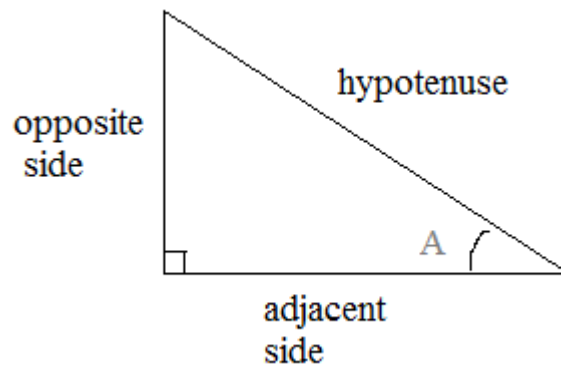


Trigonometry Introduction

Formulas, examples, and practice exercises (and solutions)



Topics include SohCahToa, finding inverse trig values, reciprocals, principal values, and more.

Trigonometry Introduction: Sine, Cosine, Tangent

Sine: $\frac{\text{opposite}}{\text{hypotenuse}}$

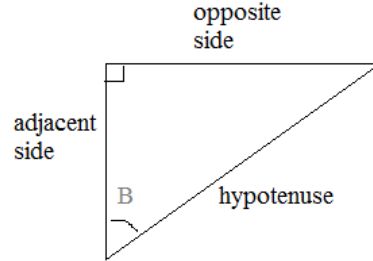
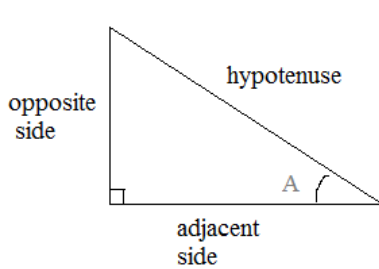
Cosine: $\frac{\text{adjacent}}{\text{hypotenuse}}$

Tangent: $\frac{\text{opposite}}{\text{adjacent}}$

Some use "Soh Cah Toa"
to memorize the ratios
(sine: opp/hyp cos: adj/hyp toa: opp/adj)

Identifying the sides of a (right) triangle:

Examples:



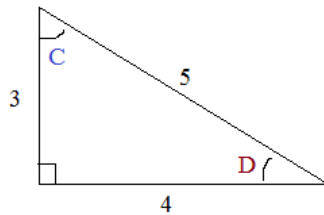
Applying the formulas:

Examples:

Sine $\angle C$ $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{4}{5}$

Cosine $\angle C$ $\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3}{5}$

Tangent $\angle C$ $\frac{\text{opposite}}{\text{adjacent}} = \frac{4}{3}$



Sine $\angle D$ $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}$

Cosine $\angle D$ $\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$

Tangent $\angle D$ $\frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}$

Note the relationship between C and D:
Sine D = Cosine C = 3/5
Sine C = Cosine D = 4/5
and, Tangent C is the reciprocal of Tangent D

Trigonometry Reciprocals: Cotangent, Secant, Cosecant :

Cotangent = $\frac{1}{\text{Tangent}} = \frac{\text{adjacent}}{\text{opposite}}$

Secant = $\frac{1}{\text{Cosine}} = \frac{\text{hypotenuse}}{\text{adjacent}}$

Cosecant = $\frac{1}{\text{Sine}} = \frac{\text{hypotenuse}}{\text{opposite}}$

A helpful memory trick to remember the pairs:

cotangent -- tangent ("tangent")
secant -- cosine ("2 syllable words")
sine -- cosecant ("the others")

Hidden Message

Solve the following trig equations to reveal the answer!

1) $\sin B = \frac{\square}{m}$

2) $\sin D = \frac{p}{\square}$

3) $\cos E = \frac{s}{\square}$

4) $\tan F = \frac{\square}{r}$

5) $\sin C = \frac{q}{\square}$

6) $\sin \square = \frac{s}{t}$

7) $\cos \square = \frac{p}{o}$

8) $\cos D = \frac{q}{\square}$

9) $\cos B = \frac{n}{\square}$

10) $\tan D = \frac{\square}{q}$

11) $\tan B = \frac{\square}{n}$

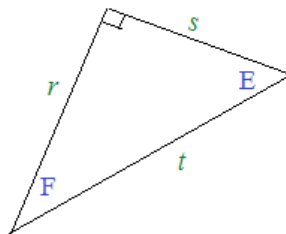
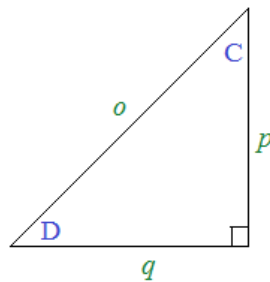
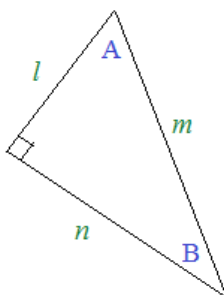
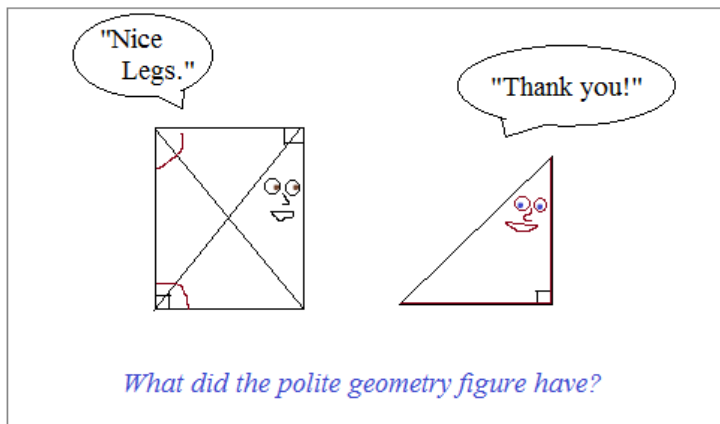
12) $\cos A = \frac{l}{\square}$

13) $\tan \square = \frac{r}{s}$

14) $\sin A = \frac{\square}{m}$

15) $\cos F = \frac{r}{\square}$

16) $\sin F = \frac{\square}{t}$



1) L

2)

3)

4)

5)

6)

7)

8)

9)

10)

11)

 E or I

12)

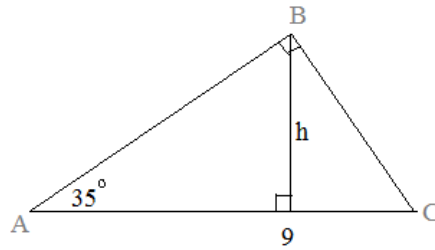
13)

14)

15)

16)

Example: Can you find h?

