6) =$ "6 less than twice the other"

$$x + (2x - 6) = 90$$

$$3x = 96$$

$$x = 32$$

$$2x - 6 = 58$$

Suggested steps:

Step 1: (If possible) draw a picture

Step 2: Label variables

Step 3: Create equation(s) that describe(s) the question

Step 4: Solve the equation

Step 5: Answer question and check!

The complementary angles are 32 and 58.

58 is "6 less than twice" of 32.... ✓

The larger angle is 58°

2) The measure of a supplement of an angle is 5 times that of the angle's complement.

Find the complement.

Let $x =$ "an angle"

Then,

$(180 - x) =$ "the supplement"

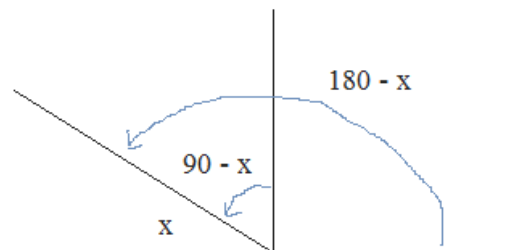
$(90 - x) =$ "the complement"

$$(180 - x) = 5(90 - x)$$

$$180 = 450 - 5x + x$$

$$4x = 270$$

$$x = 67.5$$



The complement is 22.5 and the supplement is 112.5

the supplement 112.5 is 5×22.5 ✓

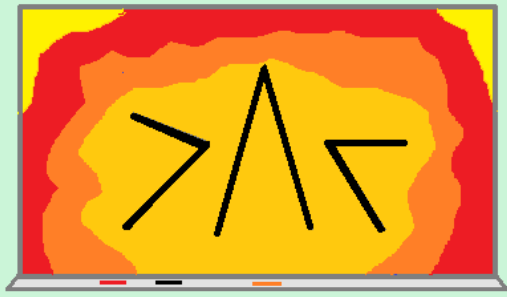
The complement is 22.5°

Townsend
School of
Geometry

Charlie's
Angles

So, Bosley, which
one is acute one?

"All three of them."



"Jill, I like
this class."

"Me too,
Sabrina!"

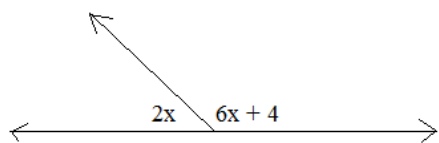
LanceAF #143 (6-18-14)
mathplane.com

This correspondence course attracts model students.

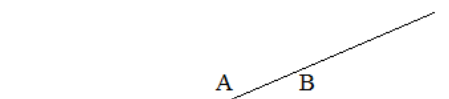
Practice Quiz-→

Angles Properties Quiz

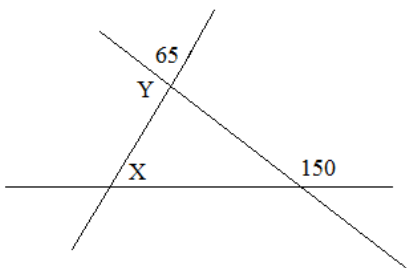
1) Find x :



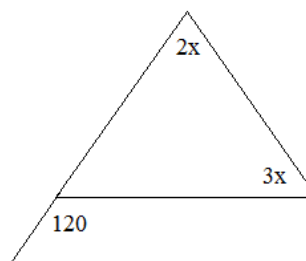
2) The ratio of A to B is $7:2$
What is the measure of angle B ?



3) Determine X and Y



4) What is the measure of the smallest angle?



5) Given: l and m are parallel.
Find the angles:

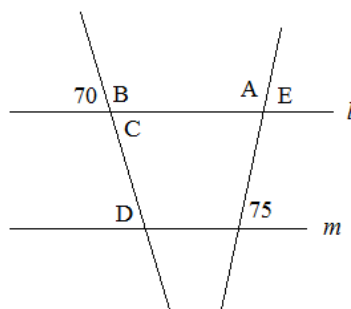
A:

B:

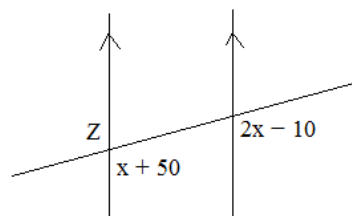
C:

