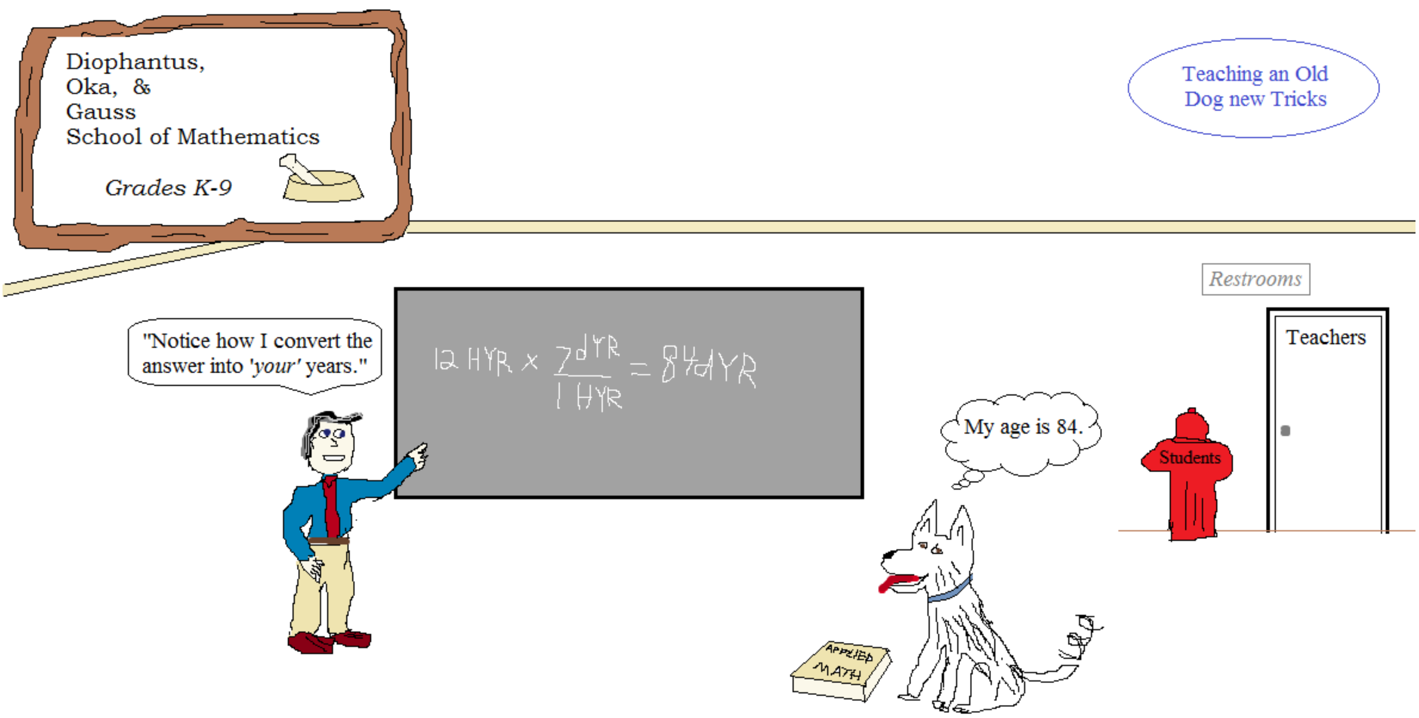
THIRTY
USEFUL
MATH
TOPICS TO
REVIEW

PREVIEW

Comics & Questions from
Lance Friedman

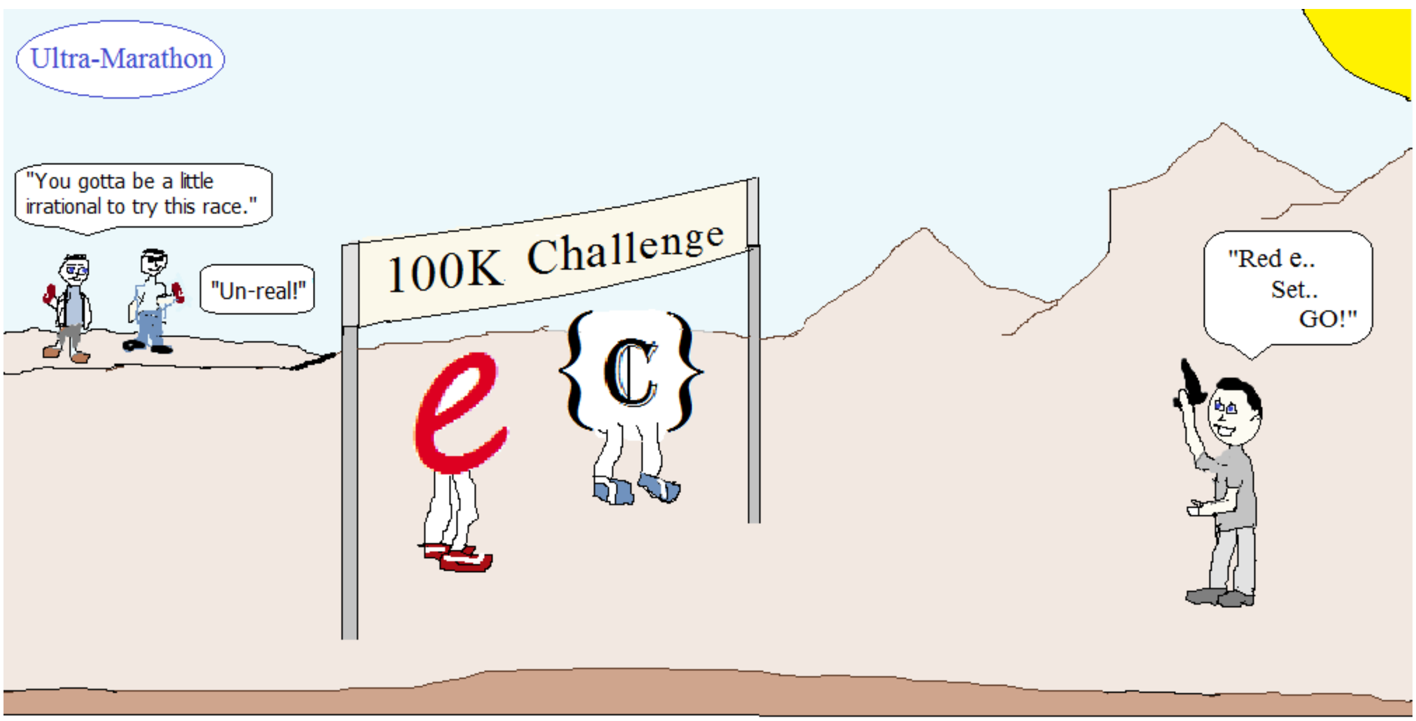


It's never too late to learn mathematics!