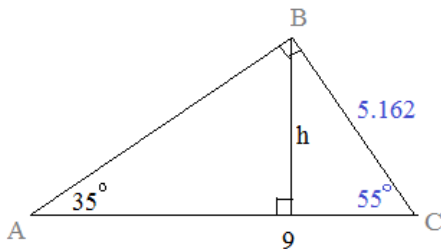


Method 1:

Since ABC is a right triangle, we can find BC using sine function.

$$\sin(35^\circ) = \frac{BC}{9}$$

$$BC = 5.162$$



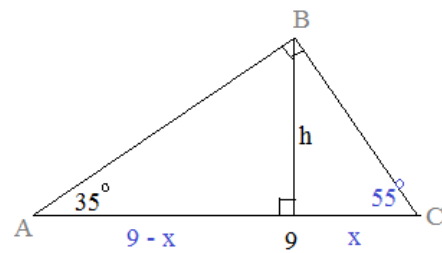
Then, we can use sine function to find h in the small right triangle:

$$\sin(55^\circ) = \frac{h}{5.162}$$

$$h = 4.23$$

Method 2:

Label the parts of the triangle(s). Then, construct 2 equations containing h:



$$\tan(35^\circ) = \frac{h}{(9-x)} \quad \tan(55^\circ) = \frac{h}{x}$$

$$h = (9-x)(\tan 35)$$

$$h = x(\tan 55)$$

using substitution,

$$(9-x)(\tan 35) = x(\tan 55)$$

$$.70(9-x) = 1.428x$$

$$6.3 = 2.128x$$

$$x = 2.96$$

$$\text{Finally, } \tan(55^\circ) = \frac{h}{2.96}$$

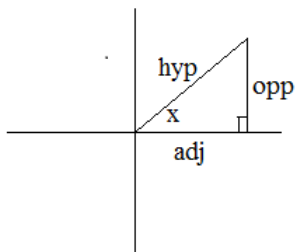
$$h = 4.23$$

Inverse Trig Functions: "Why does Sin^{-1} have a capital 'S'?"

Review: $\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$

$\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$

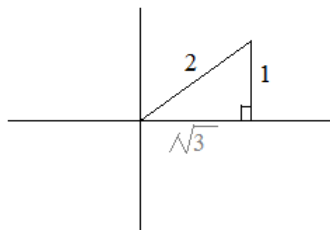
$\tan x = \frac{\text{opposite}}{\text{adjacent}}$



Inverse Trig Functions:

Suppose I ask, "what angle has a sine of $\frac{1}{2}$?"

$\sin X = \frac{1}{2}$



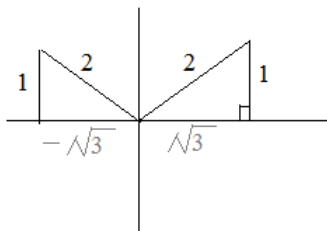
Pythagorean Theorem confirms that the adjacent side is $\sqrt{3}$.

We know the triangle is 30-60-90

therefore, $X = 30^\circ$ or $\frac{\pi}{6}$

But, WAIT!!

$\sin X = \frac{1}{2}$



$X = 30^\circ$ AND 150°

or $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

There are multiple values!!

Uniquely defined values:

Now, suppose I ask, find $\tan(\sin^{-1} \frac{1}{2})$

$(\sin^{-1} \frac{1}{2}) = 30^\circ$ or 150°

$\tan(30) = \frac{1}{\sqrt{3}}$

$\tan(150) = -\frac{1}{\sqrt{3}}$

There are 2 solutions!!

** What if we had wanted to specify a unique solution?

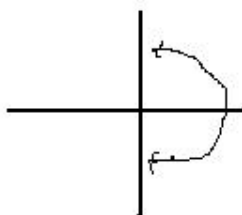
We must use *principal values*..

"Principal Values" defined:

$$y = \arctan x \quad \text{domain: all real numbers}$$

$$x = \tan y \quad \text{range: } -90^\circ \leq y \leq 90^\circ$$

$$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$



principal values of tangent and sine fall in quadrants I and IV

note: every trig value -- positive and negative -- is covered ONE time only

$$y = \arcsin x \quad \text{domain: } -1 \leq x \leq 1$$

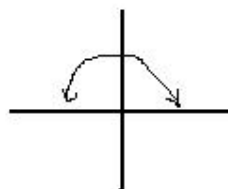
$$x = \sin y \quad \text{range: } -90^\circ \leq y \leq 90^\circ$$

$$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$$y = \arccos x \quad \text{domain: } -1 \leq x \leq 1$$

$$x = \cos y \quad \text{range: } 0 \leq y \leq 180^\circ$$

$$0 \leq y \leq \pi$$



principal values of cosine fall in quadrants I and II

To specify principal values:

- 1) use a capital letter *example: ArcSin instead of arcsin*
 Sin^{-1} instead of sin^{-1}

- 2) assume principal values when seeking a unique solution.

$$\tan(\text{sin}^{-1} 1/2) \longrightarrow \text{to find unique solution, } \tan(\text{Sin}^{-1} 1/2) = \tan(30) = \frac{\sqrt{3}}{3}$$

(consider principal values in quads I & IV only)

Study Break:
Math Snacks

LanceAF #35 6-3-12
www.mathplane.com



Preferable to ordinary computer cookies...

Essential part of a well-rounded, academic diet.

Try with (t), or any beverage...

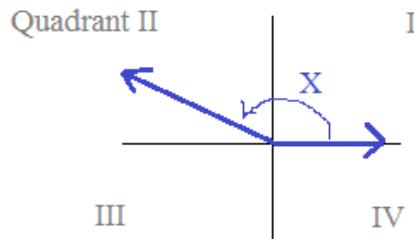
*Also, look for Honey Graham Squares
in the geometry section of your local store...*

Finding Trig values from given information

Example: $\sin X = \frac{5}{13}$ and, angle X lies in *Quadrant II*

Find the other 5 trig values of X.

