Mean, Median, Mode, and Range

Notes, Examples, and Practice Exercises

Topics include weighted average, data analysis, percentages, and more.

Mean

What is it?   An (arithmetic) mean is the average of a set of n numbers.

How do you find it?   Add all the elements in a set. Then, divide by the number of elements.

Examples: The mean of 7 and 11 is 9. $\frac{7 + 11}{2} = 9$

The mean of -6, -3, 2, 4, and 4 is .2 $\frac{(-6) + (-3) + 2 + 4 + 4}{5} = \frac{1}{5}$

(5 elements in the set)

Jim's math test scores were 78, 88, 91, and 94. What is the mean of those test scores?

$$\frac{78 + 88 + 91 + 94}{4} = 87.75$$

If Jim needs a 90 average for an A, what must he get on his final test?

$$\frac{78 + 88 + 91 + 94 + (\text{final test})}{5} = 90$$

$$351 + (\text{final test}) = 450$$

The final must be 99 or better to get an A.

Suppose the final test is worth 1/2 his grade. Does that make a difference?!?!

"Weighted Average":   A weighted average is computed when some elements carry more
importance ("weight") than others.

To calculate a weighted average, count elements with more weight more often.

So, if Jim's final is worth 1/2 his grade:

$$\frac{(78 + 88 + 91 + 94) + \text{final} + \text{final} + \text{final} + \text{final}}{8} = \text{Grade}$$

$$\frac{351 + (4 \times \text{final})}{8} = 90$$

$$351 + (4 \times \text{final}) = 720$$

$$4 \times \text{final} = 369$$

$$\text{final} = 92.25$$

If the final is 50% of Jim's grade, then
he needs a 92.25 or better to get an A!!

Median

What is it?   The <u>middle</u> number in a set.

How do you find it?   Arrange the numbers in ascending (or descending) order.   Select the term in the middle.

Examples: {3, 7, 10, 15, 100}   10 is the median.

{5, 13, -2, 0, -4, -10, 25}   arranged in order: -10, -4, -2, 0, 5, 13, 25   0 is the median.

{1, -3, -5, 12, 16, -3, -3, 4, 14} arranged in order: -5, -3, -3, -3, 1, 4, 12, 14, 16   1 is the median.

Each of the above examples has an odd number of elements, so there is one middle term.
But, if a set has an even number of terms, there are 2 middle terms. Which is the median?

"Even Sets" : If the set has an even number of terms, take the <u>average of the two</u> middle terms.

Examples: {2, 3, 7, 10, 14, 22} Two middle terms: 7 and 10.
The average (mean) of 7 and 10 is   8.5   8.5 is the median.

{-5, 4, -9, 1, 1, 0}   arranged in order: -9, -5, 0, 1, 1, 4
Two middle terms: 0 and 1   1/2 is the median.
The average is 1/2

Mode

What is it?   The element, term, class, or group that appears most often.

Examples: {3, 5, 4, 6, 5, 5, 3}   5 appears most often, so 5 is the mode

{A, B, B, B+, C, A, B, C, C, A, C}   C appear 4 times, so C is the mode

"Bimodal": A distribution that consists of 2 modes.

In this example, 50s and 80's appear most
often (and the same # of days)

50s and 80s are the modes.

of
Days

20s 30s 40s 50s 60s 70s 80s 90s

Average
Daily Temperature
Ranges
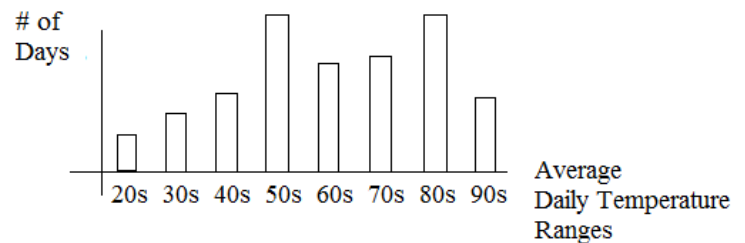
Weighted Average

Simple Average: | Total value of terms
("Mean") | Number of terms

The average of 3, 8, 12, and 20 is $\dfrac{3 + 8 + 12 + 20}{4} = \dfrac{43}{4} = 10.75$

But, suppose some terms have a greater value than others?
If the terms have different 'weights', then a simple average won't work...

Example: Your math grade is determined by this method:

homework: 20%
tests: 50%
final exam: 30%

A: 92% - 100%
A- : 90% - 92%
B+ : 88% - 90%
B: 82% - 87%
B- : 80% - 82%
etc...