Thirty Useful Math Topics to Review is a collection of comics and over 150 math questions. The subjects are selected from basic arithmetic, algebra, geometry, and trigonometry.

The various sections briefly review topics and/or introduce new concepts (while adding a bit of amusing humor!). Although formal lessons are not included, the solutions can serve as a helpful learning resource.

For extra notes and practice, visit us at mathplane.com



Testing the limits of endurance,
these math figures will run on and on...

LanceAF #87 5-24-13
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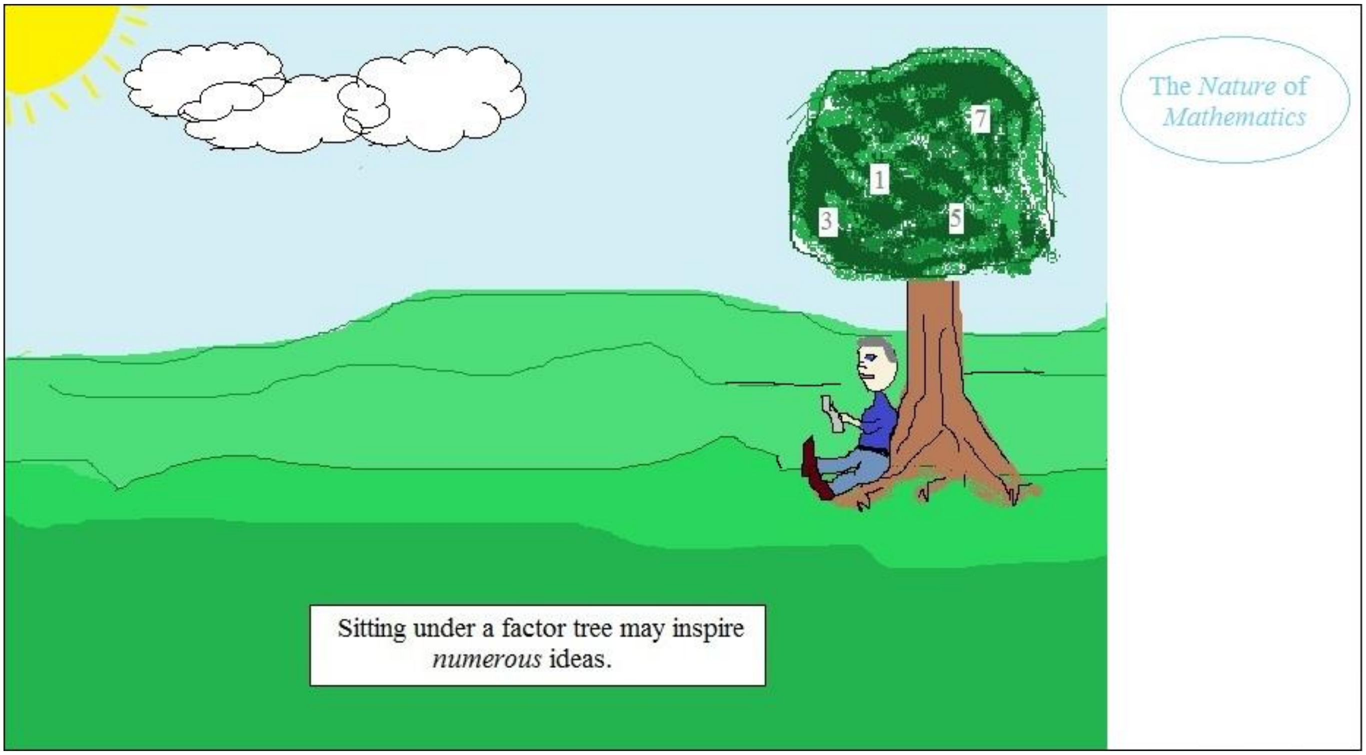
MATH TOPICS

- Factors/LCM/GCF
- Fractions
- Converting measurements
- Algebra Word Problems
- Mean/Median/Mode
- Prime numbers
- Roman Numerals
- Radicals
- Union/Intersection/Venn Diagram
- Absolute Value
- Exponents
- 2013
- 44
- Plane geometry (rays, angles, collinear)
- Area of Polygons
- Lines of Symmetry
- Bisectors
- Pythagorean Theorem
- (Clocks) central angles, sector area, arc length
- Sector area
- Special right triangles
- Surface area
- Binary/base2
- Probability
- Combinations/permutations
- Trig word problems
- Ellipse
- Sequences/Series
- Limits/Infinity
- Half-Life



Factors, Least Common Multiple (LCM), and Greatest Common Factor (GCF)

1) What is the least common multiple of the numbers in the tree? (1, 3, 5, & 7)

2) List all factors of 28

3) What is the LCM of 5 and 20?

4) What is the GCF of 5 and 20?

5) How many factors of 120 are there?

ANSWERS:

1) 105

Multiples of 1: 1, 2, 3, 4… 101, 102, 103, 104, 105…
Multiples of 3: 3, 6, 9, 12, 15… 96, 99, 102, 105…
Multiples of 5: 5, 10, 15, 20… 90, 95, 100, 105…
Multiples of 7: 7, 14, 21, 28… 91, 98, 105…

105 is the lowest number that has factors 1, 3, 5, and 7!

