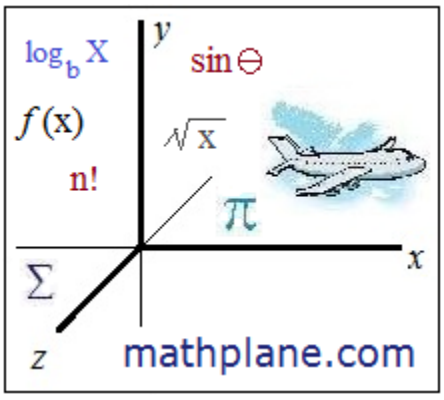
Radicals

Brief notes, quiz (w/solutions), and comic

Topics include prime factorization, rationalizing the denominator, perfect squares, and more.



Simplifying Radicals

Strategy 1: Prime Factorization

$$\sqrt{700}$$

Factor (to primes) $\sqrt{7 \cdot 2 \cdot 5 \cdot 2 \cdot 5}$

Remove "pairs" $2 \cdot 5\sqrt{7}$

Simplify $10\sqrt{7}$

Strategy 2: Using Perfect Squares

$$\sqrt{700}$$

Factor $\sqrt{7 \cdot 100}$

Remove Perfect Squares $10\sqrt{7}$

Rationalizing the Denominator

It's improper to have a radical in the denominator. So, to correct a fraction, simply rationalize the denominator.

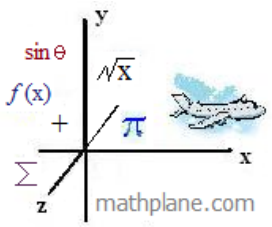
Single term denominator: $\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \longrightarrow \frac{2\sqrt{5}}{5}$

multiply by 1 radical moves to numerator

Double term denominator: use the conjugate

$$\frac{3}{4+\sqrt{6}} \cdot \frac{4-\sqrt{6}}{4-\sqrt{6}} \longrightarrow \frac{12-3\sqrt{6}}{10}$$

$\sqrt{\text{Radicals Quiz}}$



I. Simplify

a) $\sqrt{125}$

b) $\sqrt{56b^2}$

c) $\sqrt{68}$

d) $\sqrt{128ab^3}$

e) $\sqrt{99}$

II. True or False?

a) $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$

b) $\sqrt{ab} = \sqrt{a}\sqrt{b}$

c) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

III. Combine the terms

a) $\sqrt{5} + 3\sqrt{25} + 5\sqrt{125}$

b) $\sqrt{2} + \sqrt{4} + \sqrt{8} + \sqrt{16}$

c) $2\sqrt{49} - (\sqrt{64} + 14)$

Radicals Quiz (continued)