1) In the first semester, you earned an average of 70 on your homework, 89 on your tests, and 93 on your final, what math grade will you receive?

"Block Method" -- 'Set the numbers into equal blocks'

the simple average: $\dfrac{70 + 89 + 93}{3} = 84$

Since 20, 50, and 30 have factors of 10, each block will represent 10%

homework (2 'blocks'): 70   70

tests (5 'blocks'): 89 89 89 89 89

final exam (3 'blocks'): 93 93 93

Since all your scores are now *weighted*, we can find the average!

$\dfrac{70 + 70 + 89 + 89 + 89 + 89 + 89 + 93 + 93 + 93}{10} = 86.4$ B

(10 blocks @ 10% each is 100%)

Notice: the homework grade had less of an impact

What if the teacher places more emphasis (i.e. weight) on the final?

2) Suppose the final is worth 50%, the tests are worth 40%, and the homework is worth 10%...
   The weights change, and the grade increases!

$\dfrac{70 + 89 + 89 + 89 + 89 + 93 + 93 + 93 + 93 + 93}{10} = 89.1$ B+

3) Now, suppose your grades are the following: homework: 75, 83, 90, 92 , 94, 95
                                                 tests: 88, 92, 98, 97
                                                 final exam: 91

what is your final (weighted) average and grade?

Find each average:

homework average $\dfrac{(75 + 83 + 90 + 92 + 94 + 95)}{6} = 88.167$

test average $\dfrac{(88 + 92 + 98 + 97)}{4} = 93.75$

Then, use them to find the final weighted average:

final exam average $\dfrac{91}{1} = 91$

$\dfrac{2(88.167) + 5(93.75) + 3(91)}{10} = 91.81$ A-

Example: Your math grade is determined by this method: homework: 20%   A: 92% - 100%
tests: 50%   A- : 90% - 92%
final exam: 30%   B+ : 88% - 90%
B: 82% - 87%
B- : 80% - 82%
etc...

In the second semester, you had a 96 average on your homework, and 92 from your tests.
What is the lowest final exam score you'll need to earn an A ?

Notice, your 'tests' average has a greater impact on your grade than the 'final exam' and your 'homework'.
When finding the average, we apply different weights to each entry....

The "homework weights": 96   96

The "tests weights":   92   92   92   92   92

The "final exam weights": x   x   x

$$\frac{96 + 96 + 92 + 92 + 92 + 92 + 92 + x + x + x}{10}$$

Grade Average $=$

$$\frac{2(96) + 5(92) + 3(x)}{10} \geq 92$$

$$\frac{652 + 3x}{10} \geq 92$$

$$652 + 3x \geq 920$$

$$3x > 268$$

If the final exam is 30% of your grade,

$$x \geq 89.33$$

You need 89 1/3 or better on the final

Example: Suppose a bag of coins has 17% pennies, 23% nickels, 50% dimes, and 10% quarters.
What is the value of an "average coin"?
(i.e. if you randomly picked a coin, what value would you expect?)

Notice, the quantities of the coins varies, so this must be weighted.
Then, the value of each type of coin differs, so this must be weighted as well!

In this case, we'll divide the coins by type into specified shares of 17%, 23%, 50%, and 10%.

$$\frac{17(A) + 23(B) + 50(C) + 10(D)}{100} = .17(A) + .23(B) + .50(C) + .10(D)$$