Step 1: Draw a sketch

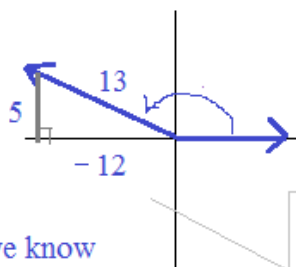


Step 2: "determine the triangle"

Since $\sin X = \frac{5}{13}$

5 is the opposite side
13 is the hypotenuse

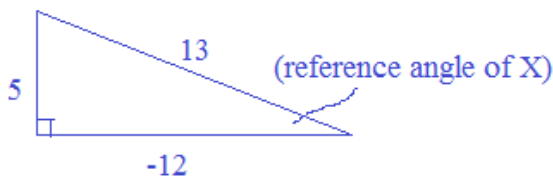
Using Pythagorean theorem, we know
the adjacent side is 12



$$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$$

****Important:** note the adjacent side is *negative* 12, because it lies on the left side of the "y-axis" (between 90° and 270°)

Step 3: Find the trig values



$$\sin X = \frac{5}{13}$$

$$\csc X = \frac{13}{5}$$

$$\cos X = \frac{-12}{13}$$

$$\sec X = \frac{13}{-12}$$

$$\tan X = \frac{5}{-12}$$

$$\cot X = \frac{-12}{5}$$

One more thing: What is the measure of angle X?

$$\sin X = \frac{5}{13} = .3846 \quad \left\{ \begin{array}{l} \arcsin(.3846) = 22.62^\circ \\ \sin^{-1}(.3846) = 22.62^\circ \end{array} \right.$$

(using inverse trig values
on a calculator)

since the reference angle lies in Quadrant II,
angle X = $180 - 22.62 = 157.38^\circ$

Example: $\tan \Theta = \frac{2}{3}$

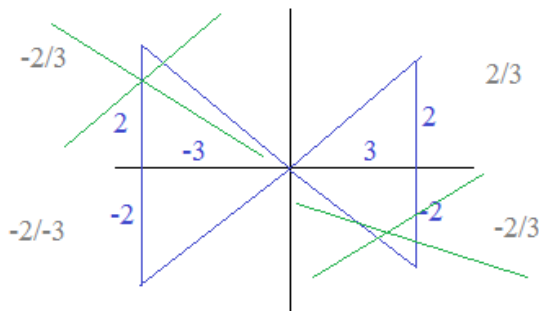
Finding Trig values from given information

$\sec \Theta < 0$

Find the 5 other trig values of Θ .

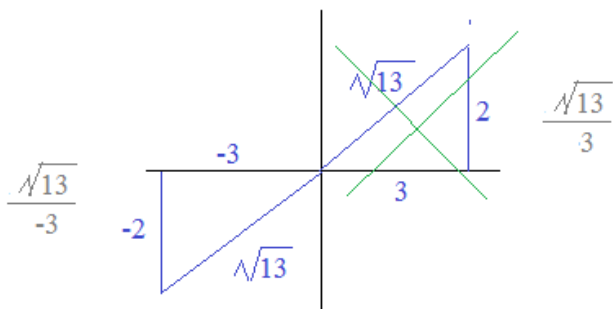
then, determine the measure of angle Θ

Step 1: Draw a sketch



Since $\tan \Theta = \frac{2}{3}$

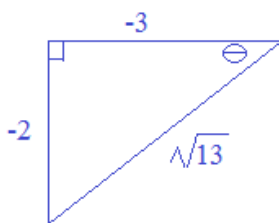
we can eliminate quadrants II and IV



Since $\sec \Theta < 0$,

we can eliminate quadrant I

Step 2: Draw the triangle and determine the trig values



$\tan \Theta = \frac{2}{3}$

$\cot \Theta = \frac{3}{2}$

$\sin \Theta = \frac{-2}{\sqrt{13}}$

$\csc \Theta = \frac{\sqrt{13}}{-2}$

$\cos \Theta = \frac{-3}{\sqrt{13}}$

$\sec \Theta = \frac{\sqrt{13}}{-3}$

Step 3: Find the angle

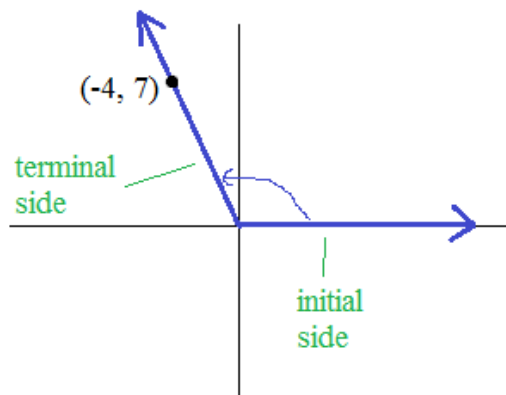
since $\tan \Theta = \frac{2}{3}$

(using a calculator) we can find the $\arctan(.667)$ or $\tan^{-1}(.667)$: 33.7 degrees..

then, recognizing that the angle is in quadrant III, angle $\Theta = 180 + 33.7 = 213.7^\circ$

The *terminal side* of an angle in *standard position* passes through the point $(-4, 7)$.
 What are the 6 trigonometric values of the angle?

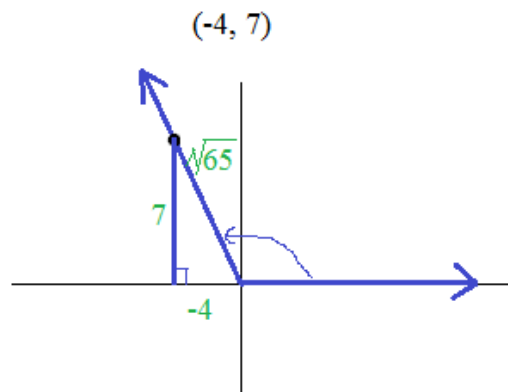
Step 1: Draw a sketch



Step 2: "Determine the triangle"

Draw a vertical line segment from the point to the x-axis.

Then, label the sides...
 (use pythagorean theorem to find measure of hypotenuse)



Step 3: Find 6 trig values

$$\sin \theta = \frac{7}{\sqrt{65}}$$

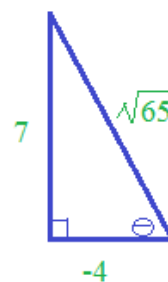
$$\csc \theta = \frac{\sqrt{65}}{7}$$

$$\cos \theta = \frac{-4}{\sqrt{65}}$$

$$\sec \theta = \frac{\sqrt{65}}{-4}$$

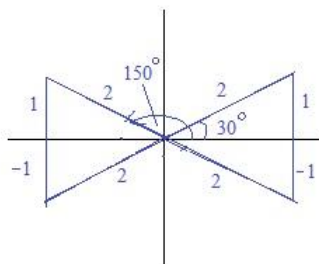
$$\tan \theta = \frac{7}{-4}$$

$$\cot \theta = \frac{-4}{7}$$



"Notes on Finding Inverse Trig Values"

1) "Triangle Method"



Therefore, for $0 \leq X \leq 2\pi$

$$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}, \frac{5\pi}{6}$$

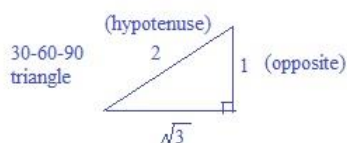
Find exact value of $X = \sin^{-1}\left(\frac{1}{2}\right)$
for $0 \leq X \leq 2\pi$

$$\sin X = \sin(\sin^{-1} 1/2)$$

$$\sin X = \frac{1}{2}$$

"Sine of what angle equals 1/2?"