D:

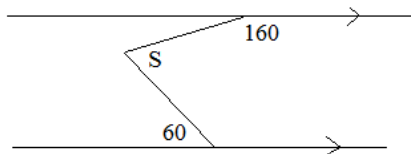
E:



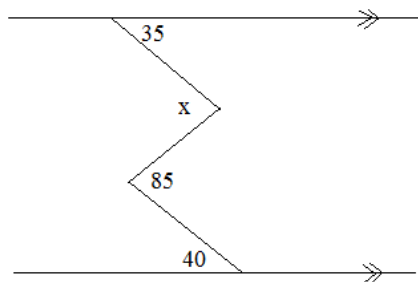
6) What is the measure of angle Z ?



7) Find S :

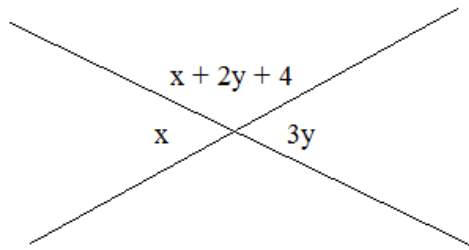


8) Find x :



Angles Properties Quiz

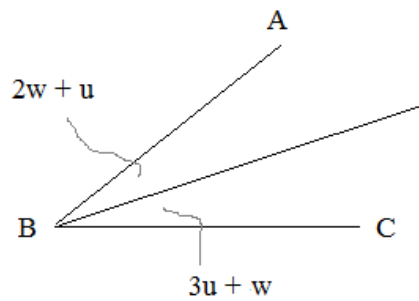
9) Find x and y :



10) $m\angle ABC = 40^\circ$

\overline{BD} bisects $\angle ABC$

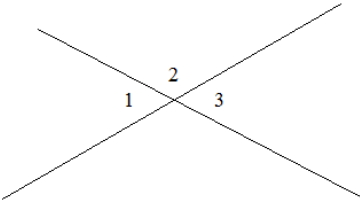
Find w and u



11) An angle is 8 less than its complement. What is the measure of the angle?

12) The supplement of an angle is ten more than twice the complement of the angle. What are the measures of the angle, complement, and supplement?

13)

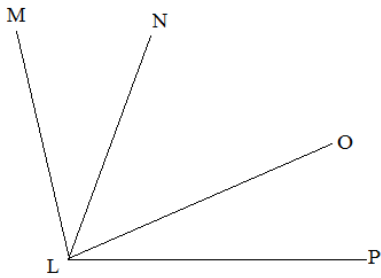


$$\angle 1 = 3x^2 - 7x + 22$$

$$\angle 3 = 4x + 26$$

What is the measure of angle 2?

14)



$$\angle MLN = x + 23$$

$$\angle NLO = x$$

$$\angle NLP = 4x - 39$$

$$\angle MLN \cong \angle OLP$$

What is the measure of angle MLP?

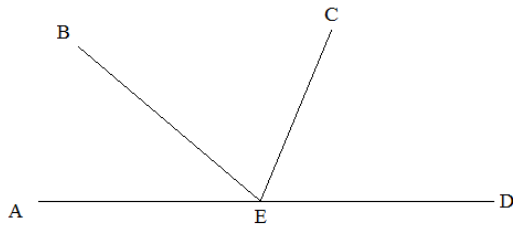
15) Draw two supplementary angles that share a vertex but are NOT adjacent...

16) Draw 2 complementary angles

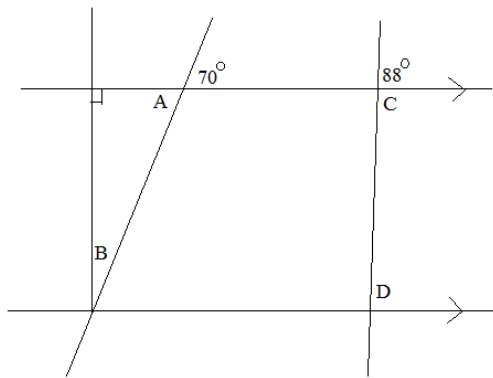
a) that are adjacent.

b) that are non-adjacent.

