Introduction to Periodic Trig Functions:

Sine Graphs

Notes/examples of trig values and the 4 components of trig graphs (amplitude, horizontal (phase) shift, vertical shift, and period).

Includes practice test (and solutions)
Contents

Sine Functions \( y = A \sin B(x - C) + D \)
- Sketching the parent function
- Vertical Shift
- Amplitude
- Reflection
- Horizontal ("phase") Shift
- Period and Cycles
- Describing a Sine Graph

Practice Test and Solutions
Periodic Functions: Sinusoidal Graphs

Sketching the Parent Function: \( y = \sin(x) \quad y = \sin(\theta) \)

(Sine = opposite \over hypotenuse)

A few examples of common angles:

\[
\begin{align*}
\sin 30^\circ &= \frac{1}{2} \\
\sin 120^\circ &= \frac{\sqrt{3}}{2} \\
\sin 330^\circ &= -\frac{1}{2}
\end{align*}
\]

The following is a table of chosen values:

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>0</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>180</th>
<th>210</th>
<th>270</th>
<th>330</th>
<th>360</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>0</td>
<td>(\frac{1}{2})</td>
<td>(\frac{\sqrt{3}}{2})</td>
<td>1</td>
<td>(\frac{\sqrt{3}}{2})</td>
<td>0</td>
<td>(-\frac{1}{2})</td>
<td>-1</td>
<td>(-\frac{1}{2})</td>
<td>0</td>
</tr>
</tbody>
</table>

Then, plot the points...

It's a periodic function -- it will repeat the pattern of \(y\)-values at a regular interval of \(360^\circ\). (The period is \(360^\circ\))

\[y = \sin \theta\]

A cycle is a complete pattern repetition. This graph contains 2 cycles.
The period is \(2\pi\) (Horizontal length of one cycle)
Sine functions: 4 components

\[ y = \sin(x) \] is the parent function.

**Vertical Shift:**

\[ y = \sin x + 4 \]

\[ y = \sin x - 3 \]

\[ y = \sin \left( \frac{x}{2} \right) \]

If \( x = \frac{\pi}{2} \):

\[ y = \sin x \quad \sin \frac{\pi}{2} = 1 \]

\[ y = \sin x + 4 \quad \sin \frac{\pi}{2} + 4 = 5 \]

\[ y = \sin x - 3 \quad \sin \frac{\pi}{2} - 3 = 2 \]

**Amplitude:**

\[ y = 4\sin x \]

\[ y = \frac{1}{2} \sin x \]

<table>
<thead>
<tr>
<th>x</th>
<th>( \sin x )</th>
<th>( 4\sin x )</th>
<th>( \frac{1}{2} \sin x )</th>
<th>(-2\sin x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{\pi}{4} )</td>
<td>( \frac{\sqrt{2}}{2} )</td>
<td>( 2\sqrt{2} )</td>
<td>( \frac{\sqrt{2}}{4} )</td>
<td>(-\sqrt{2} )</td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>1</td>
<td>4</td>
<td>( \frac{1}{2} )</td>
<td>-2</td>
</tr>
<tr>
<td>( \frac{3\pi}{2} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

When the coefficient is negative, the output is reflected over the x-axis.

\[ y = \sin x \]

\[ y = -2\sin x \]
I. Sketch one cycle of the function $f(x) = 3\sin(x) + 2$

1) Sketch the parent function 
   $y = \sin(x)$

2) Increase ("stretch") the amplitude by a factor of 3 
   $y = 3\sin(x)$

3) Shift the graph up 2 units 
   $y = 3\sin(x) + 2$

Notice, $\sin \frac{3\pi}{2} = -1$  
$3\sin \frac{3\pi}{2} + 2 = -1$

Also, $\sin(x)$ and $3\sin(x)$ intersect at 0, $\pi$, and $2\pi$.