$$\text{Sine} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2}$$



$\text{Arcsin } X \leftrightarrow \sin^{-1} X$
(Equivalent expressions)

"Principal" or "Restricted" Inverse Trigonometry Functions have the following ranges:

$$\arcsin \theta \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$\arccos \theta \quad 0 \leq \theta \leq \pi$$

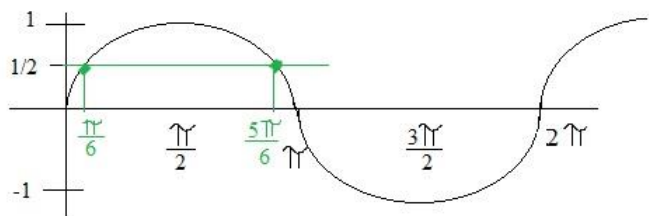
$$\arctan \theta \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$\csc^{-1} x \quad \left[-\frac{\pi}{2}, 0\right) \text{ or } \left(0, \frac{\pi}{2}\right]$$

$$\sec^{-1} x \quad \left[0, \frac{\pi}{2}\right) \text{ or } \left(\frac{\pi}{2}, \pi\right]$$

$$\cot^{-1} x \quad \left[-\frac{\pi}{2}, 0\right) \text{ or } \left(0, \frac{\pi}{2}\right]$$

2) Sine Function Graph



The diagram illustrates the points:

$$\left(\frac{\pi}{6}, \frac{1}{2}\right) \quad \left(\frac{5\pi}{6}, \frac{1}{2}\right)$$

3) Calculator

(Depending on the calculator),

Enter (1/2) or .5

Press "2nd" "Sine" (This should produce \sin^{-1} or arcsin)

The output will be 30 (degrees).. (**This calculator provided

only the principal value!)

Finally, adjust your answer:

--- convert to radians if necessary

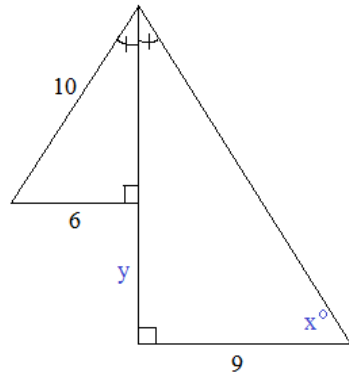
--- add values (**Since the above question wants values between

0 and 2π , you must find any other solutions)

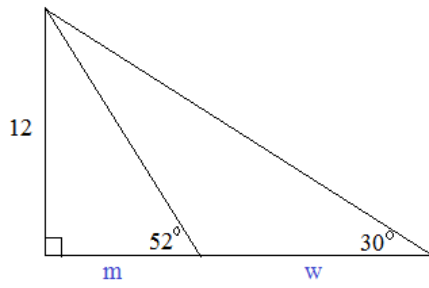
4) Ask a math teacher!

Use Trigonometry and Geometry concepts to solve.

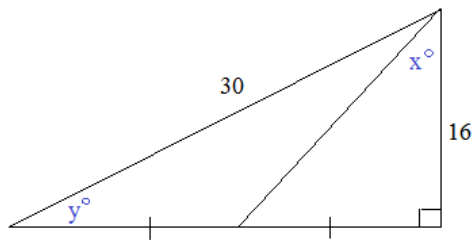
1) Find x and y :



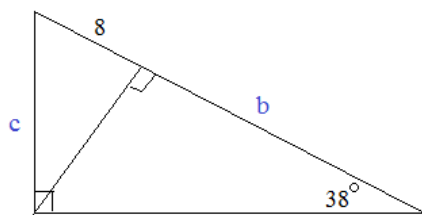
2) Find m and w :



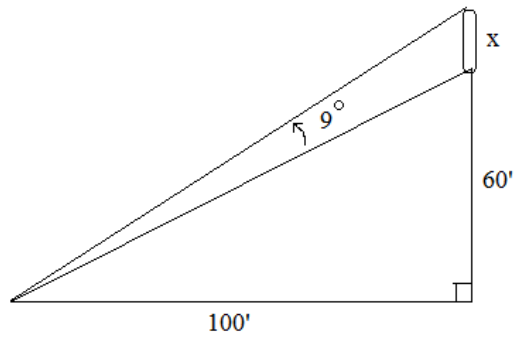
3) Find x and y :



4) Find b and c :

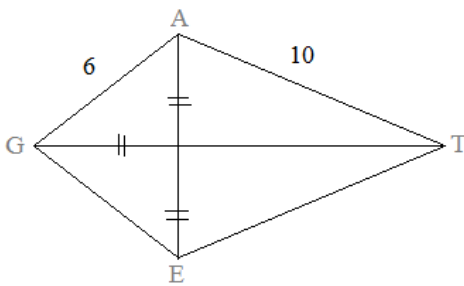


5) What is x ?



6) GATE is a kite.

What is the measure of angle $\angle ATG$?



7) $\sin(x) = .3$ What are the other 5 trig values?

$$\tan(x) =$$

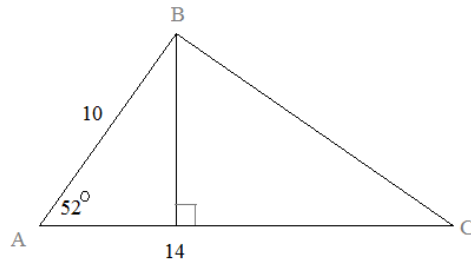
$$\cos(x) =$$

$$\csc(x) =$$

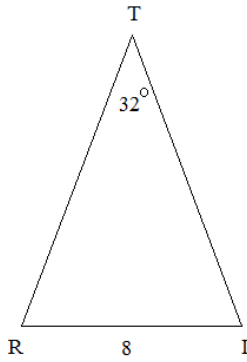
$$\sec(x) =$$

$$\cot(x) =$$

8) Find the area of triangle ABC

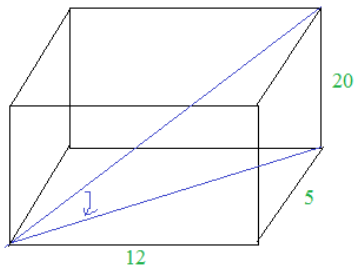


9) Find the perimeter of the isosceles triangle TRI

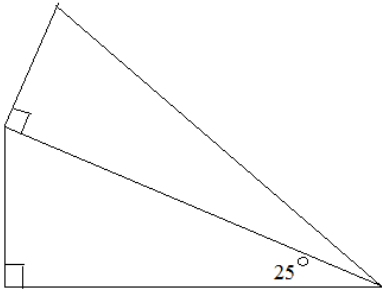


10) A 25 foot ladder extends from the ground to the window of a building.
If the window is 22 feet high, find the angle of depression from the window to the bottom of the ladder.