IV. Miscellaneous

a) List all perfect squares < 150

b) $3\sqrt{7} + 2\sqrt{28} - \sqrt{162} - \sqrt{2} =$

c) $3\sqrt{3} \cdot 6\sqrt{3} =$

d) $3\sqrt{3} + 6\sqrt{3} =$

V. Simplify (and, if necessary, rationalize the denominator)

a) $\sqrt{\frac{44}{144}}$

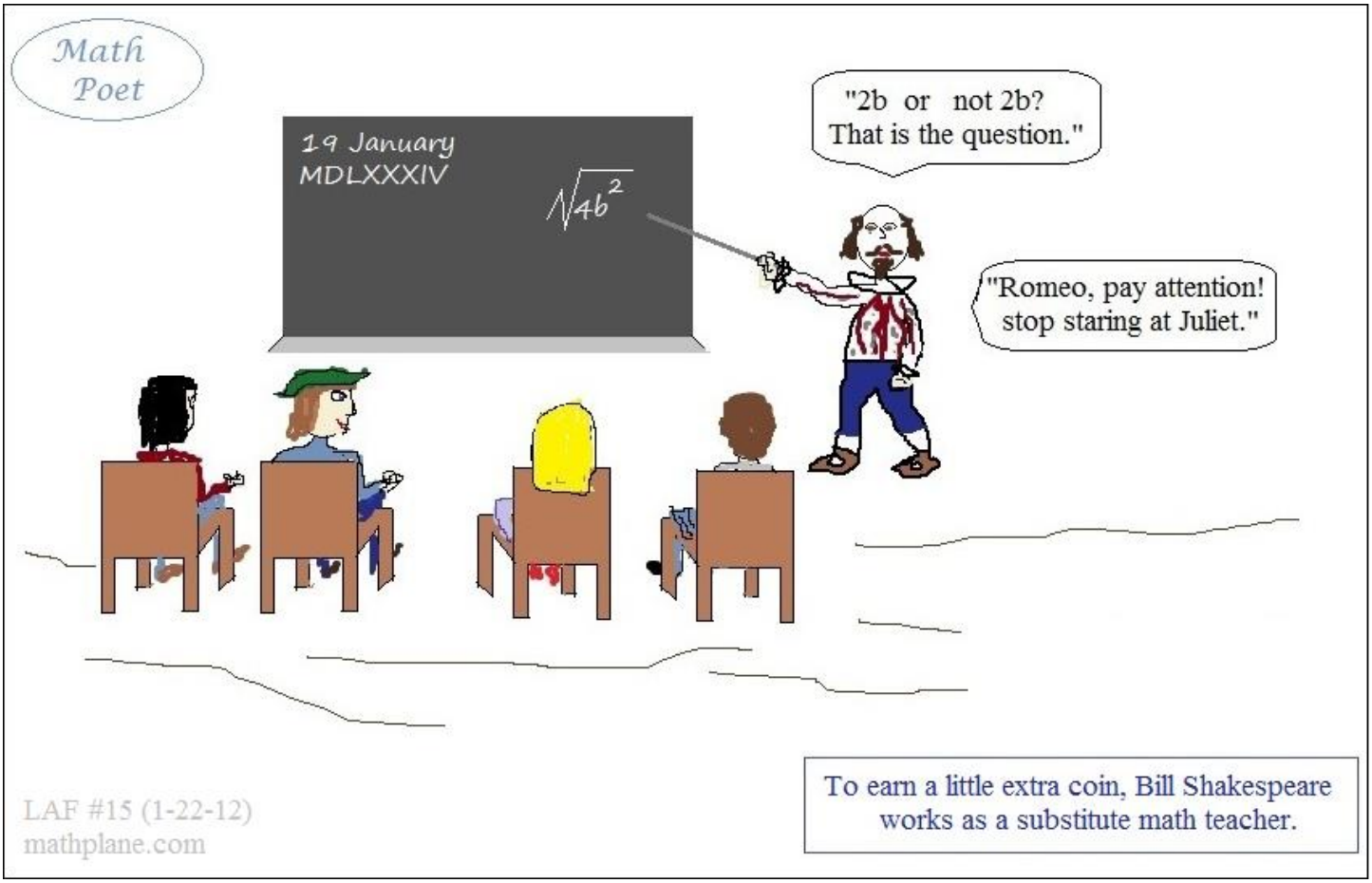
b) $\frac{(3\sqrt{7} + 8\sqrt{7})}{22}$

c) $\frac{3}{\sqrt{3}}$

d) $\frac{16}{\sqrt{17}}$

e) $\frac{3\sqrt{7} \cdot 8\sqrt{7}}{\sqrt{2} \cdot \sqrt{8}}$

f) $\sqrt{2}\,(3\sqrt{3} + 2\sqrt{2})$


Math Poet
19 January
MDLXXXIV
$\sqrt{4b^2}$
"2b or not 2b? That is the question."
"Romeo, pay attention! stop staring at Juliet."
LAF #15 (1-22-12)
mathplane.com
To earn a little extra coin, Bill Shakespeare works as a substitute math teacher.

ANSWERS-→

$\sqrt{\text{Radicals Quiz}}$

SOLUTIONS

I. Simplify

a) $\sqrt{125}$ $\quad \sqrt{5 \cdot 25} = 5\sqrt{5}$

b) $\sqrt{56b^2}$ $\quad \sqrt{4 \cdot 2 \cdot 7 \cdot b \cdot b} = 2b\sqrt{14}$

c) $\sqrt{68}$ $\quad \sqrt{2 \cdot 2 \cdot 17} = 2\sqrt{17}$

d) $\sqrt{128ab^3}$ $\quad \sqrt{2 \cdot 64 \cdot a \cdot b \cdot b^2} = 8b\sqrt{2ab}$

factor remove perfect squares

e) $\sqrt{99}$ $\quad \sqrt{9 \cdot 11} = 3\sqrt{11}$

II. True or False?

a) $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ False... EX: $a = 4 \quad b = 16$ (an exception: $a = b = 0$)

$\sqrt{20} \neq \sqrt{4} + \sqrt{16}$

b) $\sqrt{ab} = \sqrt{a}\sqrt{b}$ True...

c) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ True...

