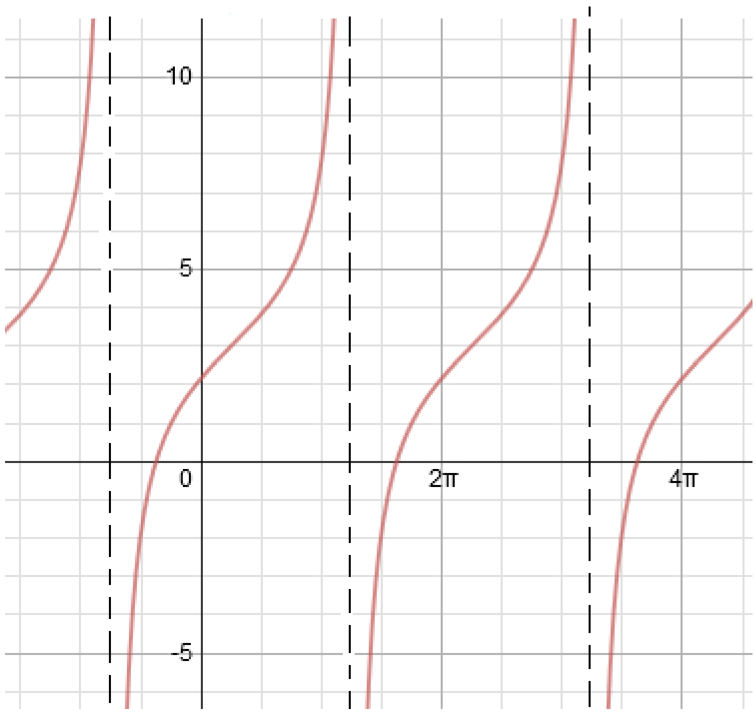
Introduction to Periodic Trig Functions:

Tangent Graphs



Notes/examples of trig values and the components of trig graphs (amplitude & reflection, horizontal (phase) shift, vertical shift, and period).

Includes practice test (and solutions)

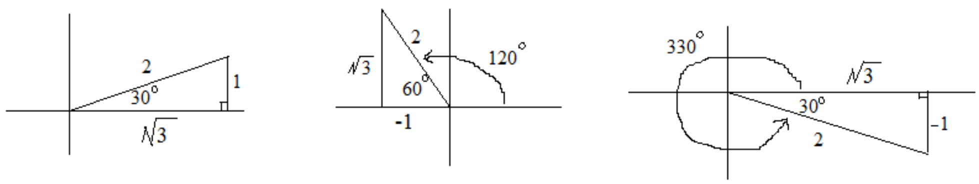
Mathplane.com

Sketching the Parent Function: y = tan(x) (radians) y = tan⊖ (degrees)

Tangent Functions

Tangent = $\frac{\text{opposite}}{\text{adjacent}}$ → $\frac{y}{x}$

A few examples of common angles:



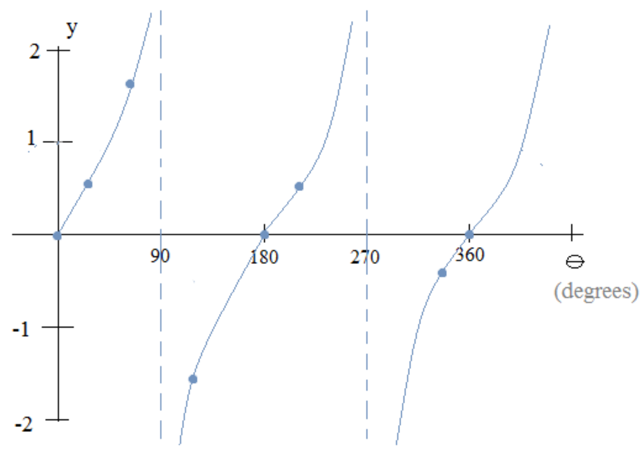
$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\tan 120^\circ = \frac{\sqrt{3}}{-1} = -\sqrt{3}$ $\tan 330^\circ = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

The following is a table of chosen values:

y = tan⊖

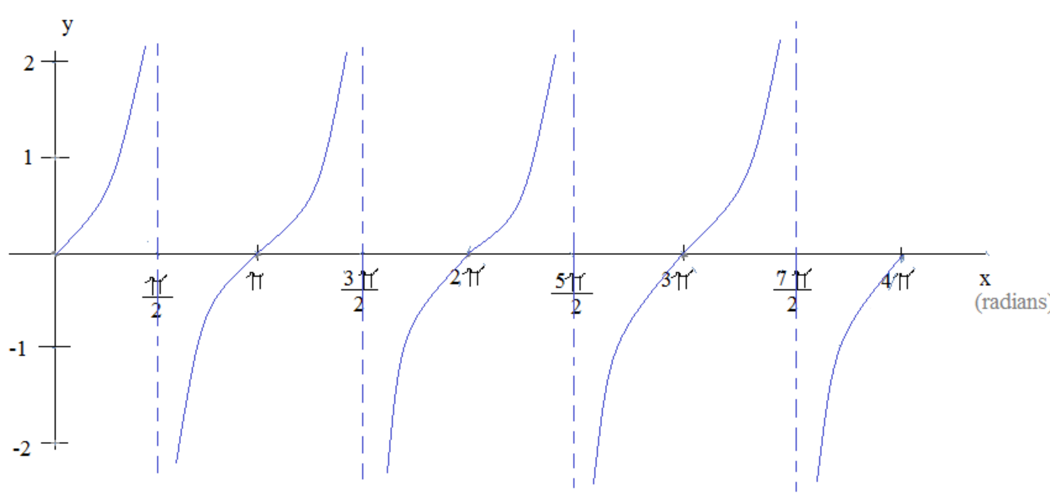
⊖	0	30	60	90	120	180	210	270	330	360
y	0	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{1}{0}$	$-\sqrt{3}$	0	$\frac{-1}{\sqrt{3}}$	$\frac{-1}{0}$	$\frac{-1}{\sqrt{3}}$	0
				Undefined				Undefined		

Then, plot the points...



It's a periodic function -- it will repeat the pattern of y- values at a regular interval of 180°. (The period is 180°)

y = tan$x$



A cycle is a complete pattern repetition. This sketch contains 4 cycles.