2) 1, 2, 4, 7, 14, 28

3) 20

Multiples of 5: 5, 10, 15, 20, 25…
Multiples of 20: 20, 40, 60…

20 is the first number that is a multiple of both…

4) 5

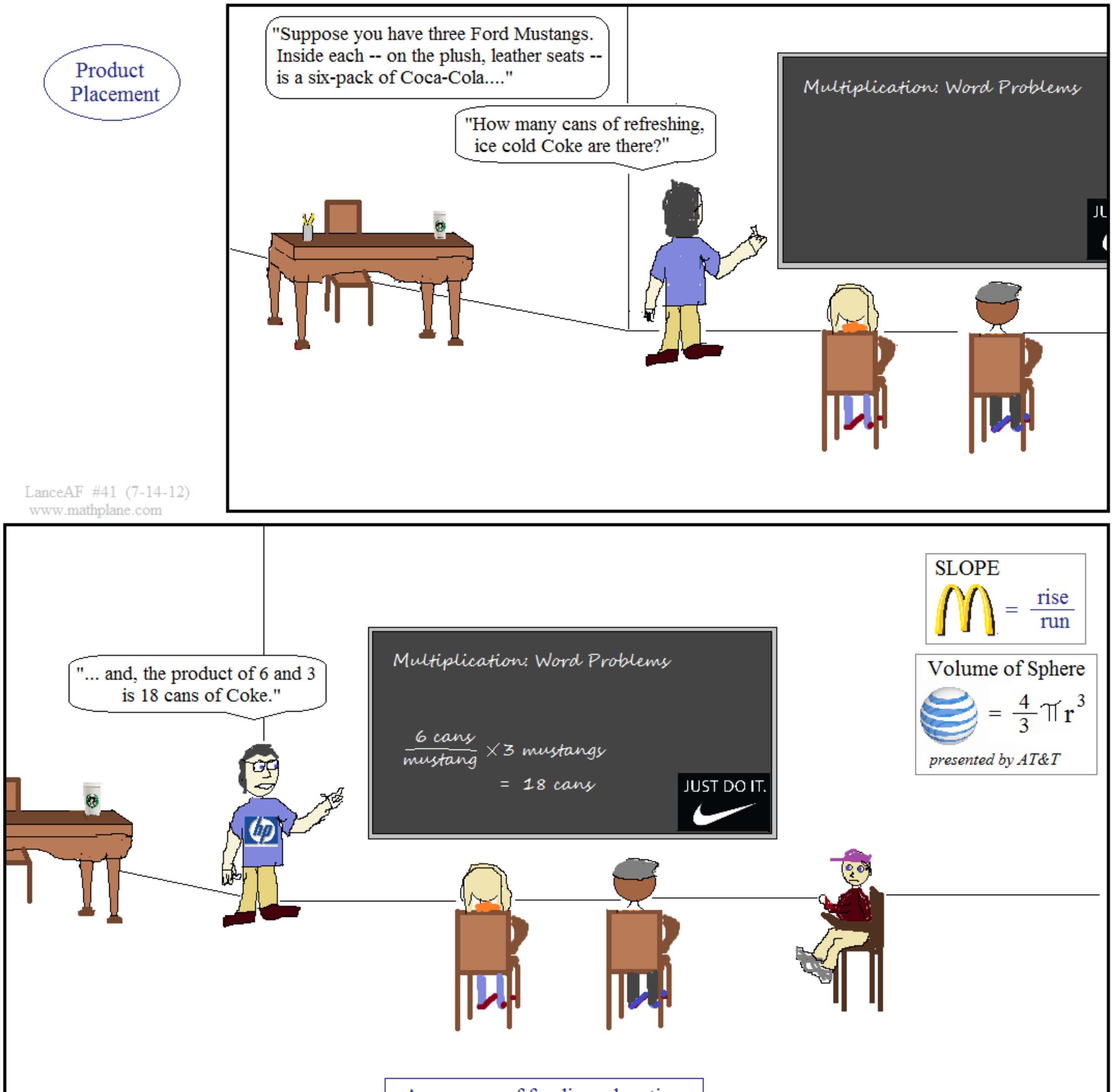
Factors of 5: 1 and 5
Factors of 20: 1, 2, 4, 5, 10, 20

1 and 5 are common factors. 5 is the greatest common factor.

5) 16

Factors of 120: 1 and 120
2 and 60
3 and 40
4 and 30
5 and 24
6 and 20
8 and 15
10 and 12

Product Placement



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Algebra Word Problems

QUESTIONS

1) A number is 8 more than another number.
If the sum of the two numbers is 70, what is the smaller number?

2) The cable company charges $59.95 per month plus $4 for each on demand movie rental. If your cable bill is $111.95, how many movies did you rent?

3) A bag containing only nickels and dimes holds 800 coins.
If the bag has $49.70, how many dimes are in the bag?

4) If a shirt cost $20 <u>*after*</u> a 10% discount, what was the original price?

5) Town A and town B are 200 miles apart.
Al leaves town A at noon and drives 30 miles/hour toward town B.
If Barney leaves town B at 2:00pm and drives 20 mph toward town A, what time will they pass each other?

SOLUTIONS

1) Let X = first number then, let X + 8 = the 2nd number

X + (X + 8) = 70
2X + 8 = 70
2X = 62
X = 31
X + 8 = 39

The smaller number is 31 (and, the larger number is 39)

2) Cable bill = monthly fixed cost + \$4M
Where M = # of movies rented

\$111.95 = \$59.95 + \$4M
\$52 = \$4M
M = 13 13 Movies

3) List variables: let d = # of dimes
n = # of nickels
So, .10d = value of the dimes
.05n = value of the nickels

Set up equations: d + n = 800
.10d + .05n = 49.70 (2 equations, 2 unknowns)

Solve: (using substitution method) d = 800 – n
.10(800 – n) + .05n = 49.7

80 - .10n + .05n = 49.7
-.05n = -30.3
n = 606
If n = 606, then d = 194

Check: 194 dimes + 606 nickels = 800 coins
$19.40 + $30.30 = $49.70

4) $20 + .1($20) = $22 is NOT the correct answer, because
10% of $22 is $2.20… So, a $22 shirt would cost $19.80 after a 10% discount!