17) Describe/Classify all the angles in the diagram.. (Hint: there are 6 angles.)

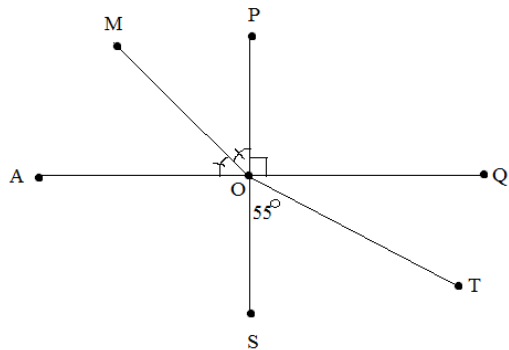


18) Identify all the angles, and describe your reasoning...



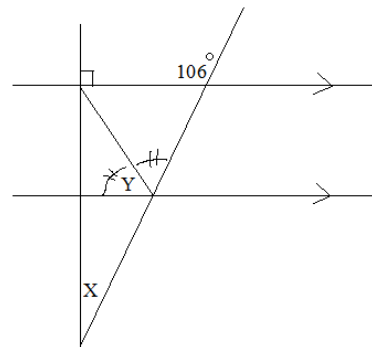
- A _____
- B _____
- C _____
- D _____

19) Determine the (acute) angle measures...



$\angle AOT$ _____

$\angle MOT$ _____



$\angle X$ _____

$\angle Y$ _____

Angles Properties Quiz

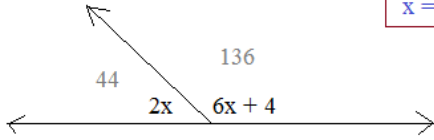
1) Find x:

(supplementary angles = 180)

$$2x + (6x + 4) = 180$$

$$8x = 176$$

$$x = 22$$



SOLUTIONS

2) The ratio of A to B is 7:2

What is the measure of angle B?

Ratio is 7:2

let $A = 7x$

$B = 2x$

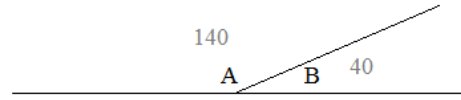
$$7x + 2x = 180 \text{ degrees}$$

$$9x = 180$$

$$x = 20$$

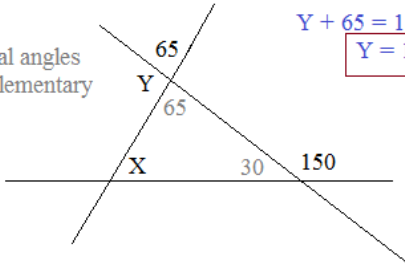
$$B = 2(20)$$

$$= 40 \text{ degrees}$$



3) Determine X and Y

65: vertical angles
30: supplementary angles



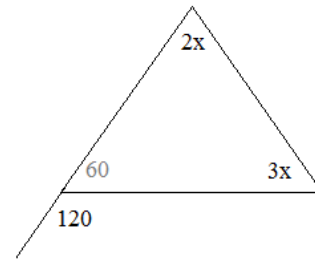
$$Y + 65 = 180$$

$$Y = 115 \text{ degrees}$$

$$65 + 30 + X = 180$$

$$X = 85 \text{ degrees}$$

4) What is the measure of the smallest angle?



$$2x + 3x = 120 \text{ degrees}$$

$$5x = 120$$

$$x = 24 \text{ degrees}$$

angles of the triangle:

48, 72, and 60

the smallest angle is

$$48 \text{ degrees}$$

5) Given: l and m are parallel.
Find the angles:

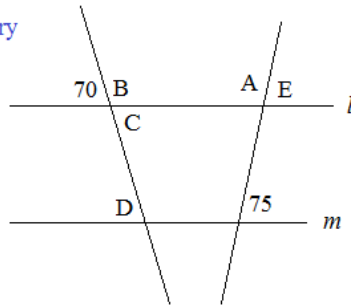
A: 105 (supplementary to angle E)

B: 110 (supplementary)

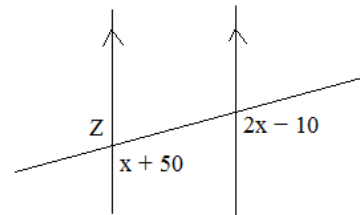
C: 70 (vertical)

D: 70 (alternate interior to C)

E: 75 (corresponding angle)



6) What is the measure of angle Z?