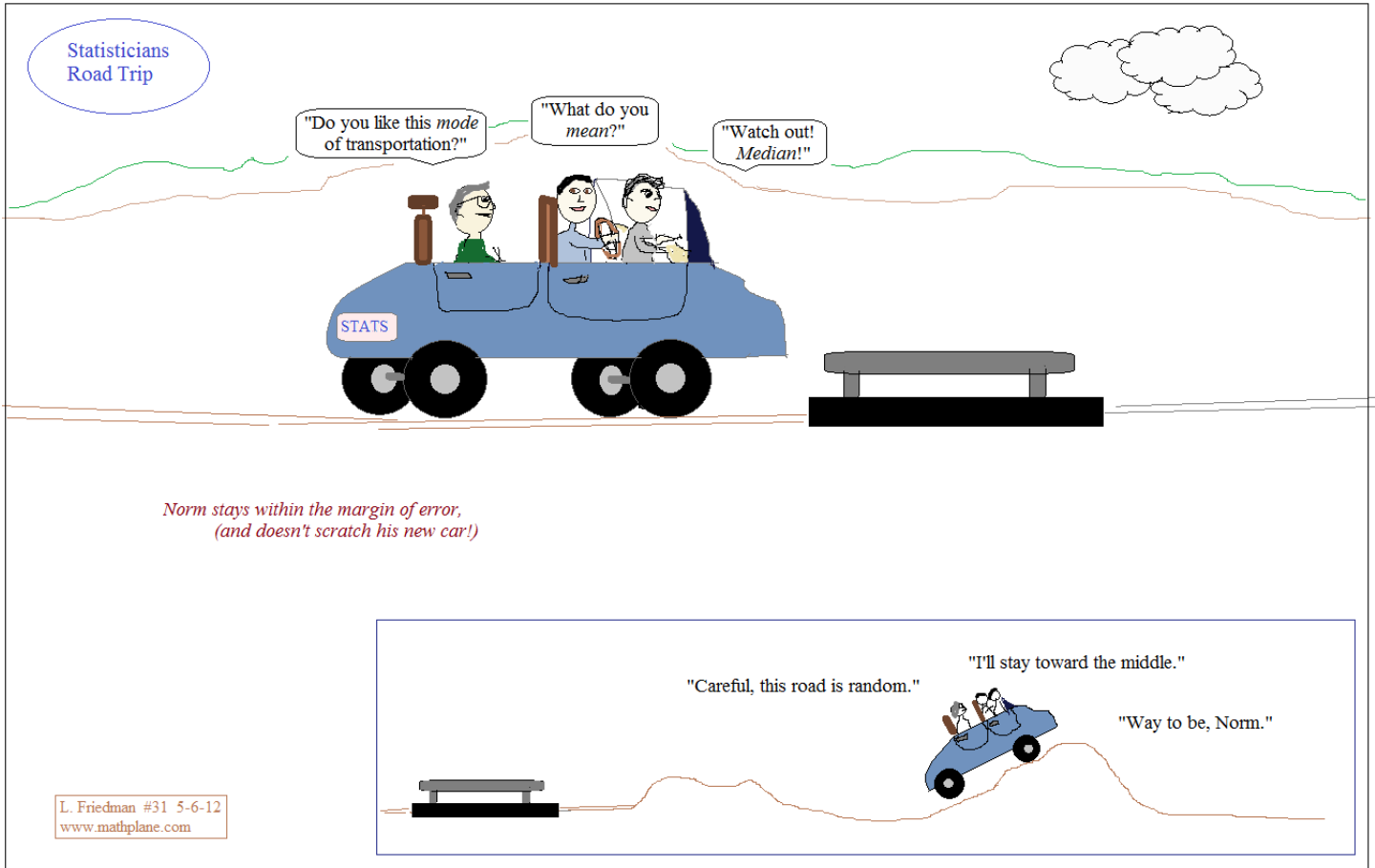Then, we make each share weighted by the value of each type of coin...

Coin   # of cents
P = 1
N = 5
D = 10
Q = 25

$$\frac{17(1) + 23(5) + 50(10) + 10(25)}{100} = .17(1) + .23(5) + .50(10) + .10(25) =$$

$$.17 + 1.15 + 5 + 2.5 = 8.82$$

Now, suppose the bag has an equal number of pennies, nickels, dimes, and quarters.
What is the value of an "average coin"?

$$\frac{25(1) + 25(5) + 25(10) + 25(25)}{100}$$

or

$$.25(1) + .25(5) + .25(10) + .25(25) = 10.25$$

Mean, Median, and Mode Quick Quiz

I. Answer the following questions.

   a) What is the mean of set A?

      A = {4, -2, 1/2, 7.5, 0}

   b) Amy's math grades are in the grid on the right.

      What is her 'homework' average?

      What is the mean of her 'quiz' grades?

homework	80	90	92	80
quiz	78	84	87	91
final exam	90			

      Amy's final grade is determined as follows: 20% homework
                                       50% quizzes
                                       30% final exam
            What is her final "weighted average"?

II. Find the median of each set.

   a) {3, -1, 9, 16, -1.5}

   b) {3, 4, 5, 5, 1, 3}

   c) {1, 5, -6, 2.4, 3.7}

III. Identify the mode(s) in each set.

   a) {9, 3, -9, 5, 2/5, 3}

   b) {2, 0, -3, 0, 1, 2, 8}

   c) {math, english, science, math, history, math, english}

IV. Miscellaneous

   Give an example of a set where the mode is less than the mean.