Let P = original price of the shirt

P – (10% of P) = $20
P - .10P = $20
.90P = $20
P = $22.22 the original price of the shirt is $22.22

Check: $22.22 would have a $2.22 discount.. So, the sale shirt would cost $20

5) Al (distance) + Barney (distance) = 200 miles
Distance = rate x time

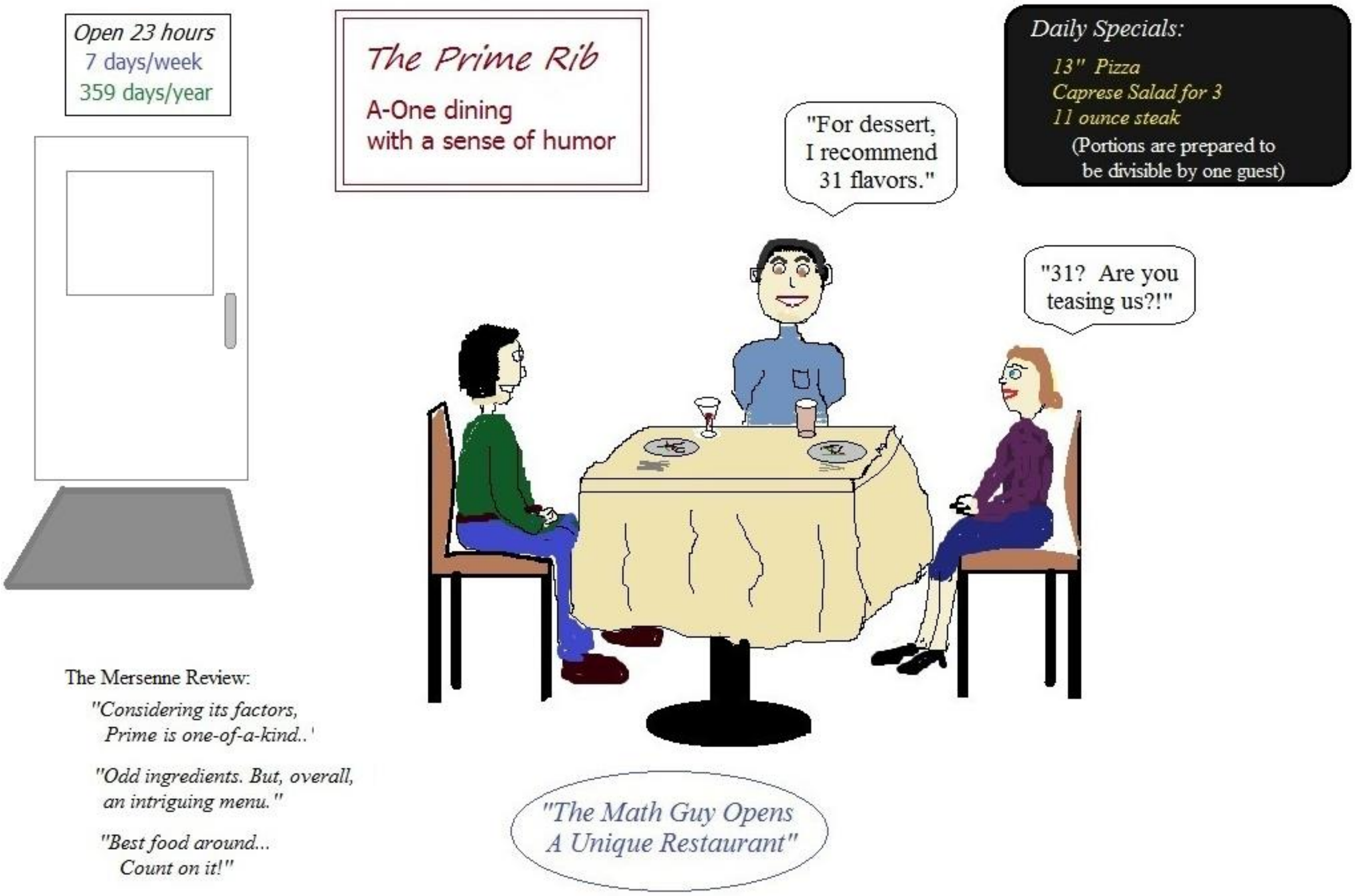
From noon to 2:00, Al traveled (30m/hour x 2 hours) = 60 miles
Barney traveled 0 miles (because he hadn't left yet)

After 2:00, they must travel 140 miles…
Since Al is going 30 miles/hour and Barney is going 20miles/hour, they are moving closer at a rate of 50miles/hour!

Distance = rate x time

140 miles = (50miles/hour) x Time
Time = 2.8 hours or 2 hours and 48 minutes.

Therefore, Al and Barney meet at 4:48 pm



PRIME NUMBERS

1) What is a “non-prime” number called?
2) How many prime numbers are between 10 and 20?
3) What is the only even prime number?
4) Is -7 a prime number?
5) What is a Marsenne Prime?
6) Is 0 or 1 a prime number?

Answers

Definition of a prime number – A natural number (i.e. positive integer) greater than 1 that has <u>no positive divisors except for 1 and itself.</u>

1) A non-prime number is a *composite number*. It has at least 3 factors.

2) Four: 11, 13, 17, 19

3) 2 is the only even prime number

4) No. Although 7 is a prime, -7 is not classified as prime.

5) A Marsenne number is a positive number that is 1 less than a power of 2.
Examples: 1, 3, 7, 15, 63, 127 (Each is 1 less than 2, 4, 8, 16, 32, 64, etc…)

So, a Marsenne prime would be a Marsenne number that is prime!
Examples: 3, 7, 31,

6) 1 is neither prime nor composite, because it only has 1 divisor: itself
0 is neither prime nor composite. It has an infinite number of factors, so it is not prime. But, since it cannot be represented by the product of 2 natural numbers, it is not a composite!

ABSOLUTE VALUE

Solve for X:

1) $|5 - 11| = X$

2) $3|X - 8| = 9$

3) $2|X + 17| - 13 = -7$

4) $|X + 15| + 10 = 5$

5) $|X + 6| + 4 < 12$

ANSWERS

1) $X = 6$

$|-6| = 6$

2) $X = 11$ OR 5

$|X - 8| = 3 \qquad X - 8 = 3 \qquad X = 11$

$X - 8 = -3 \qquad X = 5$

3) $X = -14$ OR -20

Isolate the absolute value: $2|X + 17| = 6$

$|X + 17| = 3$

Then, “branch off”: $X + 17 = 3 \qquad X = -14$

$X + 17 = -3 \qquad X = -20$

4) NO SOLUTION!!