The period is π (Horizontal length of one cycle)

Tangent Functions: 4 components

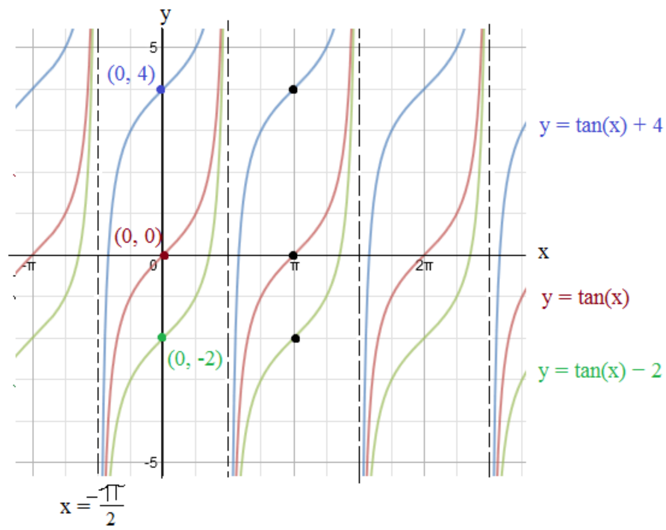
The four parts of the tangent function can stretch, shift, reflect, and compress the parent function (graph).

$y = \tan(x)$ is the parent function

$y = A\tan B(x + C) + D$

- A: Amplitude (magnitude)
- B: Period (cycles/π)
- C: Horizontal Shift
- D: Vertical Shift

Vertical Shift:



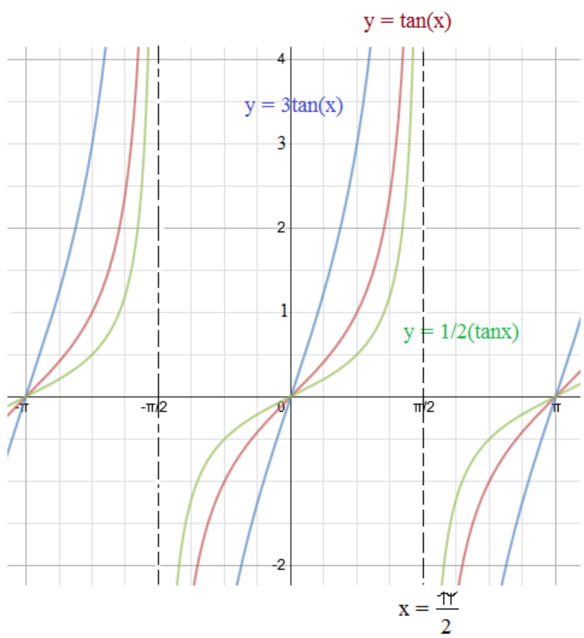
If $x = \pi$

$y = \tan(x)$ -----> $\tan(\pi) = 0$

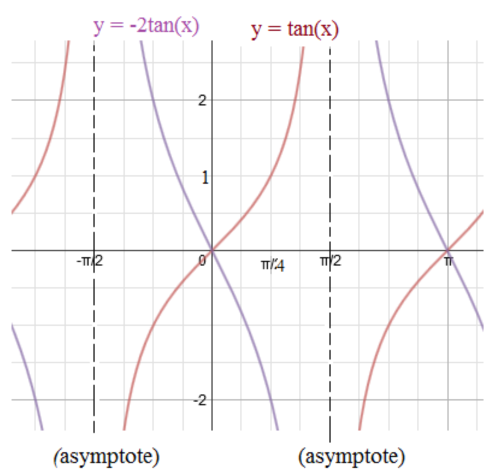
$y = \tan(x) + 4$ ----> $\tan(\pi) + 4 = 4$

$y = \tan(x) - 2$ -----> $\tan(\pi) - 2 = -2$

Amplitude:



x	tanx	3tanx	$\frac{1}{2}$ tanx	−2tanx
0	0	0	0	0
$\frac{\pi}{4}$	1	3	$\frac{1}{2}$	-2
$\frac{\pi}{2}$	undefined	undefined	undefined	
π	0	0	0	0
$\frac{3\pi}{4}$	-1	-3	$-\frac{1}{2}$	2



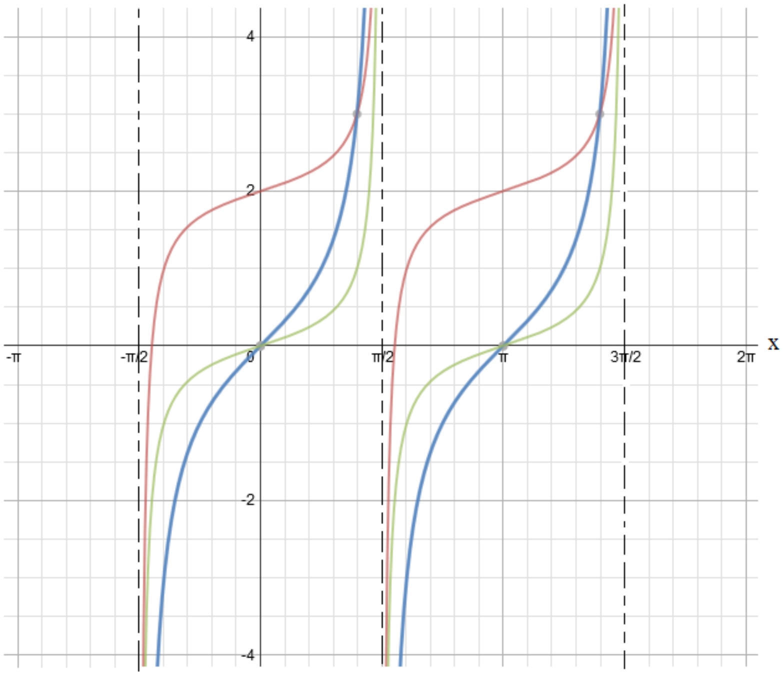
When the coefficient gets larger, the tangent curves *stretch...*

When the coefficient is negative, the output is reflected over the x-axis.