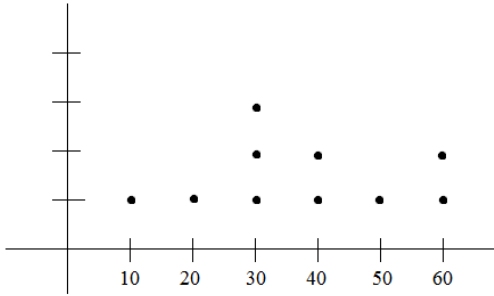   Give an example of a set where the mean and median are the same.

   {5, 0, -3, 12, 17}   If 8 is added to this set, what is the median?

   BONUS: I have a (finite) set of integers. After adding another integer,
            the mean increased. But, the median did not.
            How did this happen? (Explain and/or provide an example.)
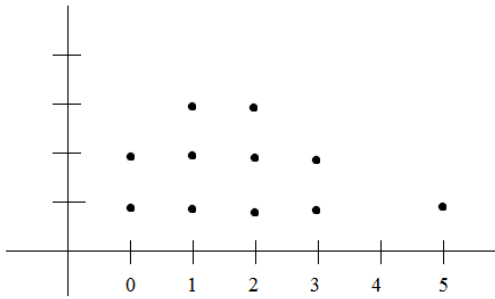
Dot Plot: Mean, Median, Mode, Range
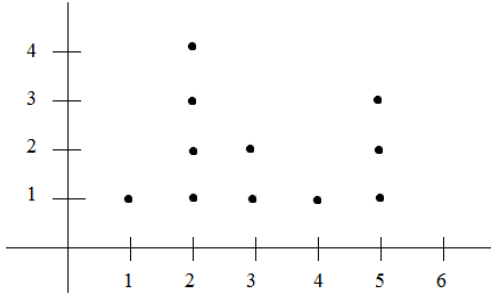
mean _____
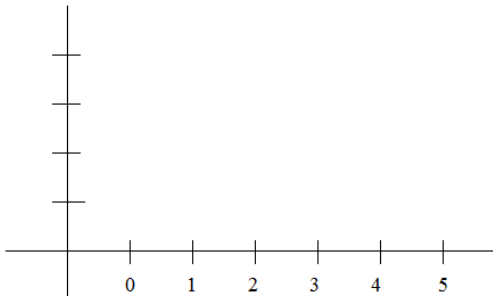
median _____

mode _____

range _____

mean _____

median _____

mode _____

range _____

mean _____

median _____

mode _____

range _____

Sample Set:

{ 1, 3, 5, 2, 3, 2, 1, 4, 4, 1, 5, 1, 3, 4, 1}

mean _____

median _____

mode _____

range _____

1) At the math school, 90% earns a student an A...
Going into the final, Sam has a 95 average.
If the final exam is 20% of the grade, what is the lowest score Sam needs
to get an A?

2) In a school raffle, 140 entries won nothing, 4 entries won $50, and one person won the $1000 grand prize.
What is the "mean" winner? What is the "median" winner?

If each ticket cost $10, is it worth it?   Explain.

If each ticket cost $5, is it worth it?   Explain.

3)   Saturday:

# of toys purchased	5	4	3	2	1	0	
# of customers	2	3	9	12	17	30	

   a) How many toys were purchased?

   b) How many toys did the average person buy?

   c) Each toy is $11. How much did the store earn?

   d) How much did the average customer spend?

4) Multiple choice: The mean and median of 8, -6, -4, 2 is

         a) mean: -5   median: -5

         b) mean: 0   median: -5

         c) mean: 0   median: -1

         d) mean: -5   median: 0

5) The following is a list of numbers: 50, 38, 33, X, Y, and 17.
   If 17 is the mode, and 29 is the median, then what is the mean of this list of numbers?

<div style="border:1px solid">Mean, Median, Mode, and Weighted Average</div>

   a) 25

   b) 27

   c) 29

   d) 30

   e) 35

6) Which is the best measure of central tendency: mean, median, or mode?