III. Combine the terms

a) $\sqrt{5} + 3\sqrt{25} + 5\sqrt{125}$ $\quad \sqrt{5} + 3\sqrt{5 \cdot 5} + 5\sqrt{5 \cdot 25} =$

$1\sqrt{5} + 3 \cdot 5 + 25\sqrt{5} = \boxed{15 + 26\sqrt{5}}$

b) $\sqrt{2} + \sqrt{4} + \sqrt{8} + \sqrt{16}$ $\quad \sqrt{2} + 2 + 2\sqrt{2} + 4 = \boxed{6 + 3\sqrt{2}}$

c) $2\sqrt{49} - (\sqrt{64} + 14)$ $\quad 2 \cdot 7 - (8 + 14) = 14 - 22$

$\boxed{= -8}$

Solutions

Radicals Quiz (continued)

IV. Miscellaneous

a) List all perfect squares < 150 1, 2, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144

b) $3\sqrt{7} + 2\sqrt{28} - \sqrt{162} - \sqrt{2} =$ $3\sqrt{7} + 4\sqrt{7} - 9\sqrt{2} - \sqrt{2} = \boxed{7\sqrt{7} - 10\sqrt{2}}$

c) $3\sqrt{3} \cdot 6\sqrt{3} =$ $3 \cdot 6 \cdot \sqrt{3} \cdot \sqrt{3} \boxed{= 54}$

d) $3\sqrt{3} + 6\sqrt{3} = \boxed{9\sqrt{3}}$

V. Simplify (and, if necessary, rationalize the denominator)

a) $\sqrt{\frac{44}{144}}$ $\frac{2\sqrt{11}}{12} = \boxed{\frac{\sqrt{11}}{6}}$

b) $\frac{(3\sqrt{7} + 8\sqrt{7})}{22}$ $\frac{11\sqrt{7}}{22} = \boxed{\frac{\sqrt{7}}{2}}$

c) $\frac{3}{\sqrt{3}} \cdot \left(\frac{\sqrt{3}}{\sqrt{3}}\right) =$ $\frac{3\sqrt{3}}{3} \boxed{= \sqrt{3}}$

d) $\frac{16}{\sqrt{17}} \cdot \left(\frac{\sqrt{17}}{\sqrt{17}}\right) =$ $\boxed{\frac{16\sqrt{17}}{17}}$

e) $\frac{3\sqrt{7} \cdot 8\sqrt{7}}{\sqrt{2} \cdot \sqrt{8}}$ $\frac{24 \cdot 7}{\sqrt{16}} = \boxed{42}$

f) $\sqrt{2}\,(3\sqrt{3} + 2\sqrt{2})$

distribute: $3\sqrt{6} + 2\sqrt{4} = \boxed{4 + 3\sqrt{6}}$

Finding square roots of numbers that aren't perfect squares (without a calculator)

1) Estimate - Get close by finding 2 perfect squares that your number is between.

2) Divide - Divide your number by one of those square roots.

3) Average - Take the average of the result and the root.

4) Repeat - Use the result of step 3 to repeat steps 2 and 3, until you get a number accurate enough for you.

Example: Calculate the square root of 10 to two decimal places.

1) $3^2 = 9$
$4^2 = 16$
So, $\sqrt{10}$ will be between 3 and 4

2) Since 10 is closer to 9, we'll use the square root of 9.

10 divided by 3 = $3.33\overline{3}$

3) Find the average of 3.000 and $3.33\overline{3}$

$(3.333 + 3)/2 = 3.1667$

4) (repeat step 2)
10 divided by 3.1667 = 3.1579

(repeat step 3)
$(3.1579 + 3.1667)/2 = 3.1623$

Check the answer: 3.1623 x 3.1623 = 10.0001

Example: Calculate $\sqrt{71}$ (without a calculator)

1) 64 and 81 are perfect squares near 71.

2) $\sqrt{64} = 8$ $\quad \frac{71}{8} = 8.875$

3) Average of 8 & 8.875 is $\frac{(8 + 8.875)}{2} = 8.4375$ $\quad$ 8.4375 x 8.4375 = 71.1914