NOTE: The amplitude (A) and vertical shift (D) are numbers *outside the function.* So, they affect changes that are *up and down.*

Tangent Functions: Amplitude and Vertical Shift Illustrations

I. Sketch two cycles of the function $f(x) = \frac{1}{3}\tan x + 2$



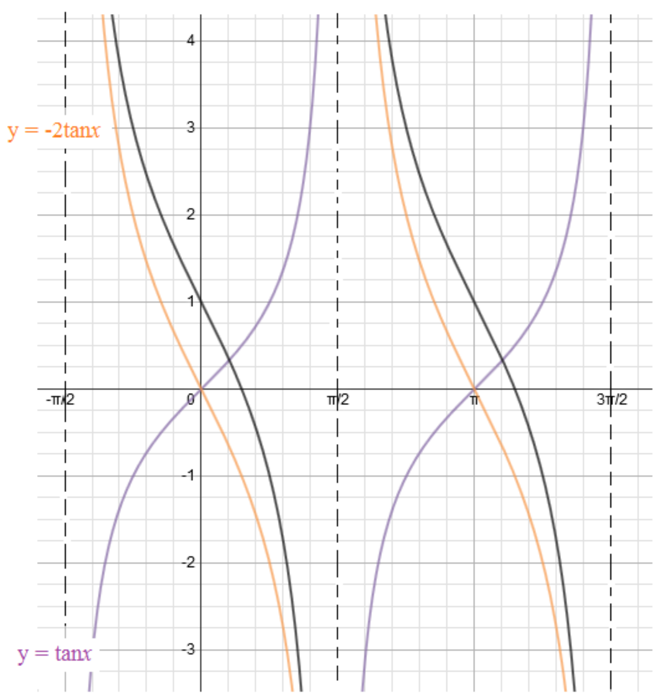
1) Sketch the parent function $y = \tan x$

2) *Decrease ("shrink") the amplitude* by a factor of 3 $y = \frac{1}{3}\tan x$

3) *Shift the graph* up 2 units $y = \frac{1}{3}\tan x + 2$

Note: The asymptotes provide a good outline for your sketch...

II. Sketch $y = -2\tan x + 1$



1) Parent Function $y = \tan x$

2) "Stretch" and "Reflect" $y = -2\tan x$

3) Vertical Shift $y = -2\tan x + 1$

Notice, the "new middle" is at $y = 1$ (instead of the x-axis)

And, because the (A) value is 'negative', the curve gets 'flipped'/'reflected' (i.e. goes from upper left to lower right)

Tangent Functions: 4 components

$y = A\tan B(x + C) + D$

A: Amplitude (magnitude)
B: Period (cycles/π)
C: Horizontal Shift
D: Vertical Shift

The four parts of the tangent function can stretch, shift, reflect, and compress the parent function (graph).

Since the A and D terms are <u>outside</u> the function, the changes affect the vertical components..
(vertical "stretch", reflection, and shift)

Since the B and C terms are <u>inside</u> the function, they will affect the horizontal shape of the graph....

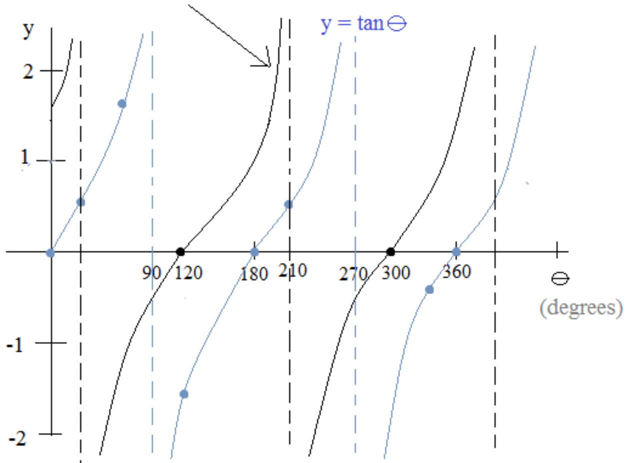
Horizontal ("Phase") Shift:

Question: If $\tan 180^\circ = 0$, then where does $\tan(\theta + 60^\circ) = 0$?

Answer: $\theta = 120^\circ$, because $\tan(120 + 60) = 0$.

Implication: $180^\circ \Rightarrow 120^\circ$ (shift 60° <u>to the left</u>)

Example I: $y = \tan(\theta + 60^\circ)$



The curves and the asymptotes shift 60° to the left

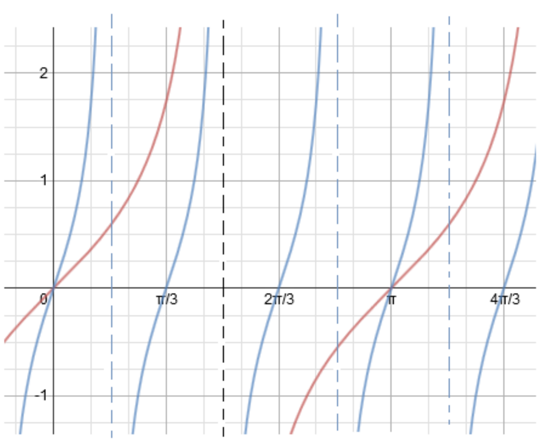
Note: The horizontal shift is the *opposite* direction of the sign.

Period: Horizontal distance required for a periodic function to complete one cycle.

$$y = \tan Bx \longrightarrow \text{period} = \frac{\pi}{B}$$

Example 2: $y = \tan 3x$ Period: $\frac{\pi}{3}$

3 cycles between 0 and π



$x = \pi/6$ $x = \pi/2$ $x = 5\pi/6$ $x = 7\pi/6$ ⇐ asymptotes

For the parent function $y = \tan x$ (where B = 1) the period is π

As B increases, the period decreases.
In other words, it takes less time to complete one cycle.

And, for $y = \tan Bx$, as B decreases, the period increases.
In other words, it takes more time to complete one cycle.

NOTE: sinx and cosx periods are 2π, but tanx is π

tanx has one cycle... (period is π)

tan3x has three cycles ... (period is $\pi/3$)

(so, 3 times as many asymptotes and intercepts)

Example: Sketch one period of the function $y = 2\tan\frac{1}{4}(x - \frac{\pi}{2}) + 3$

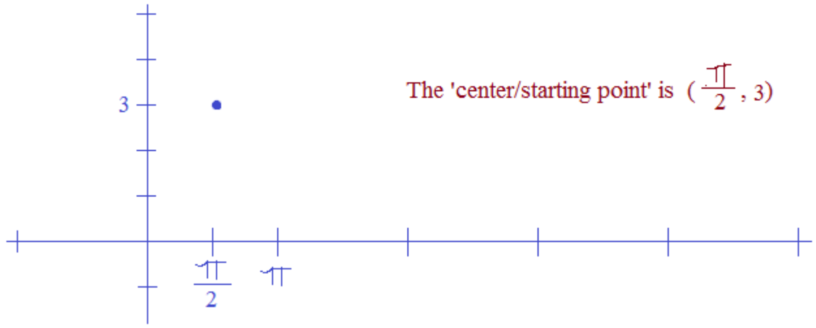
Graphing a Tangent Function: step-by-step