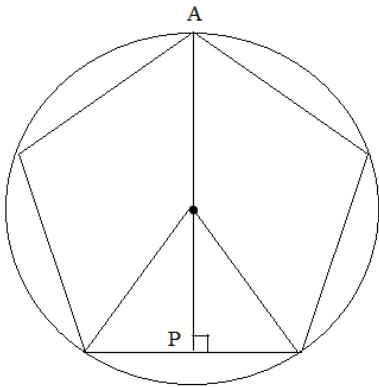
11) A rectangular box has dimensions 5, 12, and 20 inches.
Find the angle between the diagonal of the box and the diagonal of the 5 x 12 face.



- 12) If the triangles have the same perimeters,
what are the angle measures of the triangles?



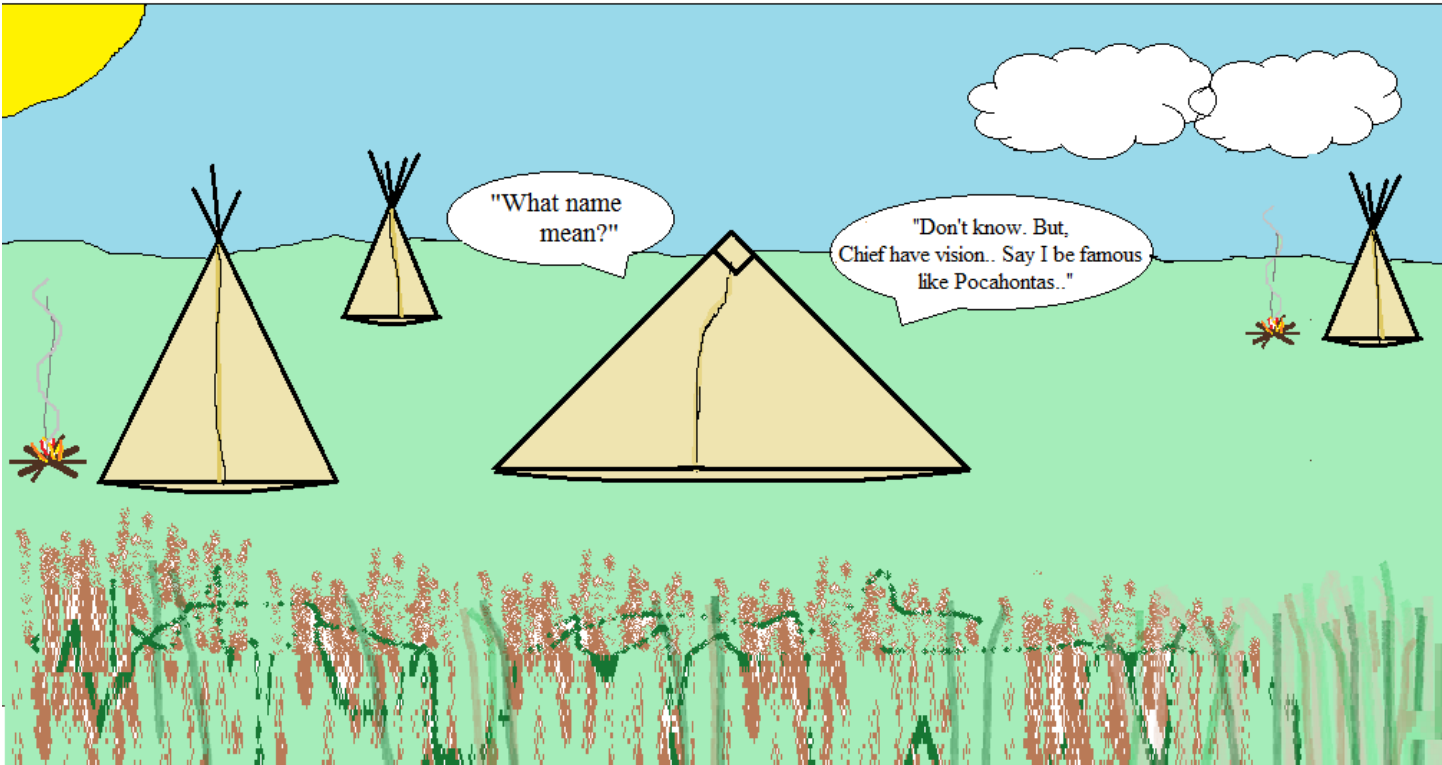
- 13) If the regular pentagon with perimeter 60 is inscribed in a circle,
what is the length of \overline{AP} ?



- 14) A 6-foot man looks at a mural that rises from the ground.
The angle of depression from his eyes to the ground is 14 degrees;
and, the angle of elevation from his eyes to the top of the mural is 27 degrees.

Approximately, how high is the mural?

(Hint: how from the mural is the man standing?)



Conversation with Princess Sochahtoa

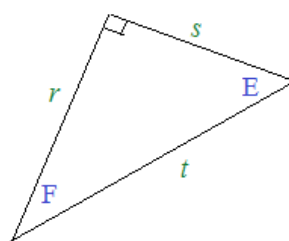
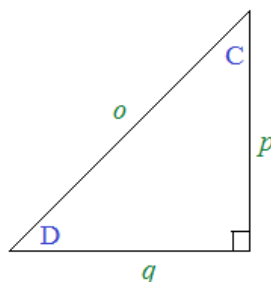
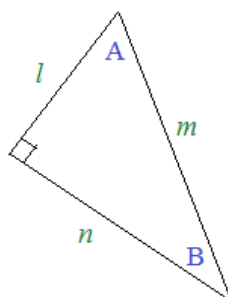
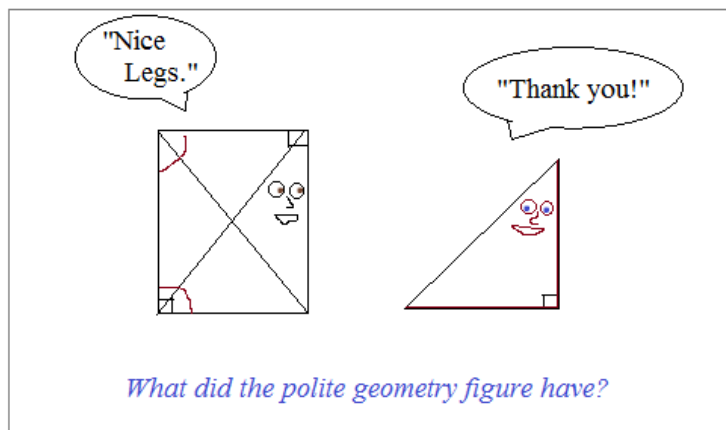
Answer Keys ->

Hidden Message

Solve the following trig equations to reveal the answer!