   a) { 2, 4, 3, 7, 6, 39, 2}

   b) flavors of ice cream choice
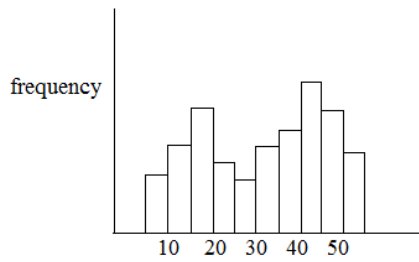   v- vanilla
       c- chocolate
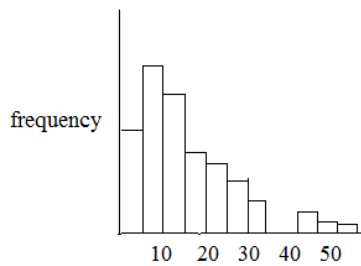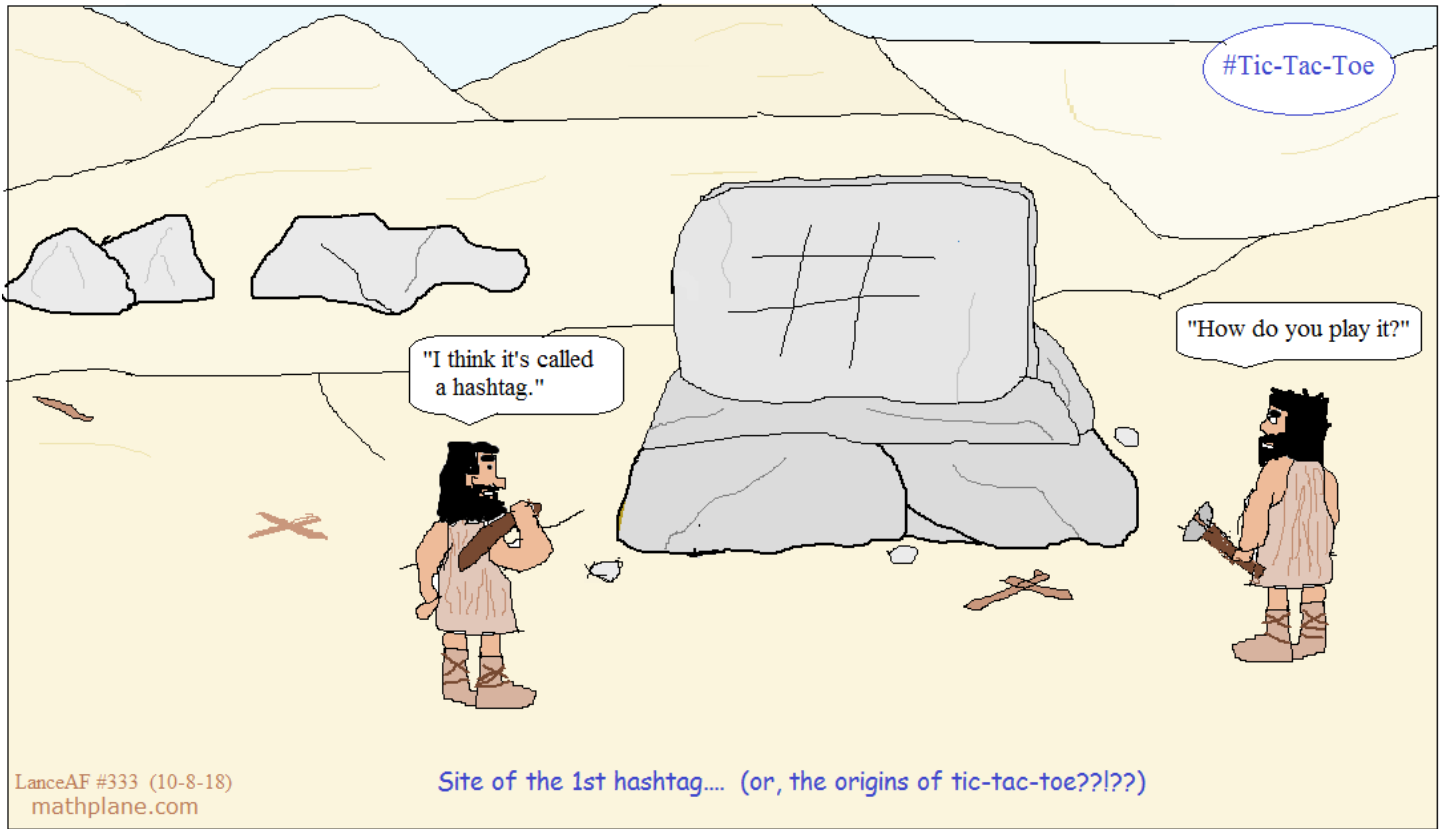   s- strawberry   {v, c, v, v, s, v, m, c, v}
       m- mint

   c) Math test scores are 85, 79, 87, 82, 91, 82

   d)

   e)

Mean, Median, and Mode Quick Quiz		ANSWERS

I. Answer the following questions.

a) What is the mean of set A?