$y = A\tan B(x + C) + D$

A: Amplitude (magnitude)
B: Period (cycles/π)
C: Horizontal Shift
D: Vertical Shift

The 'D' value is 3, so the vertical shift is up 3 units

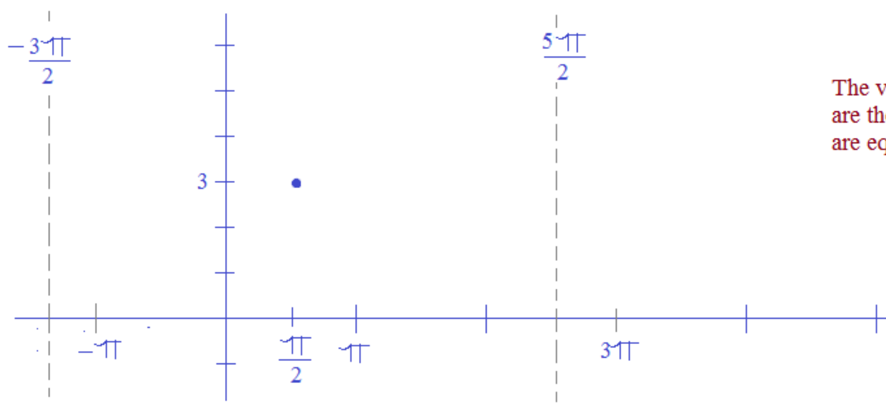
The 'C' value is $\frac{\pi}{2}$, so the horizontal shift is $\frac{\pi}{2}$ to the right



The 'center/starting point' is $(\frac{\pi}{2}, 3)$

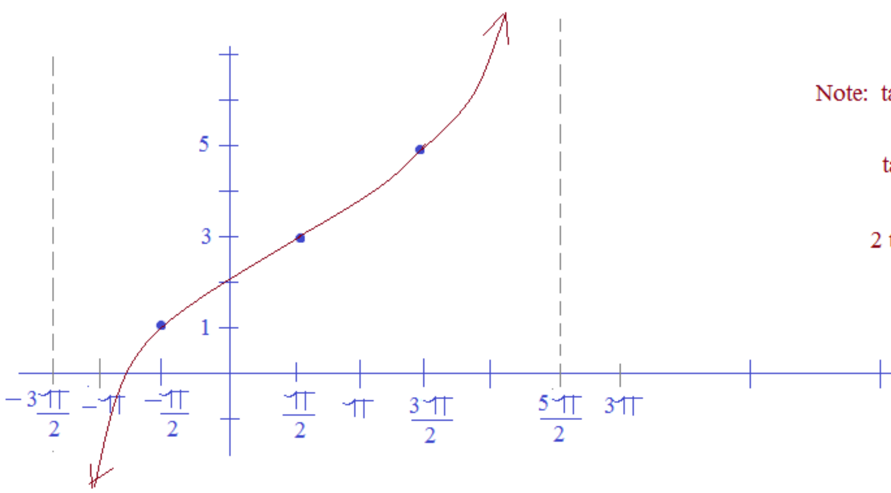
The 'B' value is $\frac{1}{4}$, so the period is $\dfrac{\pi}{\frac{1}{4}} = 4\pi$

The 'center point' is in the middle of the period... So, we can place vertical asymptotes 2π to the left... and, 2π to the right...



The vertical asymptotes are 4π apart, are the boundaries of the period, and are equidistance from the 'center point'.

The 'A' Value is 2, so the "quarter values" will be up 2 and down 2...



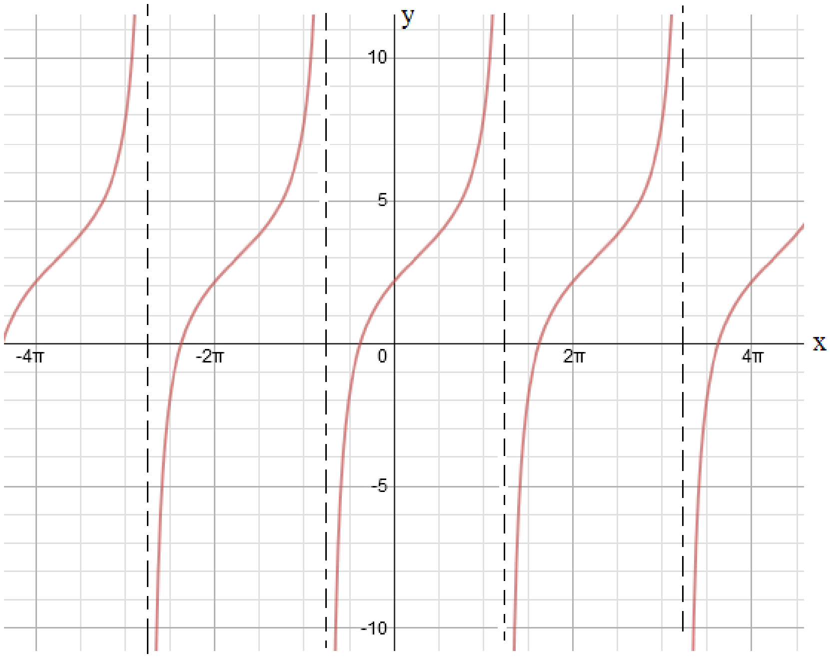
Note: $\tan\frac{\pi}{4} = 1$

$\tan\frac{1}{4}(\frac{3\pi}{2} - \frac{\pi}{2}) = 1$

$2\tan\frac{1}{4}(\frac{3\pi}{2} - \frac{\pi}{2}) + 3 = 5$

Finally, use the 3 points and asymptotes to guide your sketch....

Example: Identify the following tangent function:



$y = A\tan B(x + C) + D$

A: Amplitude (magnitude)
B: Period (cycles/π)
C: Horizontal Shift
D: Vertical Shift

Since the midpoint (i.e. the curve's point of inflection) is at 3, the vertical shift is UP 3

vertical shift (D): +3

One full cycle has a length of 2π. Since the period is 2π, $B = \dfrac{\pi}{2\pi} = \dfrac{1}{2}$

period (B): $\dfrac{1}{2}$

If the period ("B" value) is 1/2, then the asymptotes would be at π 3π 5π etc..