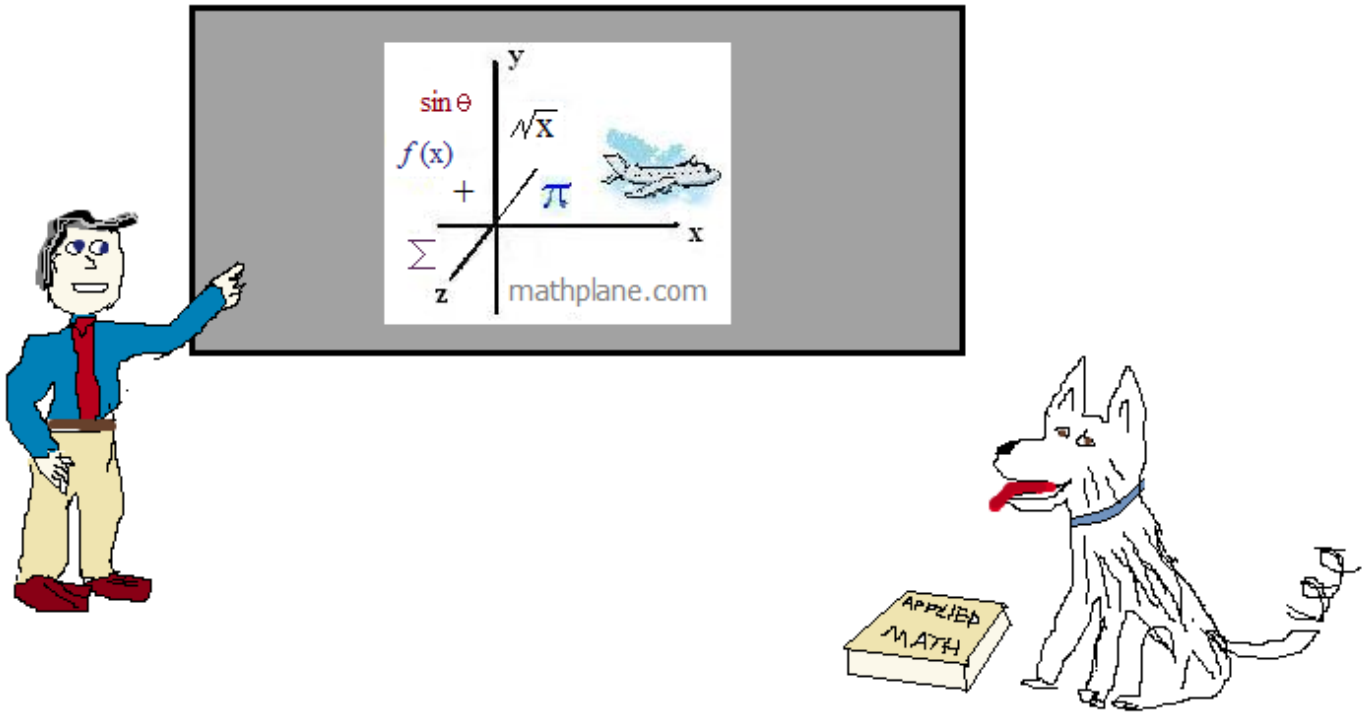
4) (repeat) $\frac{71}{8.4375} = 8.4148$

Average of 8.4148 & 8.4375 is 8.42615 $\quad$ 8.4262 x 8.4262 = 71.0008

Thanks for visiting. (Hope this quiz helped!)

If you have questions, suggestions, or requests, let us know.

Cheers,



Also, at Facebook, Google+, Pinterest, and TeachersPayTeachers

Two more questions:

Simplify:

$$\sqrt{45} + 2\sqrt{20} + \frac{1}{2}\sqrt{500}$$

Simplify

$$\frac{5\sqrt{6}}{2\sqrt{3}}$$

Solution on next page…

Simplify:

$$\sqrt{45} + 2\sqrt{20} + \frac{1}{2}\sqrt{500}$$

$$\sqrt{5 \cdot 9} + 2\sqrt{5 \cdot 4} + \frac{1}{2}\sqrt{5 \cdot 100}$$

$$\sqrt{5} \cdot \sqrt{9} + 2 \cdot \sqrt{5} \cdot \sqrt{4} + \frac{1}{2} \cdot \sqrt{5} \cdot \sqrt{100}$$

$$\sqrt{5} \cdot 3 + 2 \cdot \sqrt{5} \cdot 2 + \frac{1}{2} \cdot \sqrt{5} \cdot 10$$

$$3\sqrt{5} + 4\sqrt{5} + 5\sqrt{5}$$

$$\boxed{12\sqrt{5}}$$

Simplify

$$\frac{5\sqrt{6}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} =$$

$$\frac{5\sqrt{18}}{2\sqrt{9}} =$$

$$\frac{5\sqrt{2 \cdot 9}}{2 \cdot 3} =$$

$$\frac{15\sqrt{2}}{6} =$$

$$\boxed{\frac{5\sqrt{2}}{3}}$$