II. Sketch $y = -2\sin(x) + 5$

1) Parent Function 
   $y = \sin(x)$

2) "Stretch" and "Reflect"  
   $y = -2\sin(x)$

3) Vertical Shift 
   $y = -2\sin(x) + 5$
Sine functions: 4 components (continued)

**Horizontal ("Phase") Shift:**

**Question:** If \( \sin 90° = 1 \), then where does \( \sin (\varphi + 30°) = 1 \)?

**Answer:** \( \varphi = 60° \), because \( \sin (60° + 30°) = 1 \)

**Implication:** \( 90° \iff 60° \) (shift \( 30° \) to the left)

**Example I:** \( y = \sin (\varphi + 30°) \)

![Graph of \( y = \sin (\varphi + 30°) \)](image)

Note: The horizontal shift is the opposite direction of the sign.

**Example II:** \( y = \sin \left( x - \frac{\pi}{3} \right) \)

![Graph of \( y = \sin \left( x - \frac{\pi}{3} \right) \)](image)

\( y = \sin x \) crosses the x-axis at \( \pi \)

and

\( y = \sin \left( x - \frac{\pi}{3} \right) \) crosses at \( \frac{4\pi}{3} \)

\( y = \sin \left( x - \frac{\pi}{3} \right) = 0 \)

**y = AsinB(x - C) + D**

A: Amplitude (magnitude)
B: Period
C: Horizontal Shift
D: Vertical Shift

Compare the values of the parent function \( \sin \varphi \) to the values of \( \sin (\varphi + 30°) \)

(horizontal shift of \( 30° \) to the left)

<table>
<thead>
<tr>
<th>( \varphi )</th>
<th>( \sin \varphi )</th>
<th>( \sin (\varphi + 30°) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30</td>
<td>(-\frac{1}{2})</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>30</td>
<td>(\frac{1}{2})</td>
<td>(\frac{\sqrt{3}}{2})</td>
</tr>
<tr>
<td>60</td>
<td>(\frac{\sqrt{3}}{2})</td>
<td>1</td>
</tr>
<tr>
<td>90</td>
<td>1</td>
<td>(\frac{\sqrt{3}}{2})</td>
</tr>
</tbody>
</table>
**Sine Functions: Amplitude, Horizontal and Vertical Shifts Illustration**

**Sketch:** $y = 3\sin(\theta + 60^\circ) - 2$

1) **Parent function:**
   
   $y = \sin \theta$

2) **Amplitude** (A = 3):
   (*"Stretch by 3")
   
   $y = 3\sin \theta$

3) **Horizontal Shift** (C = $-60^\circ$):
   (*"Shift 60 degrees to the left")
   
   $y = 3\sin(\theta + 60^\circ)$

4) **Vertical Shift** (D = -2):
   (*"Shift down 2 units")
   
   $y = 3\sin(\theta + 60^\circ) - 2$

To check your sketch, plug a few points into the equation:

- $\theta = -60^\circ$: $3\sin(-60 + 60) - 2 = 3\sin(0) - 2 = -2$  \(\checkmark\)
- $\theta = 30^\circ$: $3\sin(30 + 60) - 2 = 3\sin(90) - 2 = 1$  \(\checkmark\)
- $\theta = 210^\circ$: $3\sin(210 + 60) - 2 = 3\sin(270) - 2 = -5$  \(\checkmark\)

**Period:** Horizontal distance required for a periodic function to complete one cycle.

\[ y = \sin Bx \quad \text{period} = \frac{2\pi}{B} \]

**Example:** $y = \sin 3x$

Period: $\frac{2\pi}{3}$

3 cycles between 0 and $2\pi$

For the parent function $y = \sin x$ (where $B = 1$) the period is $2\pi$

As $B$ increases, the period decreases.

In other words, it takes less time to complete one cycle.

And, for $y = \sin Bx$, as $B$ decreases, the period increases.

In other words, it takes more time to complete one cycle.
Defining Periodic Sine Graphs

Write equations to describe the graphs below:

Example I:

\[ y = A \sin(B(x - C)) + D \]

Use the above general equation as a guide:

D (vertical shift): The center that the sine wave is oscillating over is \( y = 2 \). Therefore, the vertical shift is up 2 units.

A (amplitude): The maximum y-value is 4, and the minimum y-value is 0 — a total span of 4 units. The amplitude is 1/2 of that amount: 2 units.

C (horizontal shift): Since \( y = 2 \) at 0 radians, there is no horizontal shift.