But, in the graph, the asyptotes are at $\dfrac{5\pi}{4}$ $\dfrac{13\pi}{4}$ $\dfrac{21\pi}{4}$ $\longrightarrow$

horizontal shift to the right (C):
("phase") $-\dfrac{\pi}{4}$

In a 1/4 cycle move, the value goes from 0 to 1... (i.e. $\tan(0) = 0$ $\tan(\dfrac{\pi}{4}) = 1$

In the above graph, 1/4 of a cycle is $\dfrac{\pi}{2}$ ... At $x = 0$, the output is 3

amplitude (A): 2

$$\boxed{y = 2\tan\frac{1}{2}\left(x - \frac{\pi}{4}\right) + 3}$$

At $x = \dfrac{\pi}{2}$, the output is 5

This is an increase of 2 (instead of 1)

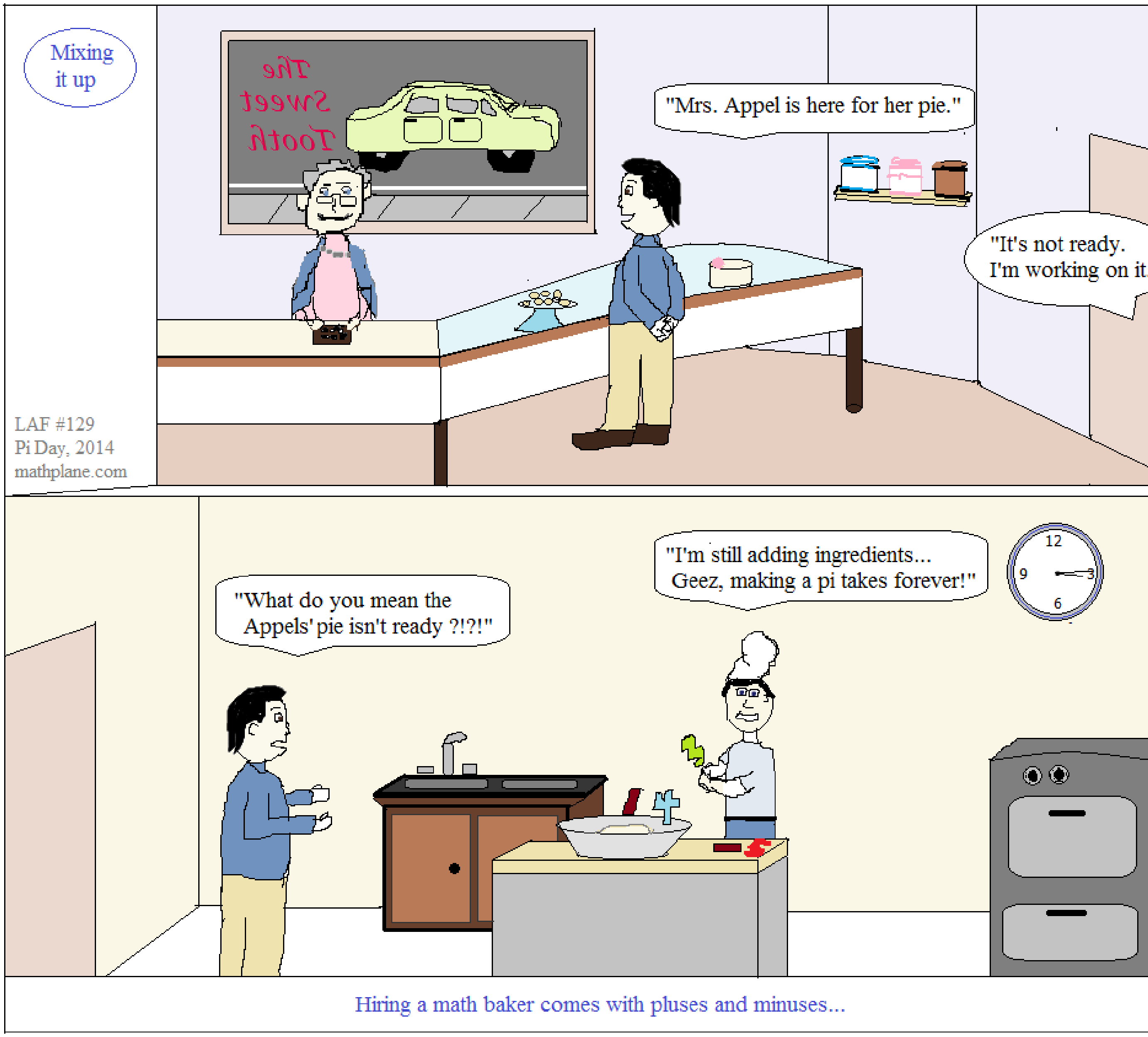
Test points to confirm your equation!

If $x = \pi$ $\quad 2\tan\dfrac{1}{2}\left(\pi - \dfrac{\pi}{4}\right) + 3 = 2\tan\dfrac{3\pi}{8} + 3$

$= 2 \cdot 2.41 + 3 \approx 7.8$ ✓

If $x = \dfrac{-\pi}{2}$ $\quad 2\tan\dfrac{1}{2}\left(\dfrac{-\pi}{2} - \dfrac{\pi}{4}\right) + 3 = 2\tan\dfrac{-3\pi}{8} + 3$

$= 2 \cdot -2.41 + 3 \approx -1.8$ ✓

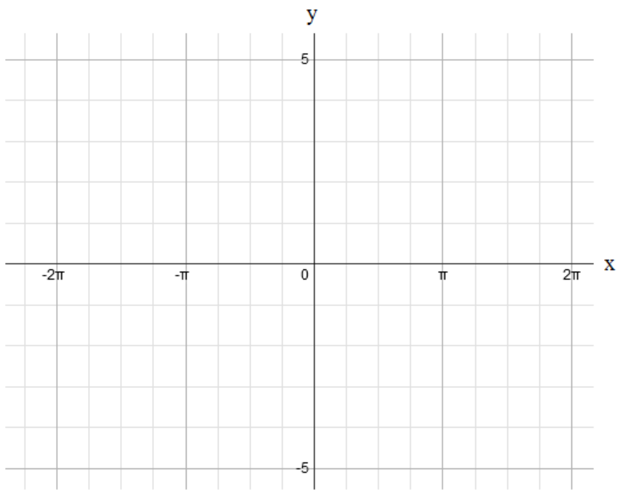

Mixing it up
The Sweet Tooth
"Mrs. Appel is here for her pie."
"It's not ready. I'm working on it."
LAF #129
Pi Day, 2014
mathplane.com
"What do you mean the Appels' pie isn't ready ?!?!"
"I'm still adding ingredients... Geez, making a pi takes forever!"
12
9
3
6
Hiring a math baker comes with pluses and minuses...