$|X + 15| = -5$

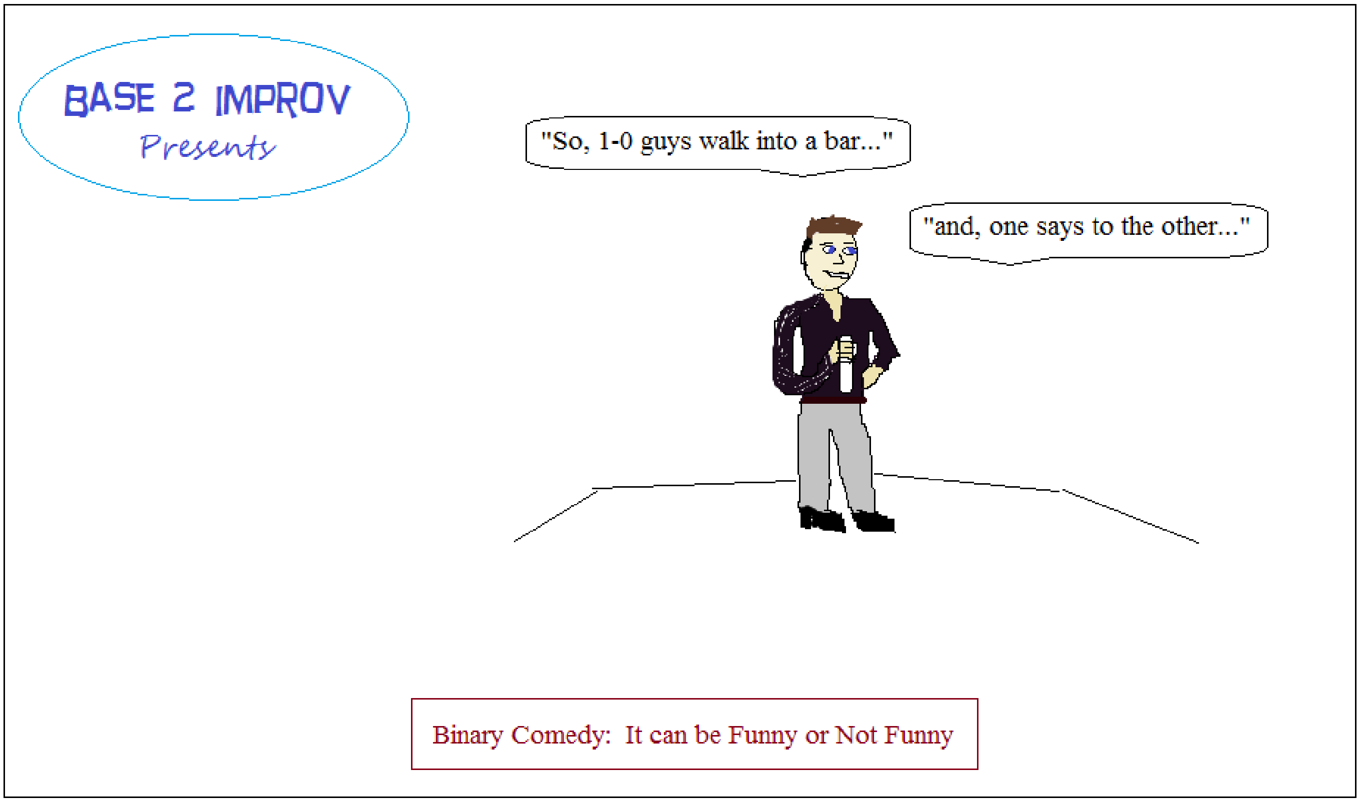
Since absolute value cannot be negative, there is no solution.

5) $-14 < X < 2$

Find ‘critical values’ or ‘boundaries: $|X + 6| = 8$ So, $X = 2$ or -14

Then, test points to determine the inequality. For $|X + 6| + 4 < 12$,

-20 and 10 are not solutions. But, 0 is a solution.



Binary Numbers

1) In base 2, how many numbers are between 10 and 100?
2) What is 101010_{base2} ?
3) Express the number 22 using binary number (i.e. base 2).
4) $111_{base2} + 111_{base2} =$

Answers

1) In base 2, there is only one number between 10 and 100: 11

2) $0 \times 2^0 = 0$

$1 \times 2^1 = 2$

$0 \times 2^2 = 0$ $\qquad$ $32 + 8 + 2 = 42$ (base 10)

$1 \times 2^3 = 8$

$0 \times 2^4 = 0$

$1 \times 2^5 = 32$

3) Looking at powers of 2:   1, 2, 4, 8, 16, 32…,

16 is the largest number that is less than or equal to 22..

$1 \times 2^4 = 16$ $\qquad$ leaving a remainder of 6

4 is the largest number that is less than or equal to 6

$1 \times 2^2 = 4$ $\qquad$ leaving a remainder of 2

2 is the largest number that is less than or equal to 2

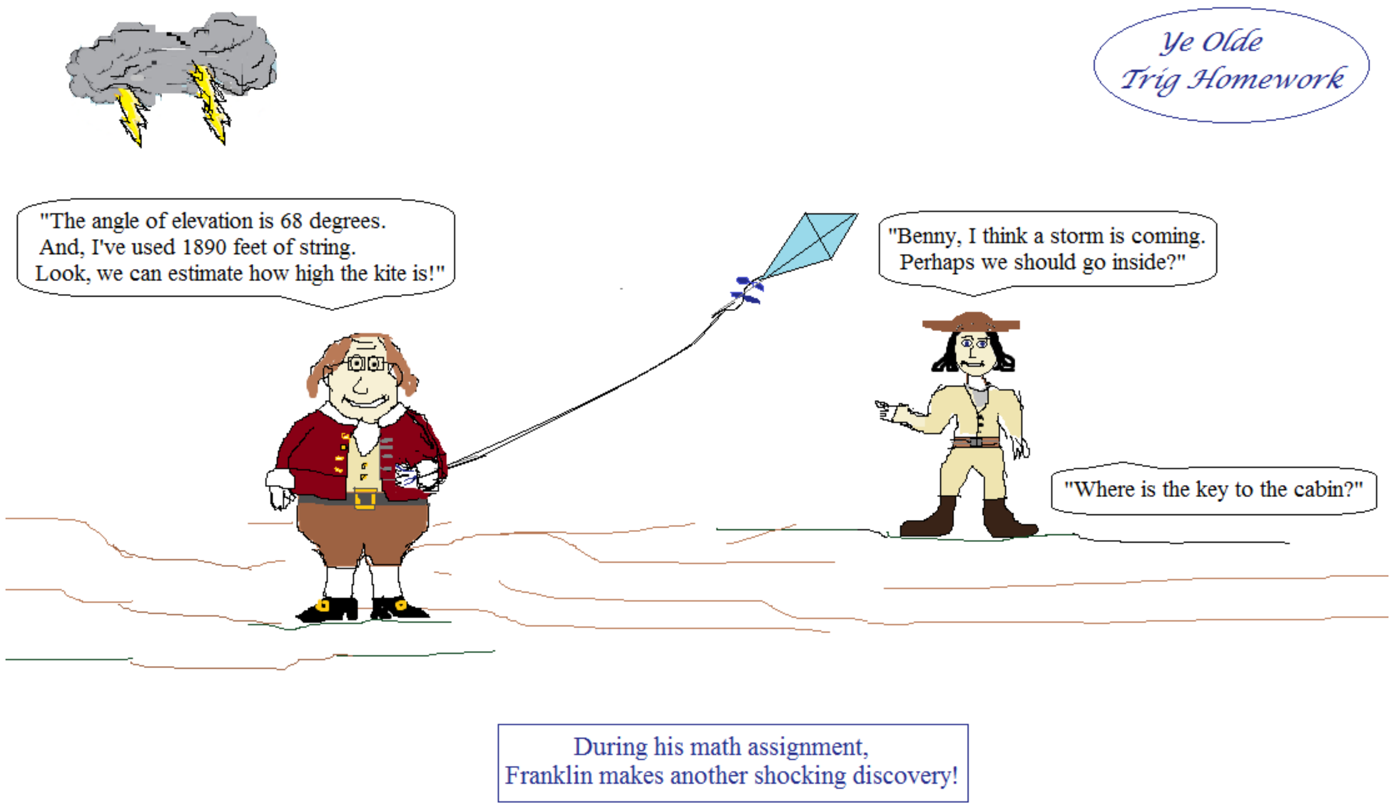
$1 \times 2^1 = 2$ $\qquad$ leaving no remainder.