SOLUTIONS

- 1) $\sin B = \frac{l}{m}$
- 2) $\sin D = \frac{p}{o}$ $\frac{\text{Opposite}}{\text{Hypotenuse}}$
- 3) $\cos E = \frac{s}{t}$ $\frac{\text{Adjacent}}{\text{Hypotenuse}}$
- 4) $\tan F = \frac{s}{r}$ $\frac{\text{Opposite}}{\text{Adjacent}}$
- 5) $\sin C = \frac{q}{o}$ $\frac{\text{Opposite}}{\text{Hypotenuse}}$
- 6) $\sin F = \frac{s}{t}$ $\frac{\text{Opposite}}{\text{Hypotenuse}}$
- 7) $\cos C = \frac{p}{o}$ $\frac{\text{Adjacent}}{\text{Hypotenuse}}$
- 8) $\cos D = \frac{q}{o}$ $\frac{\text{Adjacent}}{\text{Hypotenuse}}$
- 9) $\cos B = \frac{n}{m}$ $\frac{\text{Adjacent}}{\text{Hypotenuse}}$
- 10) $\tan D = \frac{p}{q}$ $\frac{\text{Opposite}}{\text{Adjacent}}$
- 11) $\tan B = \frac{l}{n}$ $\frac{\text{Opposite}}{\text{Adjacent}}$
- 12) $\cos A = \frac{l}{m}$ $\frac{\text{Adjacent}}{\text{Hypotenuse}}$
- 13) $\tan E = \frac{r}{s}$ $\frac{\text{Opposite}}{\text{Adjacent}}$
- 14) $\sin A = \frac{n}{m}$ $\frac{\text{Opposite}}{\text{Hypotenuse}}$
- 15) $\cos F = \frac{r}{t}$ $\frac{\text{Adjacent}}{\text{Hypotenuse}}$
- 16) $\sin F = \frac{s}{t}$ $\frac{\text{Opposite}}{\text{Hypotenuse}}$



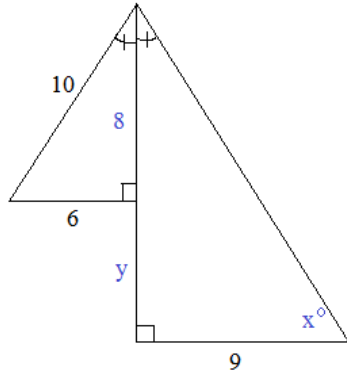
- 1) L
- 2) O
- 3) T
- 4) S
- 5) O
- 6) F
- 7) C
- 8) O
- 9) M
- 10) P
- 11) L
- E or I
- 12) M
- 13) E
- 14) N
- 15) T
- 16) S

*"Lots of Complements"
or "Lots of Compliments"*

Use Trigonometry and Geometry concepts to solve.

SOLUTIONS

1) Find x and y:



The small (left) triangle is a 6-8-10 Pythagorean Triple

Since acute angles are congruent, the right triangles are similar.
(angle-angle similarity)

$$\cos(x) = \frac{6}{10}$$

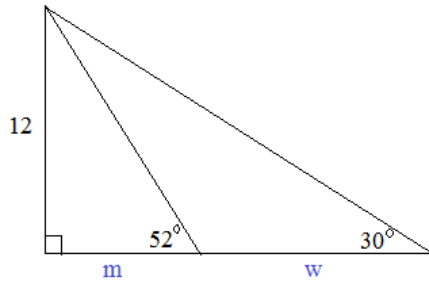
$$x = \cos^{-1}(.6) = 53.1^\circ$$

$$\frac{8+y}{9} = \frac{8}{6}$$

$$72 = 48 + 6y$$

$$y = 4$$

2) Find m and w:



$$\tan(52) = \frac{12}{m}$$

$$m = \frac{12}{1.28}$$

$$m = 9.38$$

30-60-90 triangle, so

$$(m+w) = 12\sqrt{3}$$

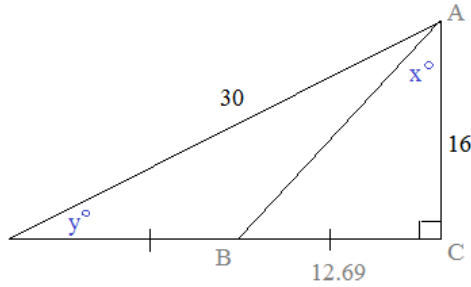
$$\text{or, } \tan(30) = \frac{12}{(m+w)}$$

$$m+w = \frac{12}{.577}$$

$$9.38 + w = 20.8$$

$$w = 11.42$$

3) Find x and y:



$$\sin(y) = \frac{16}{30}$$

$$y = \sin^{-1}(16/30)$$

$$y = 32.23^\circ$$

Using Pythagorean Theorem,

$$30^2 = 16^2 + (\text{leg})^2$$

$$\text{leg} = 25.38$$

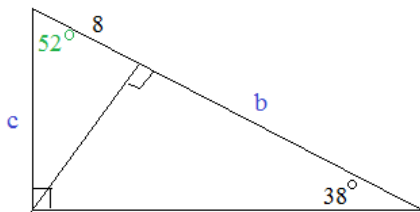
Since AB is a median,

$$BC = (1/2)(25.38) = 12.69$$

$$\tan(x) = \frac{12.69}{16}$$

$$x = 38.42^\circ$$

4) Find b and c:



$$\cos(52) = \frac{8}{c}$$

$$c = \frac{8}{\cos(52)}$$

$$c = 12.99$$

then,

$$\sin(38) = \frac{c}{(8+b)}$$

$$.616 = \frac{12.99}{(8+b)}$$

$$8+b = 21.1$$

$$b = 13.1$$

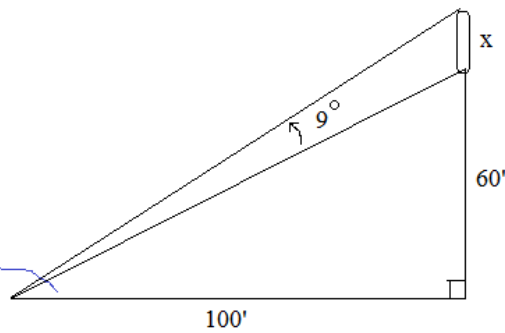
5) What is x?

