# Word Problems: Algebra 1 and 2

Notes, Examples, and Practice Exercises (with detailed solutions)

After a long day, a teacher walks into a bar and orders a

10 ounce vodka and cranberry juice. The bartender prepare the drink.

After one sip, the teacher says, "It's too strong!"

The bartender explains, "I used 2 parts vodka and 3 parts cranberry."

"Can I get one with just 15% vodka?"

"Of course," the bartender answered. "A new drink? Or, can I pour out some of this one and add cranberry juice?"

The teacher said, "I hate to see a drink go to waste. Just pour out some of this one and add cranberry juice."

Question: How much of the drink should the bartender pour out and replace with pure cranberry juice to produce a 15% vodka mixture?

		1	I
	total amount	%	vodka quatity
Original drink	10	40%	4
pour out	(10 - x)	40%	.4(10 - x)
add	x	0%	0x
final drink	10	15%	1.5

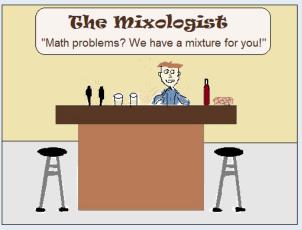
$$.4(10 - x) + 0x = 1.5$$

$$4 - .4x + 0 = 1.5$$

$$..4x = 2.5$$

x = 6.25 ounces

The bartender should pour out 6.25 ounces of the original drink, and then top it off with cranberry juice....



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Topics include translating words to operations, linear systems, mixture, work, and rate problems, area, and more.

#### Basic Algebra Word Problems

1) Write each expression and solve. ("find the number")

b) 8 plus the product of 21 and a number is 71

a) The sum of 14 and a number is 41.

	c) The difference between 104 and a number is 79
	d) five minus the product of two and a number is 12
	e) Twelve is one-eighth of a number
2)	Answer carefully.  a) A number is eight more than another number.  If the sum of the 2 numbers is 70, what is the <i>smaller</i> number?
	b) A number is 10 less than another number.  If their sum is 26, what is the <i>product of the two</i> numbers?
3)	Draw and label each figure. Then, solve.
	a) A triangle's height is half the length of its base. If the base is 8, what is the area?
	b) The length of a rectangle is twice the width.  If the width is 6, what is the perimeter?
	c) The length of a rectangle is four times its width. If the area is 100 square feet. What is the width?

Basic Algebra Word Problems (continued)

4)	50 cars and one locomotive weigh 4825 tons. (Each car is identical.)  If the locomotive weighs 225 tons, how much does each car weigh?
5)	A car's tank holds 16 gallons of gas. At 1 gallon, you stop at the gas station to refuel. If the car uses 3 gallons per hour, when will you refuel?
6)	A farmer has \$755. One cow costs \$500 and a flock of chicks costs \$20. If the farmer buys one cow, how many flocks of chicks can he afford to buy?
7)	If 9 friends take a trip, it will cost \$810. However, during the off-season, there is a discount. If it costs the 9 friends \$729, how much was the discount <i>per person</i> ? (Bonus: What is the <i>percentage</i> discount?)
8)	The sum of 3 consecutive integers is 234. What is the middle integer?
9)	The cable company charges \$59.95 per month plus \$4 for each movie rental. If your cable bill is \$111.95, how many movies did you rent?

- 1) Write each expression and solve. ("find the number")
  - a) The sum of 14 and a number is 41.

b) 8 plus the product of 21 and a number is 71

c) The difference between 104 and a number is 79

$$104 - y = 79$$
 $-104$ 

$$-y = -25$$
  $y = 25$ 

(also, if "the number" is larger than 104, then z - 104 = 79...

d) five minus the product of two and a number is 12

$$5 - (2 \cdot d) = 12$$

$$\frac{-2d}{-2} = \frac{7}{-2}$$

and z = 183

e) Twelve is one-eighth of a number

$$12 = \frac{1}{8} \text{ r}$$

- 2) Answer carefully.
  - a) A number is eight more than another number. If the sum of the 2 numbers is 70, what is the smaller number?