$A = \{4, -2, 1/2, 7.5, 0\}$ $\dfrac{4 + (-2) + 1/2 + 7.5 + 0}{5} = \boxed{2}$

b) Amy's math grades are in the grid on the right.

homework	80	90	92	80
quiz		78	84	87	91
final exam		90			

What is her 'homework' average? $\dfrac{80 + 90 + 92 + 80}{4} = \boxed{85.5}$

What is the mean of her 'quiz' grades? $\dfrac{78 + 84 + 87 + 91}{4} = \boxed{85}$

Amy's final grade is determined as follows: 20% homework   85.5 85.5   (2 out of 10: 20%)
50% quizzes   85 85 85 85 85 (5 out of 10: 50%)
30% final exam   90 90 90   (3 out of 10: 30%)

What is her final "weighted average"?

$\dfrac{(2 \times 85.5) + (5 \times 85) + (3 \times 90)}{10} = \boxed{86.6}$

II. Find the median of each set.

a) $\{3, -1, 9, 16, -1.5\}$ -1.5   -1 $\boxed{3}$ 9   16

b) $\{3, 4, 5, 5, 1, 3\}$ 1   3 $\boxed{3\ \ 4}$ 5   5   the median is 3.5

c) $\{1, 5, -6, 2.4, 3.7\}$ -6, 1, $\boxed{2.4,}$ 3.7, 5

III. Identify the mode(s) in each set.

a) $\{9, 3, -9, 5, 2/5, 3\}$ mode is 3

b) $\{2, 0, -3, 0, 1, 2, 8\}$ bimodal: modes are 0 and 2

c) {math, english, science, math, history, math, english}   math

IV. Miscellaneous

Give an example of a set where the mode is less than the mean. $\{3, 3, 3, 19\}$ mode: 3   mean: 7

Give an example of a set where the mean and median are the same. $\{1, 3, 5\}$ mean: 3   median: 3

$\{5, 0, -3, 12, 17\}$ If 8 is added to this set, what is the median?   -3, 0, 5, 8, 12, 17   median: 6.5
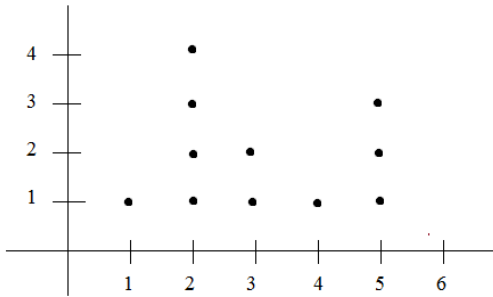
BONUS: I have a (finite) set of integers. After adding another integer, the mean increased. But, the median did not. How did this happen? (Explain and/or provide an example.)

The set has an EVEN number of terms, where the middle terms are the <u>same</u>.
$\{1, 3, 3, 9\}$ mean: 4   median 3
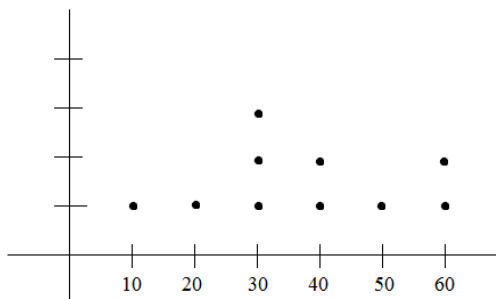$\{1, 3, 3, 9, 9\}$ mean: 5   median 3

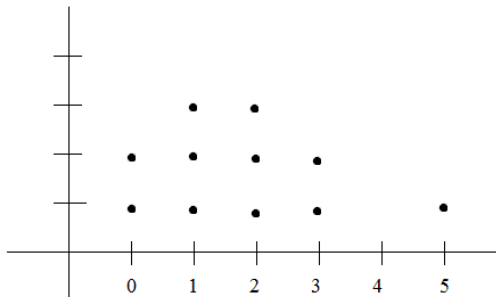Dot Plot: Mean, Median, Mode, Range

Mean: average

Median: middle value

Mode: most often

Range: difference between
maximum and minimum

mathplane.com

1
2
2
2
2
3
3
4
5
5
5

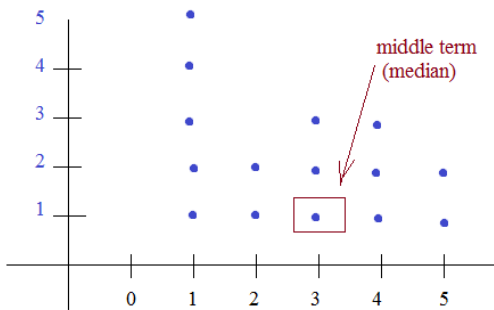total: 34

mean __34/11 = 3.1 (approx.)__

median __3__

mode __2__

range __4__   max: 5   min: 1

total: 370
number of elements: 10

10
20
30
30
30
40
40
50
60
60

mean __370/10 = 37__

median __35__   (since there are "two middles",
we take the average of them!)