Practice ->

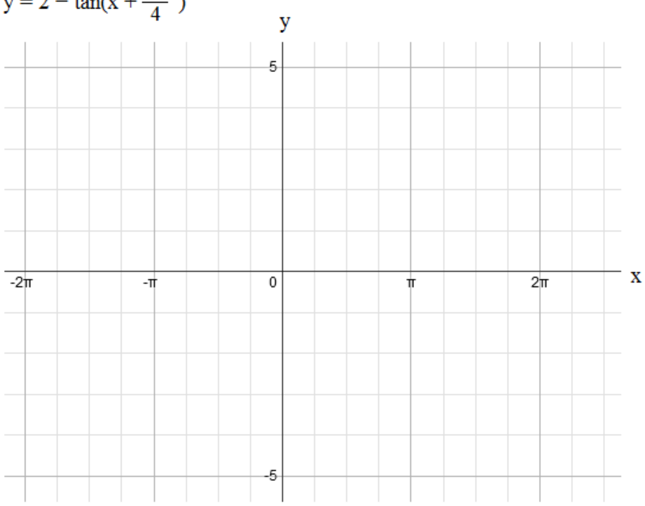
Tangent Function Practice

I. Graphing: Sketch each of the following equations. (Include at least 2 periods. And, label the asymptotes.)

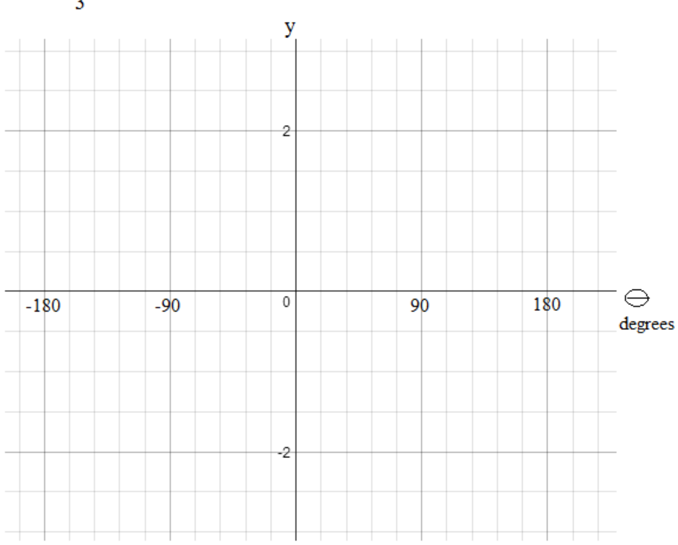
A) $y = 2\tan x - 3$



B) $y = 2 - \tan(x + \frac{\pi}{4})$



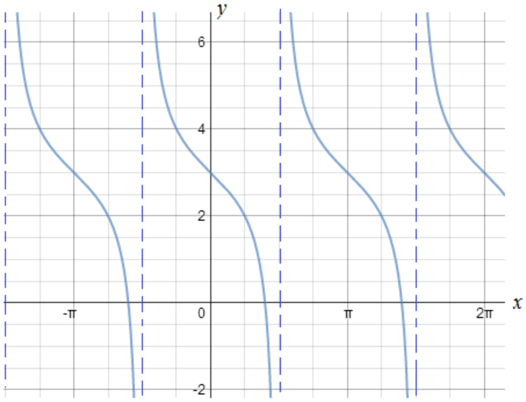
C) $y = \frac{\tan 2\theta}{3}$



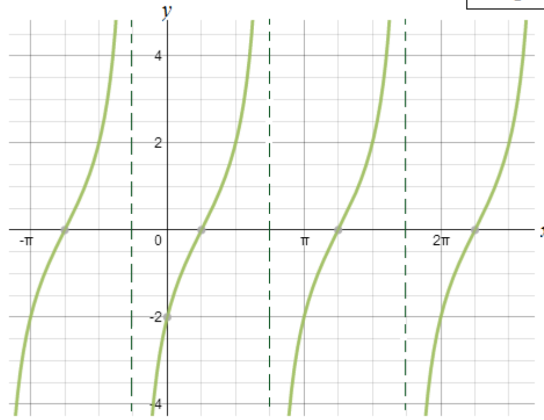
Tangent Function Practice

II. Identifying: Determine the equations of the following.

A)



B)



III. For the function $f(x) = \tan(x - \frac{\pi}{2})$, determine the

a) domain

b) range

c) maximum

d) minimum

e) x-intercepts (or, zeros)

f) y-intercept

Challenge Question: Solve algebraically. Then, graph to confirm your solution.

$\text{Cos}\,x = \text{Tan}\,x$ (in the interval $0^{\circ} < x < 360^{\circ}$)

Skip got a disappointing C+ on the math assignment...
(but, he did see 2 awesome reruns of Magnum P.I.)

Answers-→

Tangent Function Practice

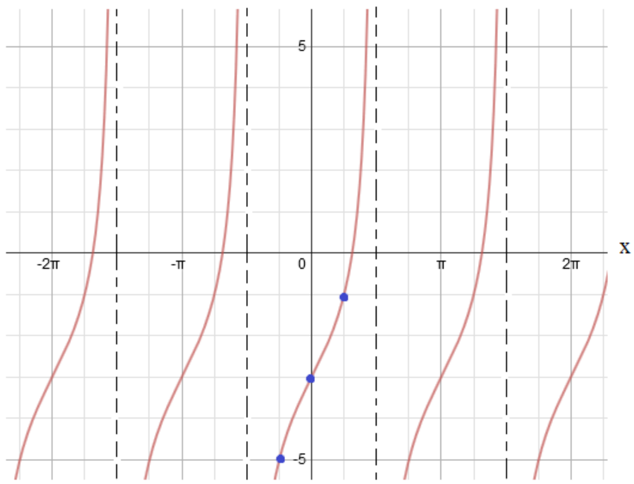
SOLUTIONS

I. Graphing: Sketch each of the following equations. (Include at least 2 periods. And, label the asymptotes.)

$y = A\tan B(x + C) + D$

A: Amplitude (magnitude)
B: Period (cycles/π)
C: Horizontal Shift
D: Vertical Shift