Recognizing the place values, $10110_{base2} = 22$

4) $111_{base2} = 1 + 2 + 4 = 7$

$7 + 7 = 14$ $\qquad$ and, 14 in binary is 1110_{base2}

$$
\begin{array}{r}
{\scriptstyle 1\,1\,1} \\
111 \\
+\ 111 \\
\hline
1110
\end{array}
$$



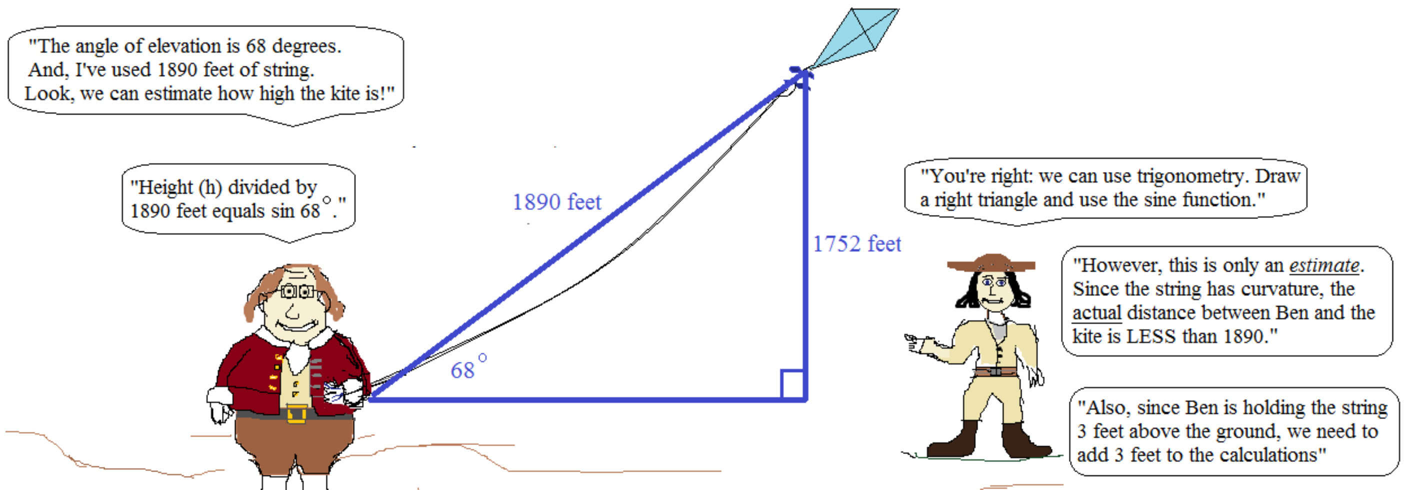
Trigonometry Word Problems

1) (In the comic above), estimate the height of the kite. (HINT: the year of Franklin's famous kite experiment?)

2) Suppose a 40-foot tree casts a shadow of length 60 feet. What is the angle of elevation (with respect to the ground) from the end of the shadow to the top of the tree?

3) A windmill blade is 16 feet long.
If it goes around at a rate of 12 rotations/minute,

> a) What is its angular speed (in degrees per minute)?
> b) What is its linear speed (in feet per minute)?

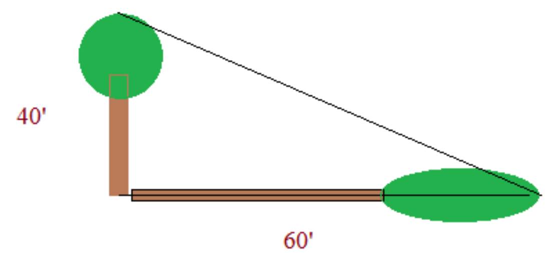


$$\text{Sine } (68^\circ) = \frac{h}{1890 \text{ feet}}$$

$$.927184 \text{ (1890 feet)} \cong 1752.4 \text{ feet}$$

**Franklin's famous kite experiment occurred in June 1752

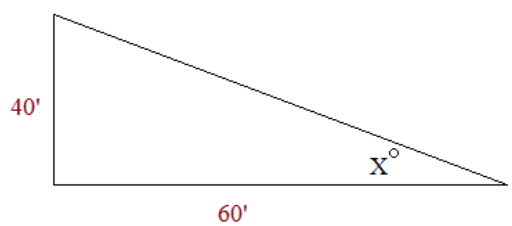
2) Draw a sketch:



$$\text{Tan X} = \frac{40'}{60'}$$

$$\text{ArcTan } .667 = \boxed{33.7^\circ}$$

"Isolate Triangle" and Solve:



3) a) angular speed is the amount of distance covered *measured in degrees.*

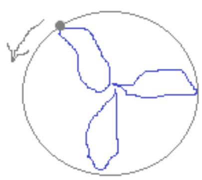
Every rotation will be 360 degrees covered. (or 2π radians)