mode __30__   appears most often

range __50__   max: 60   min: 10

list of data

0
0
1
1
1
2
2
2
3
3
5

total: 20

mean __20/11 = 1.8 (approx.)__

median __2__

mode __1 and 2__ (bimodal)

range __5__

Sample Set:

{ 1, 3, 5, 2, 3, 2, 1, 4, 4, 1, 5, 1, 3, 4, 1}

middle term
(median)

mean __40/15 = 2.67 (approx.)__

median __3__

mode __1__

range __4__

1) At the math school, 90% earns a student an A...
   Going into the final, Sam has a 95 average.
   If the final exam is 20% of the grade, what is the lowest score Sam needs
   to get an A?

Since the final exam is worth 20%, will apply the following weights:

$$\frac{F + 95 + 95 + 95 + 95}{5} \geq 90$$

$$\frac{.20F + .80(95)}{1.00} \geq 90$$

$$F + 95 + 95 + 95 + 95 \geq 450$$

$$\boxed{F \geq 70}$$

2) In a school raffle, 140 entries won nothing, 4 entries won $50, and one person won the $1000 grand prize.
   What is the "mean" winner? What is the "median" winner?

   The median winner is 0. (If you listed all the winners, including those that won nothing, the middle person would win 0)

   If each ticket cost $10, is it worth it? Explain.

   If each ticket cost $5, is it worth it? Explain.

   The mean winner is the (weigthed) average:

   We needed to find the *expected value* of each ticket
   (i.e. how much is each ticket 'worth')

   $$\frac{140(0) + 4(50) + 1(1000)}{145} = \frac{1200}{145} = 8.28$$

   So, the average ticket is 'worth' $8.28..
   Therefore, a $10 ticket is not worth it (because you would expect to lose money)...
   However, a $5 is worth it.
   (although the school would lose money!)

3) Saturday:

# of toys purchased	5	4	3	2	1	0
# of customers	2	3	9	12	17	30

   a) How many toys were purchased?

   b) How many toys did the average person buy?

   c) Each toy is $11. How much did the store earn?

   d) How much did the average customer spend?

   a) $2(5) + 3(4) + 9(3) + 12(2) + 17(1) + 30(0) = 90$ toys

   2 customers
   5 toys each

   b) average person = $\frac{\text{total toys purchased}}{\text{total customers}} = \frac{90 \text{ toys}}{73 \text{ customers}} = 1.23 \frac{\text{toys}}{\text{customer}}$

   c) 90 toys x $11/toy = $990

   d) ave. customer spending = $\frac{\text{total spent}}{\text{customers}} = \frac{\$990}{73 \text{ customers}} = \$13.56$ per person

4) Multiple choice: The mean and median of 8, -6, -4, 2 is

   $11/toy x 1.23 toys/customer =
   $13.56 per customer

   a) mean: -5   median: -5

   b) mean: 0   median: -5

   c) mean: 0   median: -1

   d) mean: -5   median: 0

5) The following is a list of numbers: 50, 38, 33, X, Y, and 17.
   If 17 is the mode, and 29 is the median, then what is the mean of this list of numbers?

a) 25

b) 27

c) 29

d) 30

e) 35

Since 17 is the mode, Y or X is 17...

50, 38, 33, X, 17, 17..

then, the middle numbers are 33 and X..
Since the median is 29,
X must equal 25

Therefore, the mean of the set is

$$\frac{50 + 38 + 33 + 25 + 17 + 17}{6} = 30$$

SOLUTIONS

6) Which is the best measure of central tendency: mean, median, or mode?

a) { 2, 4, 3, 7, 6, 39, 2}   since 39 is an extreme outliner, it would have an extra affect on the mean...
and, the mode isn't necessarily a good indicator of the sample..
answer: MEDIAN

b) flavors of ice cream choice
   v- vanilla
   c- chocolate
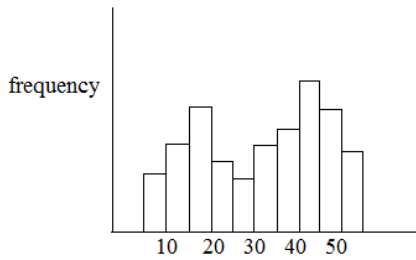   s- strawberry
   m- mint