(corresponding angles are congruent)

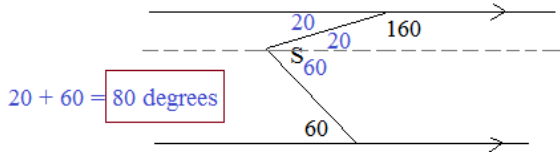
$$x + 50 = 2x - 10$$

$$x = 60$$

each angle is 110.

therefore, angle Z is 110°

7) Find S:

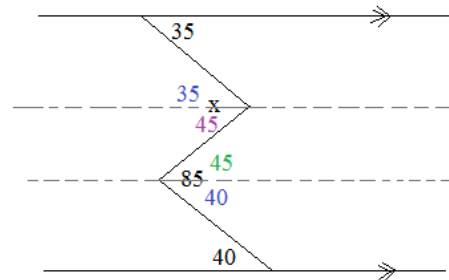


$$20 + 60 = 80 \text{ degrees}$$

Draw a line segment parallel to the given lines; then, utilize theorem -- alt. interior angles are congruent.

(20 and 160 are supplementary)

8) Find x:



35 (alt. interior)

40 (alt. interior)

45 (addition postulate: 40 + 45 = 85)

45 (alt. interior)

$$x = 35 + 45$$

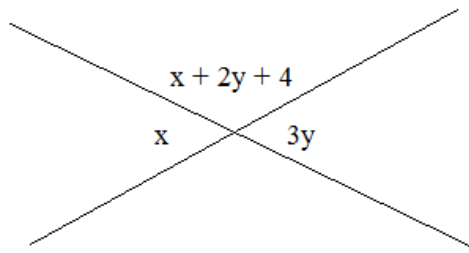
$$= 80 \text{ degrees}$$

Add auxiliary parallel lines and then fill in the values.

Angles Properties Quiz

SOLUTIONS

9) Find x and y:



$$x = 3y \quad (\text{vertical angles})$$

$$(x + 2y + 4) + x = 180 \quad (\text{supplementary angles})$$

Two equations, two unknowns... Solve the system using substitution:

$$x = 3y \quad 2(3y) + 2y = 176$$

$$2x + 2y = 176 \quad 8y = 176$$

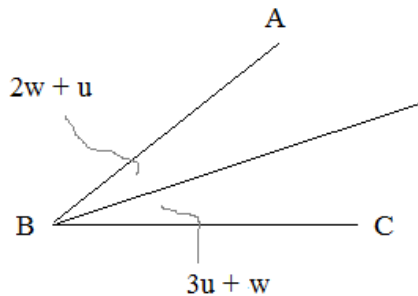
$$y = 22$$

$$x = 66$$

10) $m\angle ABC = 40^\circ$

\overline{BD} bisects $\angle ABC$

Find w and u



since BD is bisector,
 $2w + u = 3u + w$

also, $\angle ABC = 40$
therefore, $(2w + u) + (3u + w) = 40$

$$w = 2u$$

$$3w + 4u = 40$$

$$3(2u) + 4u = 40$$

$$u = 4$$

$$w = 8$$

11) An angle is 8 less than its complement. What is the measure of the angle?

Plug in numbers to check!

Let x = "an angle"

then, $(90 - x)$ = the complement

$$x + 8 = 90 - x$$

$$2x = 82$$

$$x = 41 \quad \text{and, the complement} = 49$$

12) The supplement of an angle is ten more than twice the complement of the angle. What are the measures of the angle, complement, and supplement?