The windmill goes at 12 rotations/minute ---- $12 \dfrac{\text{rotations}}{\text{minute}} \cdot 360 \dfrac{\text{degrees}}{\text{rotation}} = \boxed{4320 \text{ degrees/minute}}$

or 24π radians/minute

$\cong 75.4$ radians/minute

b) linear speed is the distance covered from a spot *on the tip of the blade.*



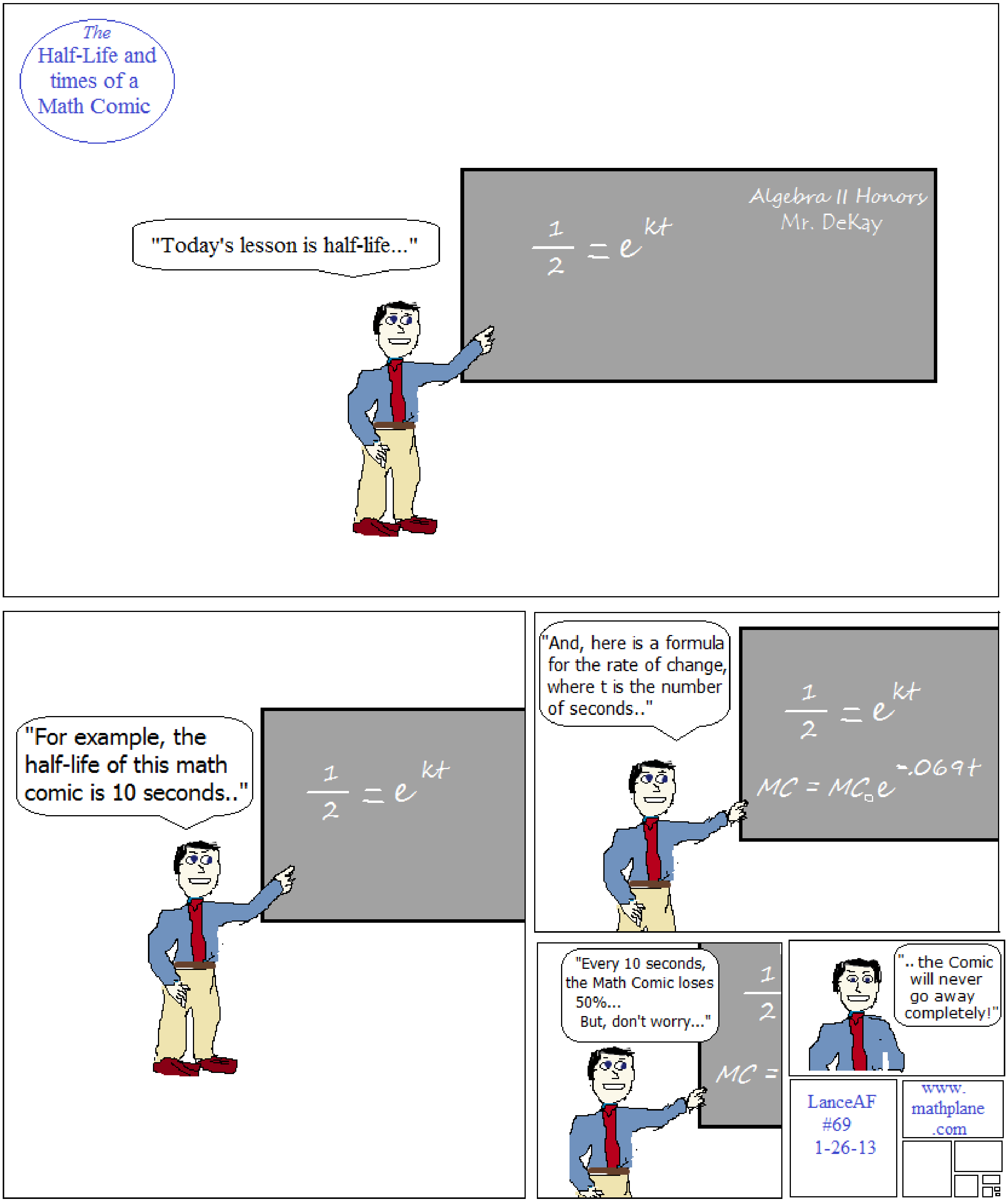
The spot will travel the circumference of the circle during one rotation...

circumference = $2\pi r$ = $2\pi 16$ feet

= 32π feet per rotation

Therefore, the linear speed is 384π feet/minute

$\boxed{\text{approx. 1206 feet/minute}}$


The Half-Life and times of a Math Comic
"Today's lesson is half-life..."
$\frac{1}{2} = e^{kt}$
Algebra II Honors
Mr. DeKay
"For example, the half-life of this math comic is 10 seconds.."
"And, here is a formula for the rate of change, where t is the number of seconds.."
$MC = MC_0 e^{-.069t}$
"Every 10 seconds, the Math Comic loses 50%... But, don't worry..."
".. the Comic will never go away completely!"
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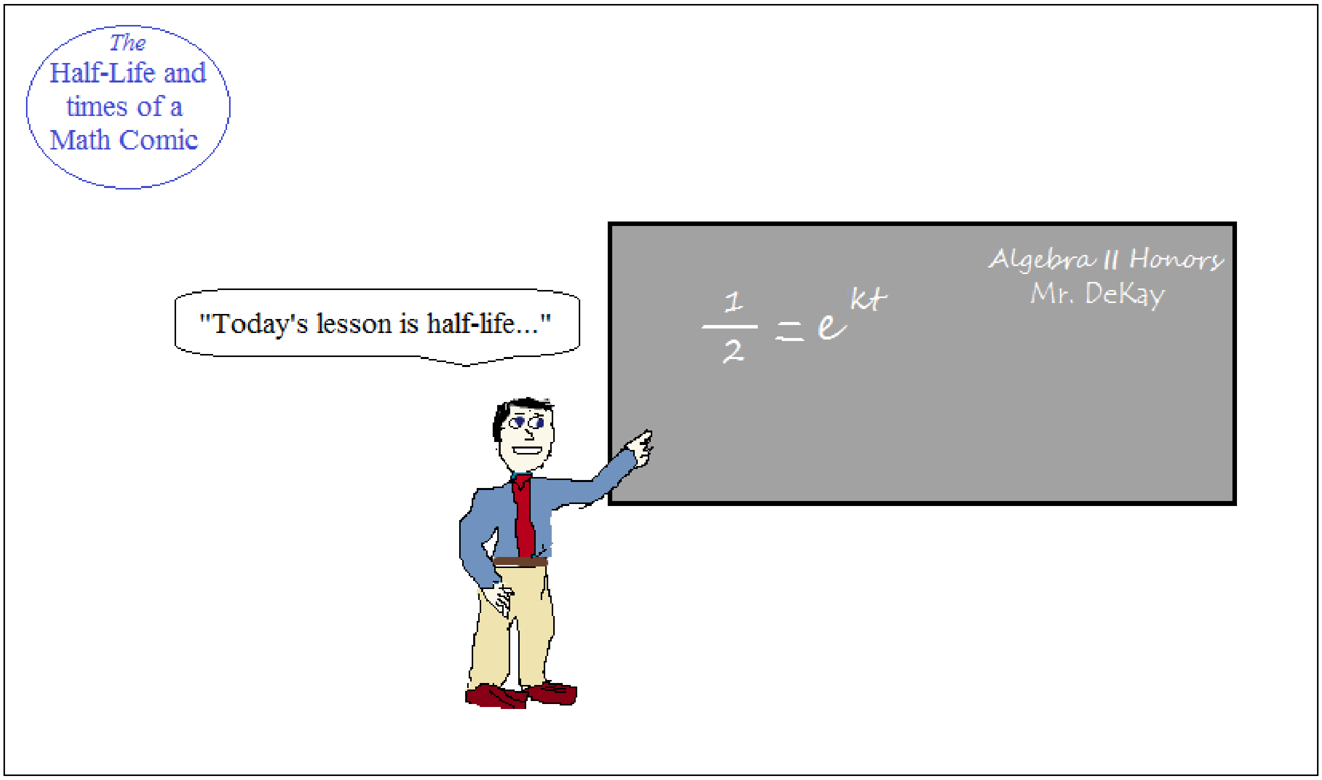