{v, c, v, v, s, v, m, c, v}

(It's categorial)
Since the elements cannot be ranked, the best
method of evaluating expectation is using the MODE
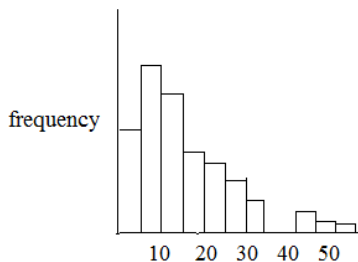
c) Math test scores are 85, 79, 87, 82, 91, 82

Since there are no outliers, and the scores are apparently
evenly distributed, the MEAN would be best indicator.

d)

frequency

10 20 30 40 50

The distribution is reasonably balanced.
The MEAN would identify the measure well.
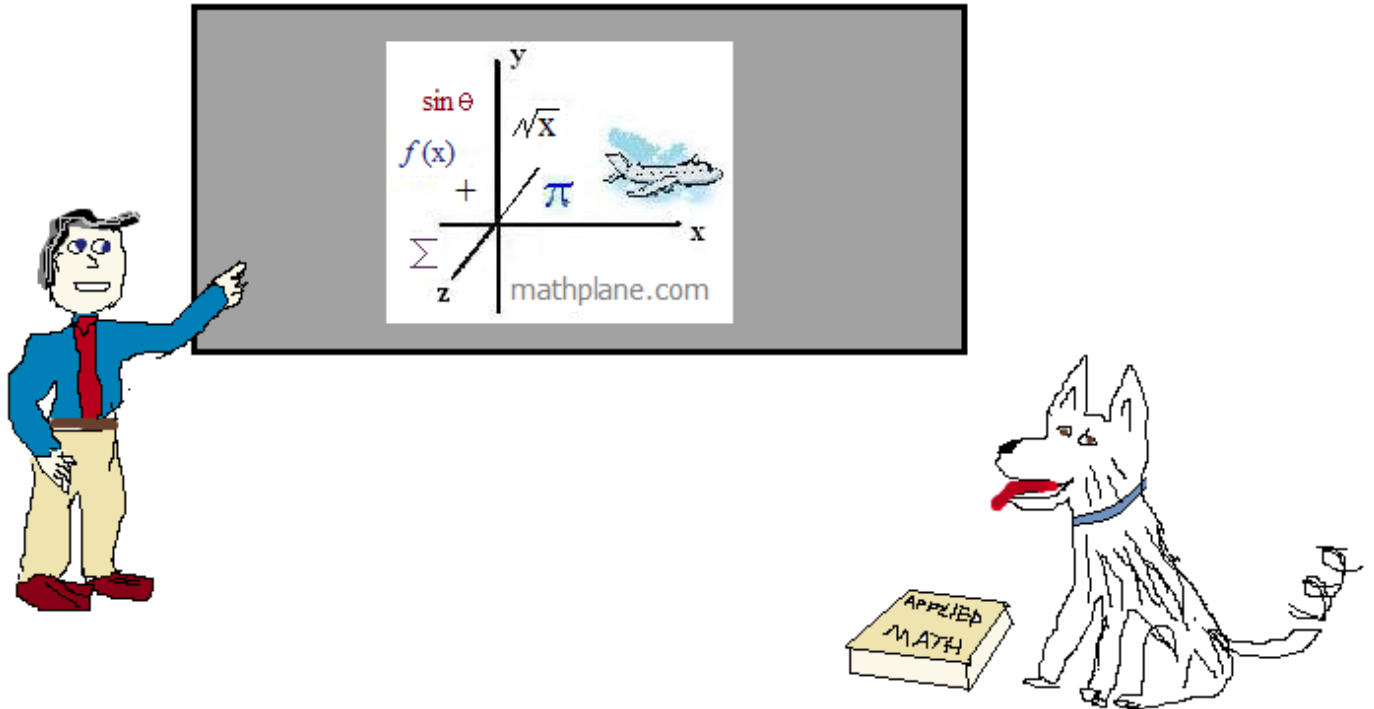
e)

frequency

10 20 30 40 50

Since this data set is skewed to the right, it pulls the mean to the right.
So, the MEDIAN is more effective.

Thanks for visiting! (Hope it helped.)

If you have questions, suggestions, or requests, let us know.

Cheers

Also, find our materials at our store in TeachersPayTeachers.com

Or, visit us at Mathplane.ORG for mobile and tablets