Let A = "an angle"

since $A + \text{"supplement"} = 180$,
"supplement" = $180 - A$

since $A + \text{"complement"} = 90$,
"complement" = $90 - A$

"supplement" = $10 + 2(\text{"complement"})$

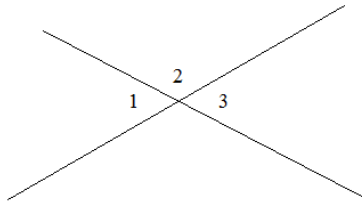
$$180 - A = 10 + 2(90 - A)$$

$$180 - A = 10 + 180 - 2A$$

$$A = 10$$

Angle: 10
Supplementary Angle: 170
Complementary Angle: 80

13)



$$\angle 1 = 3x^2 - 7x + 22$$

$$\angle 3 = 4x + 26$$

What is the measure of angle 2?

Angle 1 equals Angle 3 (vertical angles congruent)

SOLUTIONS

Angles Properties Quiz

$$3x^2 - 7x + 22 = 4x + 26$$

$$3x^2 - 11x - 4 = 0$$

$$(3x + 1)(x - 4) = 0$$

$$x = -1/3 \text{ or } 4$$

If $x = -1/3$: $\angle 1 = 3(-1/3)^2 - 7(-1/3) + 22$

$$= 3/9 + 7/3 + 22 = 24\frac{2}{3}$$

$$\angle 2 = 4(-1/3) + 26 = 24\frac{2}{3} \checkmark$$

Then, $m\angle 3 = 180 - 24\frac{2}{3} = 155\frac{1}{3}$

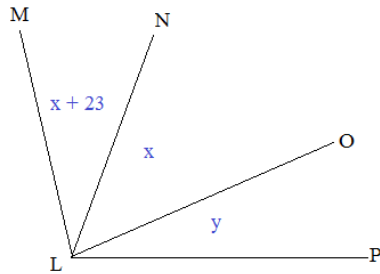
If $x = 4$: $\angle 1 = 3(4)^2 - 7(4) + 22$

$$= 48 - 28 + 22 = 42$$

$$\angle 2 = 4(4) + 26 = 42 \checkmark$$

Then, $m\angle 3 = 180 - 42 = 138$

14)



$$MLN = x + 23$$

$$NLO = x$$

$$NLP = 4x - 39$$

$$\angle MLN \cong \angle OLP$$

What is the measure of angle MLP?

Since $MLN = OLP$, $x + 23 = y$

and, since $NLO + OLP = NLP$ (addition postulate)

$$x + y = 4x - 39$$

Then, solve the system of equations:

$$\begin{aligned} y &= x + 23 \\ y &= 3x - 39 \end{aligned}$$

$$\begin{aligned} x + 23 &= 3x - 39 \\ 62 &= 2x \\ 31 &= x \\ \text{then,} \\ y &= 54 \end{aligned}$$

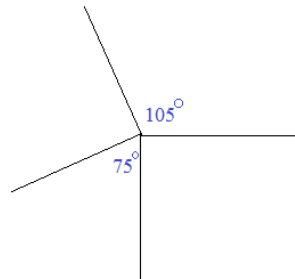
$$MLN = 54$$

$$NLO = 31$$

$$OLP = 54$$

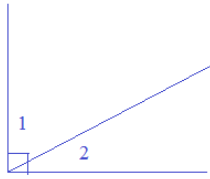
therefore, $MLP = 139$

15) Draw two supplementary angles that share a vertex but are NOT adjacent...

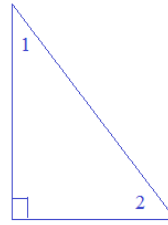


16) Draw 2 complementary angles

a) that are adjacent.

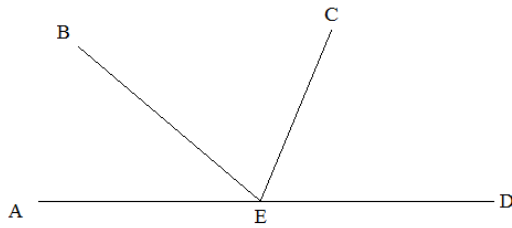


b) that are non-adjacent.



SOLUTIONS

17) Describe/Classify all the angles in the diagram.. (Hint: there are 6 angles.)