SOLUTIONS

$$\tan(Y) = \frac{60}{100}$$

$$Y = \tan^{-1}(.6)$$

$$Y = 30.96^\circ$$



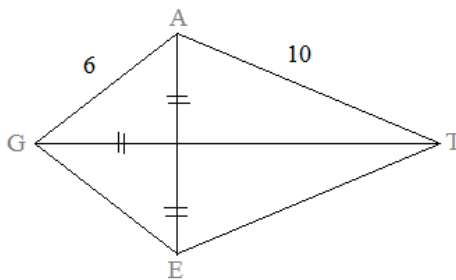
Therefore, the "big triangle" has angle 39.96 and sides $100'$ and $(60' + x)$

$$\tan(39.96) = \frac{(60' + x)}{100'}$$

$$.838 \times (100') = 60' + x$$

$$x = 23.8 \text{ feet}$$

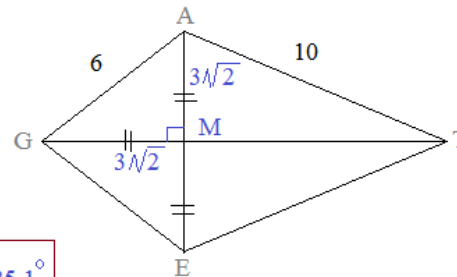
6) GATE is a kite.

What is the measure of angle $\angle ATG$?

$$\sin(\angle ATG) = \frac{3\sqrt{2}}{10} \longrightarrow \angle ATG = 25.1^\circ$$

Since GATE is a kite, GT is a perpendicular bisector of AE...

Also, $\overline{GM} \cong \overline{AM} \cong \overline{EM}$ Using Pythagorean Theorem (or 45-45-90 properties)
 \overline{AM} is $3\sqrt{2}$

7) $\sin(x) = .3$ What are the other 5 trig values?

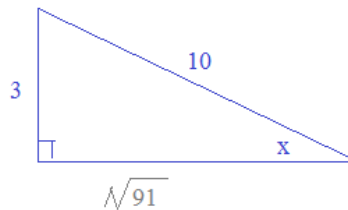
$$\tan(x) = \frac{3\sqrt{91}}{91}$$

$$\cos(x) = \frac{\sqrt{91}}{10}$$

$$\csc(x) = \frac{10}{3}$$

$$\sec(x) = \frac{10\sqrt{91}}{91}$$

$$\cot(x) = \frac{\sqrt{91}}{3}$$



The ratio of the 'opposite' over the 'hypotenuse' is .3

(so, any 2 numbers equal to 3/10 or .3 will work)

then, use pythagorean theorem:

$$3^2 + b^2 = 10^2$$

$$b = \sqrt{91}$$

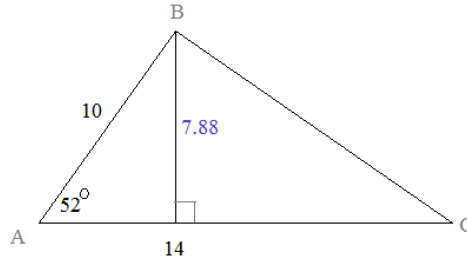
8) Find the area of triangle ABC

$$\sin(52) = \frac{\text{height}}{10}$$

$$\text{height} = 7.88$$

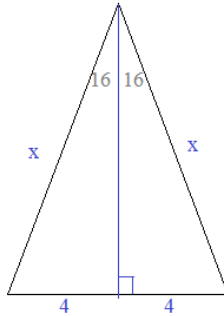
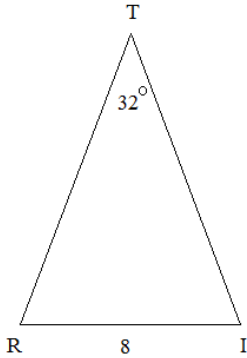
$$\text{area of ABC} = \frac{1}{2}(14)(7.88)$$

$$= 55.16$$



SOLUTIONS

9) Find the perimeter of the isosceles triangle TRI



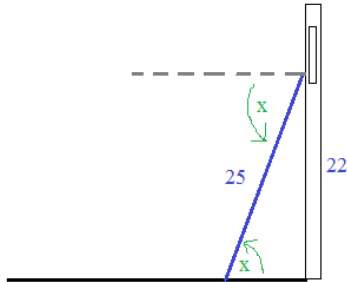
$$\sin(16) = \frac{4}{x}$$

$$x = 14.51$$

so, perimeter of TRI is

$$14.51 + 14.51 + 8 = 37.02$$

10) A 25 foot ladder extends from the ground to the window of a building.
If the window is 22 feet high, find the angle of depression from the window to the bottom of the ladder.



$$\sin(x) = \frac{22}{25}$$

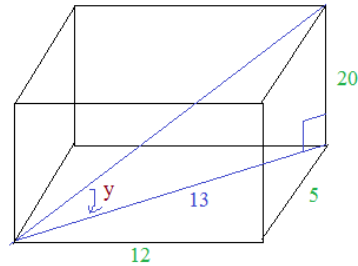
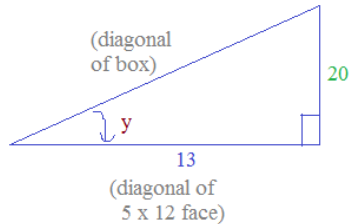
$$x = 61.65 \text{ degrees}$$

11) A rectangular box has dimensions 5, 12, and 20 inches.
Find the angle between the diagonal of the box and the diagonal of the 5 x 12 face.

$$\tan(y) = \frac{20}{13}$$

$$y = \tan^{-1}(1.538)$$

$$y = 57 \text{ degrees}$$



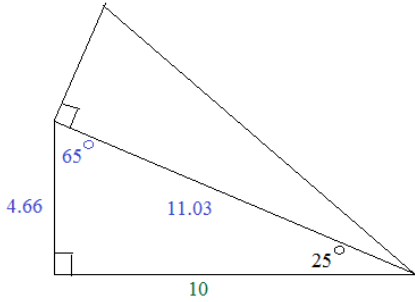
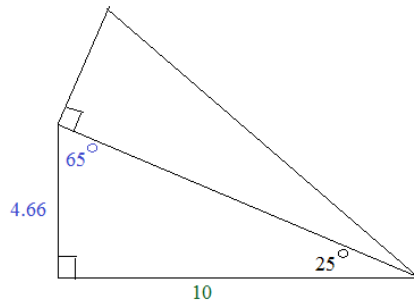
- 12) If the triangles have the same perimeters, what are the angle measures of the triangles?

SOLUTIONS

Let's assign a value and see what happens...
Assume the base of the lower triangle is 10 units...