Decay and Half-Life Questions:

Assume the above comic has an area of 32 square inches.

If the half-life is 10 seconds,

a) What is the area after 10 seconds?

b) What is the area after 30 seconds?

c) What is the area after 16 seconds?

d) When will the area decay to 20 square inches?

e) When will the comic's area go to zero?

ANSWERS

time (seconds)	area (sq. inches
0	32
?	20
10	16
16	?
20	8
30	4
40	2
50	1
60	1/2

a) what is the area after 10 seconds?

After 10 seconds, the areas size is 1/2: $\boxed{\text{16 square inches}}$

b) what is the area after 30 seconds?

(see table) after 30 seconds, the area has been cut in half 3 times..

$\boxed{\text{4 square inches}}$

c) what is the area after 16 seconds?

First, we need to find the 'rate of decay'..

The rate of decay is continuous,
so we can use this formula: $A = Pe^{rt}$

$$16 = 32e^{r(10)}$$

$$\frac{1}{2} = e^{10r}$$

To solve, use natural logs:

$\ln\frac{1}{2} = \ln e^{10r}$ (power rule)

$\ln\frac{1}{2} = 10r \cdot \ln e$ ($\ln e = 1$)

$-.693 \cong 10r$

$r \cong -.069$

Equation is

$A = 32e^{-.069t}$

so, after 16 seconds:

$A = 32e^{-.069(16)}$ $\boxed{\cong \text{ 10.6 square feet}}$

d) when will the area decay to 20 square inches?

$A = Pe^{rt}$

$A = 32e^{-.069t}$

substitute values: $20 = 32e^{-.069t}$

$.625 = e^{-.069t}$

$\ln .625 = \ln e^{-.069t}$

$-.47 = -.069t$

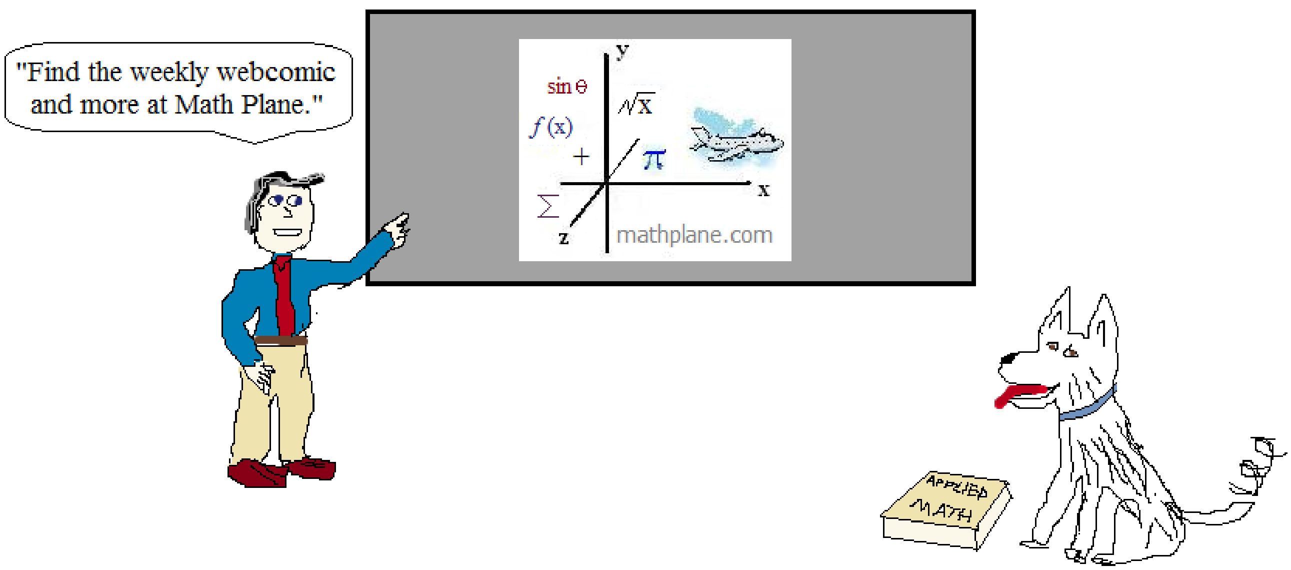
$\boxed{t \cong 6.8 \text{ seconds}}$

e) when will the comic's area go to zero?

Never.. Although the area approaches zero and gets very small, it doesn't reach zero...

Thanks for checking out this PREVIEW. To see more comics and math problems, download the product file or visit mathplane.com. If you have questions, suggestions, or feedback, let us know.

Cheers,
Lance



We appreciate your support! (All proceeds go to the site and treats for my dog, Oscar…)