A) $y = 2\tan x - 3$



3 points: $\left(-\frac{\pi}{4}, -5\right)$ $(0, -3)$ $\left(\frac{\pi}{4}, -1\right)$

Vertical shift (D): DOWN 3 units eg: a point of inflection at (0, -3)

Period (B = 1): π

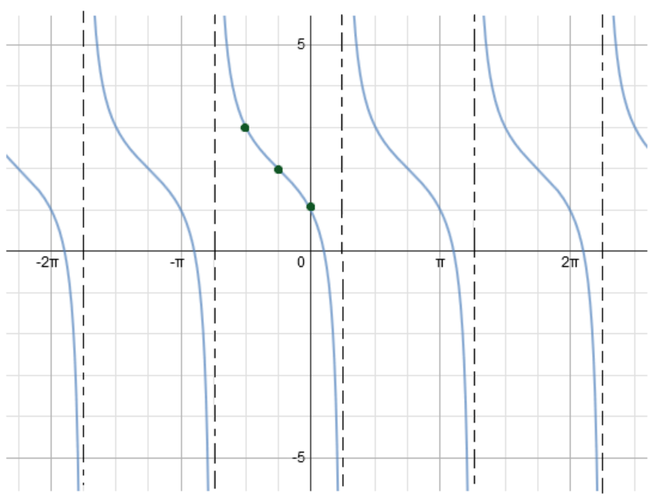
Asymptotes at $x = \frac{-\pi}{2}$ $x = \frac{\pi}{2}$ $x = \frac{3\pi}{2}$ etc...

$x = \frac{\pi}{2} + n\pi$ where n is an integer

Horizontal shift (C): None

Amplitude (A): Stretch by 2 at $\frac{\pi}{4}$ the function increases by 2 units (instead of 1)

B) $y = 2 - \tan\left(x + \frac{\pi}{4}\right)$



rewrite the equation:

$y = -\tan\left(x + \frac{\pi}{4}\right) + 2$

"A" = -1 amplitude is 1, and the graph is *reflected over the x-axis*

"B" = 1 the period is $\frac{\pi}{1}$

"C" = $\frac{\pi}{4}$ the graph shifts $\frac{\pi}{4}$ to the left

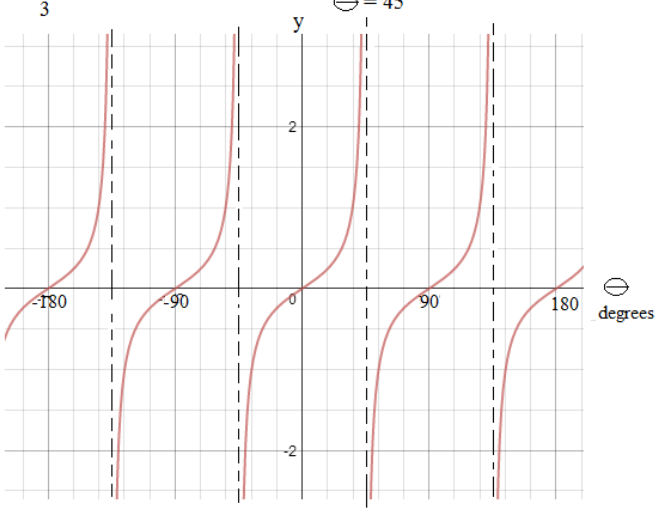
"D" = 2 the graph shifts 2 units UP

3 points on the graph: $\left(-\frac{\pi}{2}, 3\right)$ $\left(-\frac{\pi}{4}, 2\right)$ $(0, 1)$

asymptotes: $x = \frac{\pi}{4} + n\pi$ where n is any integer

C) $y = \frac{\tan 2\theta}{3}$

$\theta = 45^\circ$



$y = \frac{1}{3}\tan 2\theta$

"D" = 0 no vertical shift

"C" = 0 no horizontal shift

"A" = 1/3 the graph will "shrink" (up and down)

"B" = since B = 2, the period is cut in half..
It only takes 90 degrees for one cycle!

Since period is 90 degrees, there are asymptotes spaced 90 degrees apart...

when

$\theta = \ldots -135^\circ, -45^\circ, 45^\circ, 135^\circ \ldots$

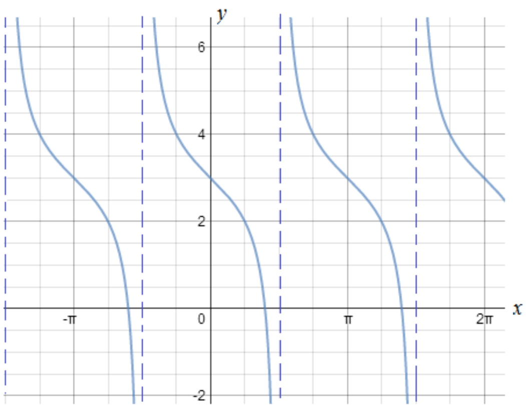
Test random points to check your answer!

II. Identifying: Determine the equations of the following.

SOLUTIONS

Tangent Function Practice

A)



Period is π B = 1

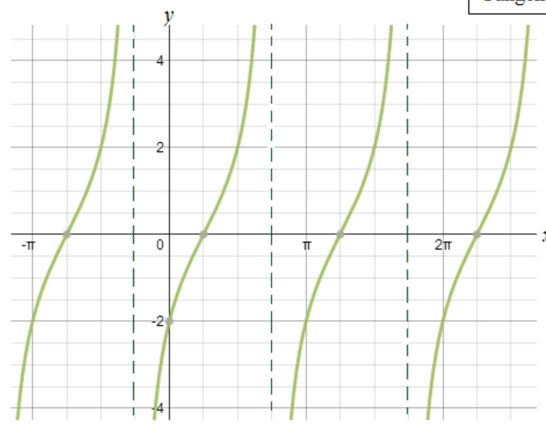
The 'center' or point of inflection occurs at y = 3   Vertical shift D = + 3

$y = -\tan x + 3$

Asymptotes are at $\frac{\pi}{2} + \pi k$ No horizontal shift C = 0

At $\frac{\pi}{2}$, the graph moved down 1, so the amplitude A = −1

B)



$y = 2\tan(x - \frac{\pi}{4})$

Vertical asymptote moved from $-\frac{\pi}{2}$ to $-\frac{\pi}{4}$ horizontal shift right   C = $-\frac{\pi}{4}$