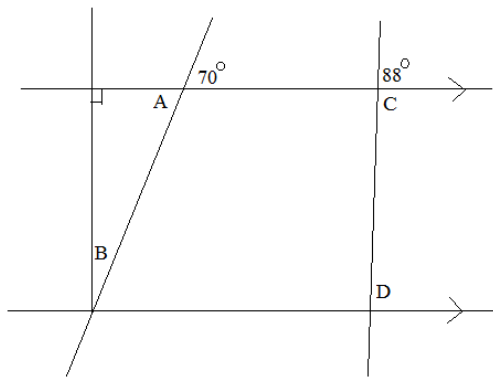


Acute angles: $\angle AEB$ $\angle BEC$ $\angle CED$

Obtuse angles: $\angle AEC$ $\angle BED$

Straight Angle: $\angle AED$

18) Identify all the angles, and describe your reasoning...



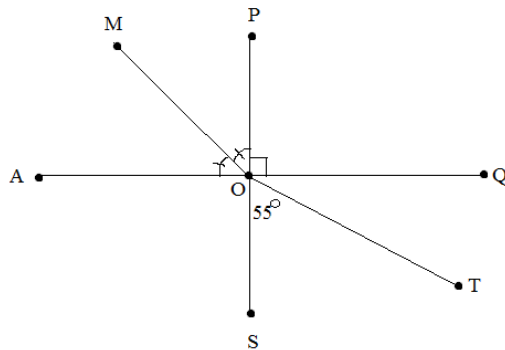
A 70 vertical angles

B 20 (sum of triangle interior angles = 180)

C 92 supplementary angles

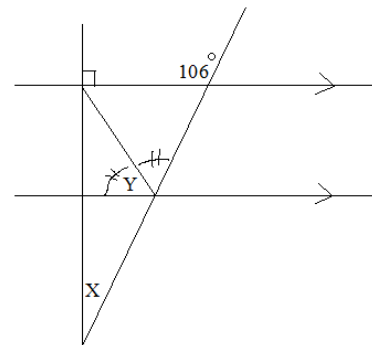
D 88 corresponding angles (parallel lines cut by transversal)

19) Determine the (acute) angle measures...



$\angle AOT$ 145 degrees $90 + 55$

$\angle MOT$ 170 degrees $45 + 90 + 35$

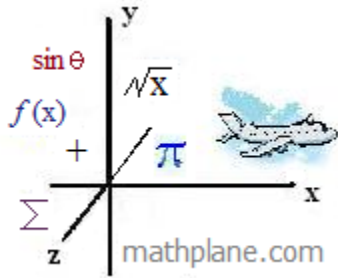


$\angle X$ 16 degrees triangle contains angles 90, 74, and X

$\angle Y$ 53 degrees $Y = 1/2(106)$ corresponding angles.. then, bisected..

Thanks for visiting the site. Hope it helped!

If you have questions, suggestions, or feedback, just let us know!



Also, at mathplane *express* for mobile at mathplane.ORG

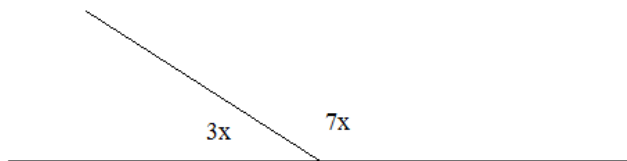
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One more geometry question: The ratio of an angle to its supplement is 3:7. What is the ratio to its complement?

(Answer on the next page)

The ratio of an angle to its supplement is 3:7.

What is the ratio to its complement?



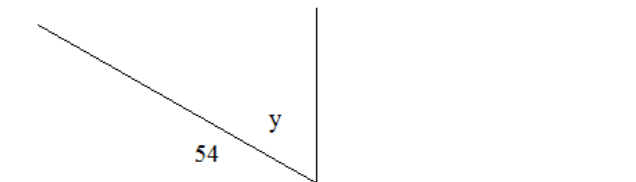
$$3x + 7x = 180 \text{ degrees}$$

$$10x = 180 \text{ degrees}$$

$$x = 18 \text{ degrees..}$$

$$3x = 54 \text{ degrees}$$

$$7x = 126 \text{ degrees}$$



$$54 + y = 90 \text{ degrees}$$

$$y = 36 \text{ degrees}$$

$$54:36 = 3:2$$