B (period): The horizontal distance of one cycle is \( 4\pi \). Since \( \frac{2\pi}{B} = \frac{4\pi}{B} \), \( B = \frac{1}{2} \)

\[ y = 2 \sin \left( \frac{1}{2} x \right) + 2 \]

Vertical Shift: The center of the sine wave is \( y = -3 \). So, \( D = -3 \)

Amplitude: The height from max to min is 2 units. 1/2 of the height is 1

Then, because the cycle goes down then up, it is negative. \( A = -1 \)

Horizontal Shift: At \( y = -3 \), the first positive value is 45°

(45° shift to the right)

\( C = 45° \)

Period: 360 degrees to complete one cycle

\[ B = 1 \]

\[ y = -\sin(\Theta - 45°) - 3 \]

Example II:

Example III:

Vertical Shift: None (center is \( y = 0 \))

Horizontal Shift: None (Cycle begins at \( (0, 0) \))

Amplitude (“Stretch”): Magnitude of 3

\( A = 3 \)

Period: \( \frac{2\pi}{B} = \) horizontal distance of one cycle

\[ \frac{2\pi}{B} = 8 \]

\[ B = \frac{\pi}{4} \]

\[ f(x) = 3 \sin \frac{\pi}{4}x \]

To check the equation, plug in a few points:

\( (2, 3) \): \( f(2) = 3 \sin \frac{\pi}{4}(2) = 3 \sin \frac{\pi}{2} = 3 \) \( \checkmark \)

\( (6, -3) \): \( f(6) = 3 \sin \frac{\pi}{4}(6) = 3 \sin \frac{3\pi}{2} = -3 \) \( \checkmark \)
Study Break: Math Snacks

Preferable to ordinary computer cookies...

Essential part of a well-rounded, academic diet.

Try with (f), or any beverage...

Also, look for Honey Graham Squares in the geometry section of your local store...

Sine Function Practice Test

(Next page)
Graph the following function: \[ 4\sin\left(x - \frac{\pi}{2}\right) + 3 \]

Identify the following sine functions:

1) 

2)
Graph the following Sine Functions. Then, use the given points to check your answers algebraically and graphically.

A) \( y = -5\sin x + 3 \)

Check: \( x = \frac{\pi}{4} \)
\[ x = \frac{3\pi}{2} \]

B) \( y = \sin(2x + \frac{\pi}{2}) \)

Check: \( x = \frac{\pi}{4} \)
\[ x = \frac{3\pi}{2} \]

C) \( y = 3|\sin \Theta| \)

Check: \( \Theta = 90^\circ \)
\[ \Theta = 270^\circ \]
A) For the graph \( y = \sin x \),

1) Domain:

2) Range:

3) \( x \)-intercepts:

4) \( y \)-intercept:

5) the graph of the function is positive on the intervals that correspond to which quadrants?

B) \( f(x) = a \sin(b(x - c)) + d \)

Write an equation where \( a < 0 \).

Write an equation where \( a > 0 \).
"The angle of elevation is 68 degrees. And, I’ve used 1890 feet of string. Look, we can estimate how high the kite is!"

"Benny, I think a storm is coming. Perhaps we should go inside?"

"Where is the key to the cabin?"

During his math assignment, Franklin makes another shocking discovery!

Practice Test SOLUTIONS

(Next page)
Graph the following function: \(4\sin(x - \frac{\pi}{2}) + 3\)

\[
y = 4\sin(x - \frac{\pi}{2}) + 3
\]

Amplitude (A) = 4
Period (B) = \(\frac{2\pi}{1}\)

1 cycle per \(2\pi\)

Horizontal Shift (C) = \(\frac{\pi}{2}\)
Vertical Shift (D) = 3

The vertical shift is up 3 units. (The center of the sine function will be \(y = 3\))

Since the amplitude is 4, the function is stretched by a factor of 4. Therefore, the maximum will be 7 (center 3 + 4) and the minimum will be -1 (center 3 - 4).