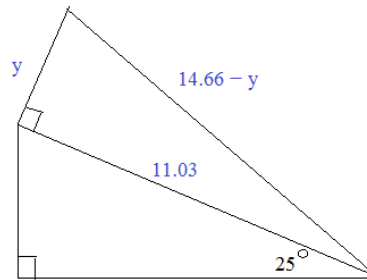
$$\tan(25) = \frac{x}{10}$$

$$x = 4.66$$



$$4.66^2 + 10^2 = 11.03^2$$

so, the perimeters are 25.69



$$11.03^2 + y^2 = (14.66 - y)^2$$

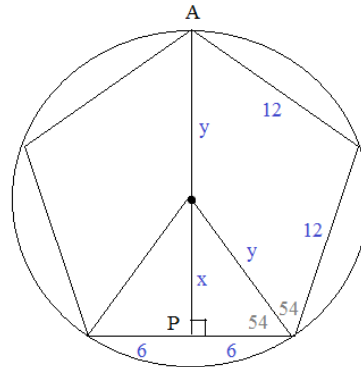
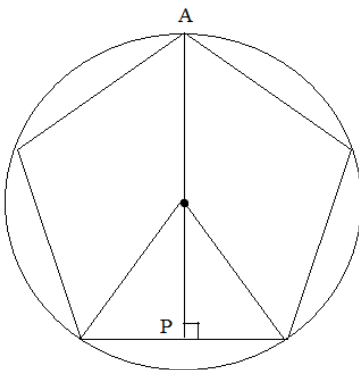
$$y = 3.18$$

$$\tan(z) = 3.18/11.03$$

$$z = 16.08^\circ$$

$$\text{and, } 73.92^\circ$$

- 13) If the regular pentagon with perimeter 60 is inscribed in a circle, what is the length of AP?



sum of interior angles of a pentagon are 540 degrees...
so, each interior angle of regular pentagon is 108 degrees

$$\tan(54) = \frac{x}{6} \quad x = 8.26$$

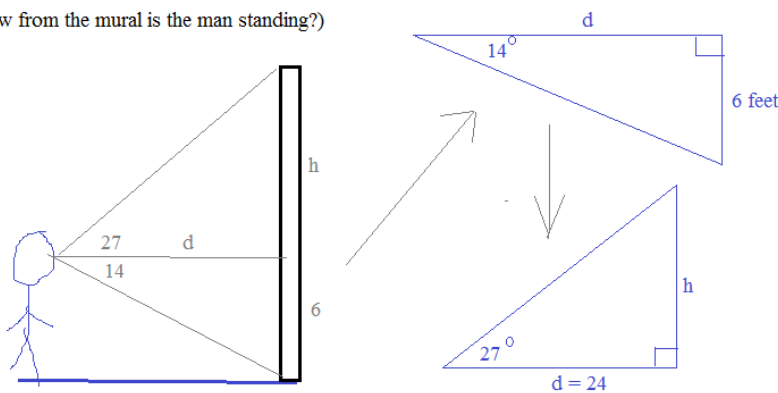
$$\cos(54) = \frac{6}{y} \quad y = 10.21$$

$$AP = x + y = 18.47$$

- 14) A 6-foot man looks at a mural that rises from the ground. The angle of depression from his eyes to the ground is 14 degrees; and, the angle of elevation from his eyes to the top of the mural is 27 degrees.

Approximately, how high is the mural?

(Hint: how from the mural is the man standing?)



to get d,

$$\tan(14) = \frac{6}{d} \quad \text{distance to mural: } 24 \text{ feet}$$

then, to get h,

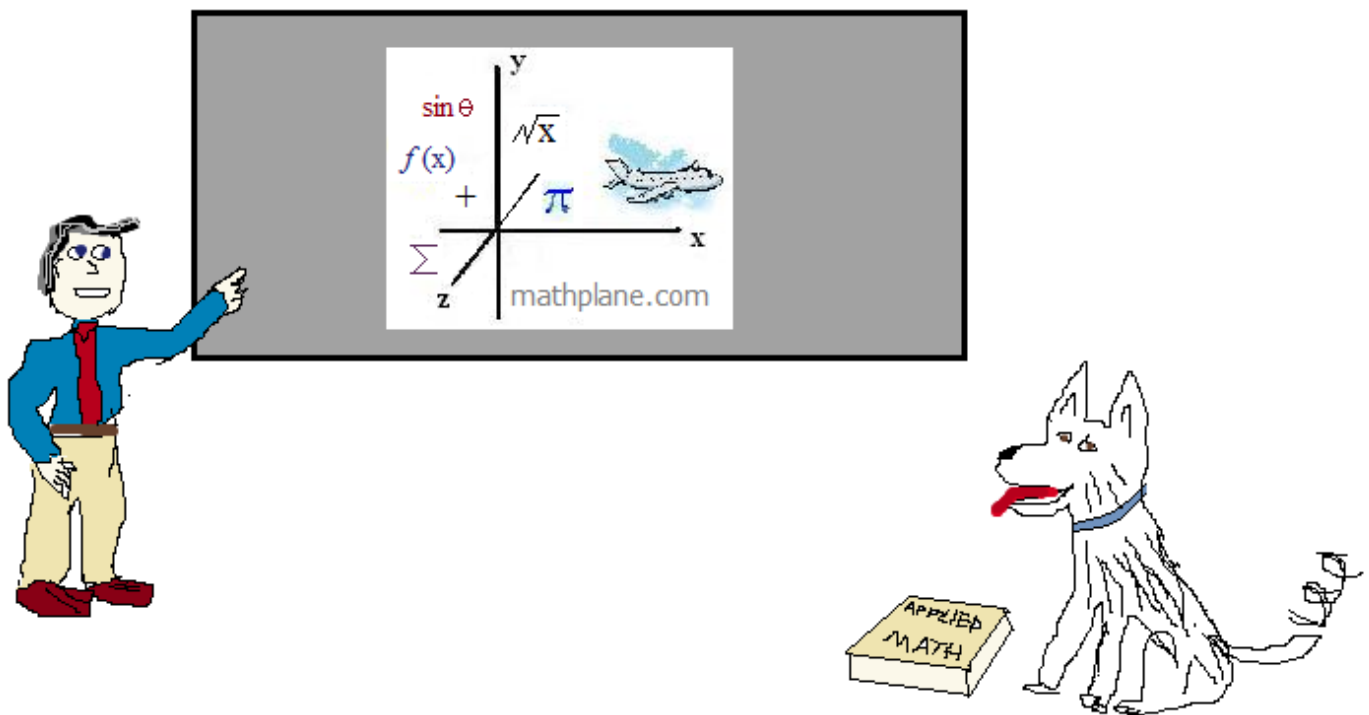
$$\tan(27) = \frac{h}{24} \quad h = 12.24$$

the mural is approximately 36.24 feet

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