Center/point of inflection is on x-axis No vertical shift D = 0

Period is π B = 1

at $\frac{\pi}{4}$ output is 0   at $\frac{\pi}{2}$ output is 2 (instead of 1) this implies the amplitude is A = 2

III. For the function $f(x) = \tan(x - \frac{\pi}{2})$, determine the

a) domain all real numbers EXCEPT πk (where k is an integer)

b) range $(-\infty\ ,\ \infty)$ i.e. all real numbers

c) maximum None

d) minimum None

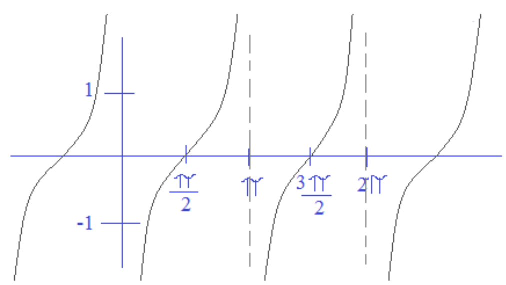
e) x-intercepts (or, zeros) $\frac{\pi}{2} + \pi k$ (where k is an integer)

f) y-intercept Does not exist

Using a sketch and table of values as a guide:

x	y
0	undefined
$\frac{\pi}{4}$	-1
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	-1
π	undefined
$\frac{3\pi}{2}$	0

(tangent function shifted $\frac{\pi}{2}$ to the right)



Challenge Question: Solve algebraically. Then, graph to confirm your solution.

Cos$x$ = Tan$x$ (in the interval $0^\circ < x < 360^\circ$)

$\frac{\cos x}{1} = \frac{\sin x}{\cos x}$ (Trig identity)

$\cos^2 x = \sin x$ (Cross multiply)

$1 - \sin^2 x = \sin x$ Trig identity)

$\sin^2 x + \sin x - 1 = 0$ (Quadratic formula)

Sin$x$ = .618 and -1.618

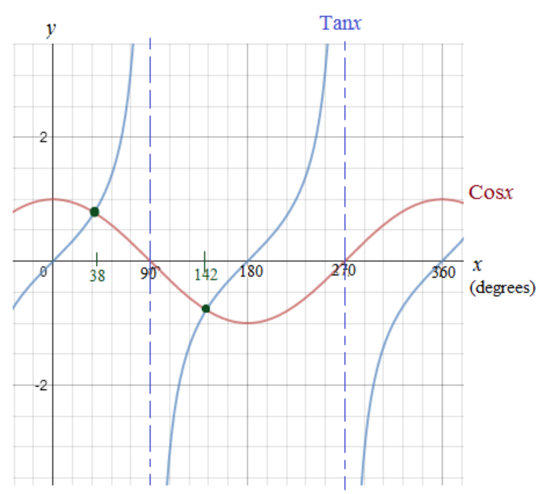
$\boxed{x = 38.2 \text{ degrees}}$ (-1.618 is extraneous...)

Then, since tangent and cosine have same signs ('negative') in Quadrant II

Reference angle: 38.2 degrees ----> 180 - 38.2 = $\boxed{141.8 \text{ degrees}}$

Quick check: Cos(38.2) = .786 (approx)

Tan(38.2) = .787 (approx)

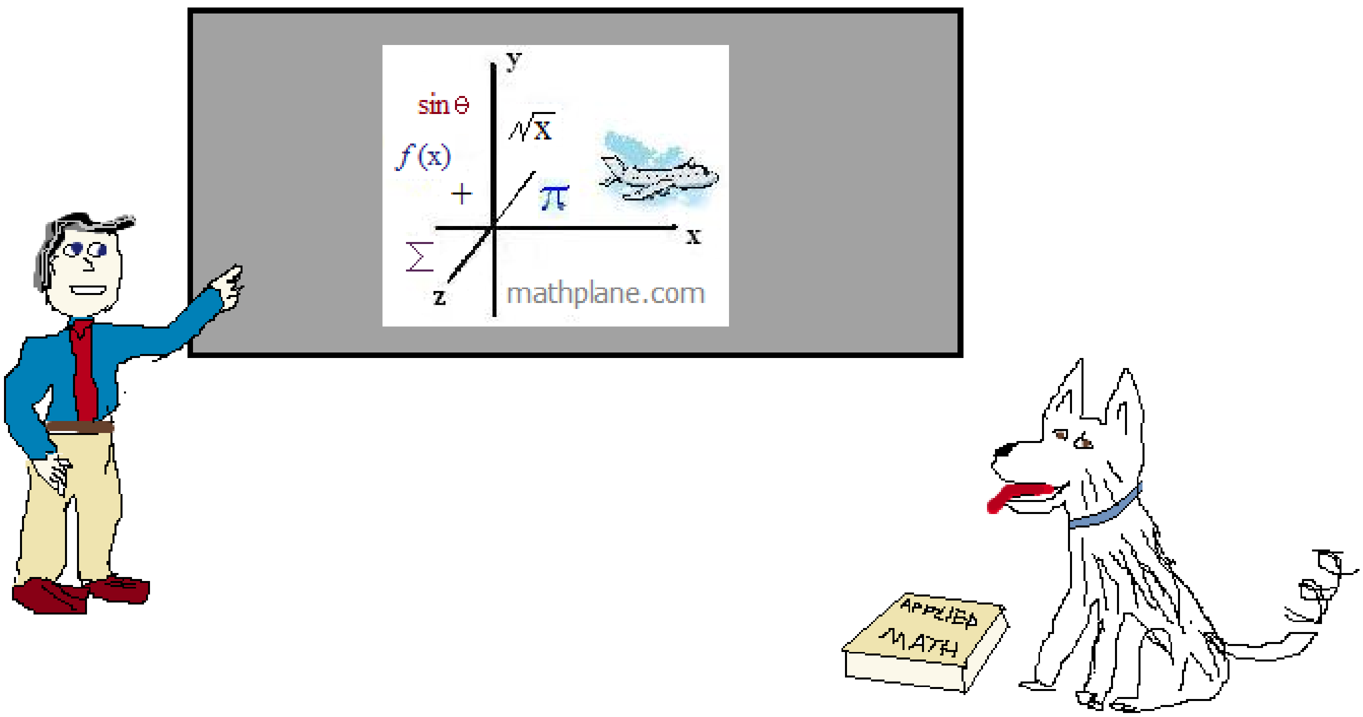


The two points of intersection (in the interval) represent the solution to the equation.

Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Enjoy.



All proceeds go to site maintenance and improvement. (Plus, treats for Oscar the Dog!)

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