The shift is \(\frac{\pi}{2}\) to the right. So, we can begin the sketch at \(x = \frac{\pi}{2}\) And, the period is \(2\pi\)

Steps:

1) Identify the center
   - Max y-value: 5
   - Min y-value: -1
   - D = 2
   - Midpoint is 2, so vertical shift is +2

2) Find the amplitude
   - The span of the wave is 6 units (from peak to bottom). The amplitude is 1/2 that value \(\rightarrow\) 3
   - A = 3

3) Horizontal shift? None, because at 0°, \(y = 2\) (the center)
   - C = 0

4) Period? Since there is 1 cycle from 0° to 360°, the period is 360°.
   - B = 1

2) Identify the center:
   - Max value \(\rightarrow\) 5
   - Min value \(\rightarrow\) 3
   - \(y = 4\)

2) Amplitude (‘stretch’): the height of the wave is 2. And, the amplitude is 1/2 that value.
   - A = 1

3) Horizontal shift: \(\frac{3\pi}{4}\) is a starting point of the cycle.
   - C = \(\frac{3\pi}{4}\)

4) Period: the length of one cycle is \(\pi\).
   - (there are 2 cycles every \(2\pi\))

\[
y = \sin(2(x - \frac{3\pi}{4})) + 4
\]
A) \( y = -5 \sin x + 3 \)
- **Amplitude**: \( A = -5 \) (negative, function faces down)
- **Vertical Shift**: \( D = 3 \)
- **Horizontal Shift**: \( C = \text{none} \)
- **Period**: \( \frac{2\pi}{B} = 2\pi \)
- **Maximum**: 8, **Minimum**: -2

Check:
- \( x = \frac{\pi}{2} \)
  - \( y = -5 \sin \frac{\pi}{2} + 3 = -5(1) + 3 = 3 \) \( \checkmark \)

B) \( y = \sin(2x + \frac{\pi}{2}) \)
- **Amplitude**: \( A = 1 \)
- **Horizontal Shift**: \( C = -\frac{\pi}{4} \) (shift to the left)
- **Vertical Shift**: \( D = \text{none} \)
- **Period**: \( \frac{2\pi}{2} = \pi \)
- **Maximum**: 1, **Minimum**: -1

Check:
- \( x = \frac{\pi}{4} \)
  - \( y = \sin(2 \cdot \frac{\pi}{4} + \frac{\pi}{2}) = \sin(\frac{3\pi}{2}) = 0 \) \( \checkmark \)

C) \( y = 3|\sin \Theta| \)
- **Amplitude**: \( A = 3 \)
- **Period**: 180 degrees
- **Vertical Shift**: none
- **Horizontal Shift**: none
- **Maximum**: 3, **Minimum**: 0

Check:
- \( \Theta = 90^\circ \)
- \( \Theta = 270^\circ \)
  - \( y = 3|\sin 90^\circ| = 3(1) = 3 \) \( \checkmark \)
  - \( y = 3|\sin 270^\circ| = 3(-1) = -3 \) \( \checkmark \)
Characteristics of Sine Function

A) For the graph \( y = \sin x \),

1) Domain: all the \( x \)-values: all Real Numbers

2) Range: all the \( y \)-values: \([-1, 1]\) or \(-1 \leq y \leq 1\)

3) \( x \)-intercepts: The points where the function crosses the \( x \)-axis: \((\pi k, 0)\) where \( k \) is any integer

4) \( y \)-intercept: The point where the function crosses the \( y \)-axis: \((0, 0)\)

5) the graph of the function is positive on the intervals that correspond to which quadrants?

Sine is positive in quadrants I and II

B) \( f(x) = a \sin (b(x - c)) + d \)

Write an equation where \( a < 0 \).

If \( a < 0 \), then sin function will go down first... Here is one possibility...

\[ f(x) = -1 \sin 2(x - \frac{\pi}{4}) + 4 \]

Write an equation where \( a > 0 \).

If \( a > 0 \), then sin function will go up first.

Here is one possibility...

\[ f(x) = 1 \sin 2(x - \frac{3\pi}{4}) + 4 \]

(To check equations, test points on the graph)

amplitude is 1 ---- 'a' can be 1 or -1
vertical shift is up 4 ---- 'd' will be +4
period is \( \pi \) ---- 'b' will be 2

***the horizontal shift will correspond to where the graph starts
Thanks for visiting. (Hope it helped!)

To learn about Cosine Functions, visit the trig section at mathplane.com. We appreciate your support!

Lance...