let n =the number then, n - 8 = another number

$$n + (n - 8) = 70$$
  $2n = 78$   $n = 39$   
+8 +8  $2$  and  $n - 8 = 31$ 

31 is the smaller number

d = -7/2 or -3.5

b) A number is 10 less than another number. If their sum is 26, what is the product of the two numbers?

let 
$$x = a$$
 number  
then,  $x + 10 =$  another number

$$x + (x + 10) = 26$$
  $2x = 16$ 

$$\frac{2x}{2} = \frac{16}{2}$$

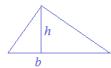
$$x = 8$$
$$x + 10 = 18$$

the product of 8 and 18 is 144

= 16

sq. units

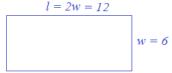
- 3) Draw and label each figure. Then, solve.
  - a) A triangle's height is half the length of its base. If the base is 8, what is the area?



area of triangle = 1/2(bh)

$$b = 8$$
  
 $h = 1/2 \cdot b = 4$  area = 1/2(8)(4)

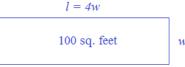
b) The length of a rectangle is twice the width. If the width is 6, what is the perimeter?



perimeter = 2l + 2w

$$= 2(12) + 2(6) = 36$$

c) The length of a rectangle is four times its width. If the area is 100 square feet. What is the width?



area = lw

100 sq. feet = 
$$4w \cdot w$$
  

$$\underbrace{100 \text{ sq. ft}}_{4} = \underbrace{4w^{2}}_{4}$$

$$\sqrt{25 \text{ sq. ft}} = \sqrt{w^{2}}$$
(length must be positive)

n = 77

4) 50 cars and one locomotive weigh 4825 tons. (Each car is identical.) If the locomotive weighs 225 tons, how much does each car weigh?

Total weight = loco + cars
let c = number of cars
Weight = 225 tons + c(wt)

$$4825 tons = 225 tons + 50 (weight of each car)$$

$$-225 tons - 225 tons$$

$$-225 tons - 225 tons$$

$$-225 tons - 225 tons$$

$$92 tons = weight of each car$$

5) A car's tank holds 16 gallons of gas. At 1 gallon, you stop at the gas station to refuel. If the car uses 3 gallons per hour, when will you refuel?

Fill-up = total gas - gas use 
$$1g = 16g - t(3g)$$
 
$$1g = 16g - t(3g)$$
 
$$-16g - 16g$$
 
$$1 gallon = 16 gallon - t hours(3 gallons/hour)$$
 
$$t = 5 hours$$

6) A farmer has \$755. One cow costs \$500 and a flock of chicks costs \$20. If the farmer buys one cow, how many flocks of chicks can he afford to buy?

7) If 9 friends take a trip, it will cost \$810. However, during the off-season, there is a discount. If it costs the 9 friends \$729, how much was the discount per person? (Bonus: What is the percentage discount?)

$$discount/person = \frac{total \ discount}{total \ persons} = \frac{\$810 - \$729}{9 \ people} = \frac{\$81}{9 \ people}$$
 discount: \\$9/person

\$810 cost for 9 people... then, cost per person: \$90... If the discount is \$9, then the percentage discount is 10%

8) The sum of 3 consecutive integers is 234. What is the middle integer?

let n = first integer 
$$n + (n + 1) + (n + 2) = 234$$
 so, the three integers are  $n + 2 =$  third (consecutive) integer  $n + 2 =$  third (consecutive) integer  $n + 3n + 3 = 234$  so, the three integers are  $n + 2 =$  third (consecutive) integer  $n + (n + 1) + (n + 2) = 234$  so, the three integers are  $n + 2 =$  third (consecutive) integer  $n + (n + 1) + (n + 2) = 234$  so, the three integers are  $n + 2 =$  third (consecutive) integer  $n + 3n + 3 = 234$  so  $n + 3n +$ 

9) The cable company charges \$59.95 per month plus \$4 for each movie rental. If your cable bill is \$111.95, how many movies did you rent?

cable bill = monthly fixed cost + \$4M where M = # of movies rented 
$$\frac{\$52}{\$4} = \frac{\$4M}{\$4}$$

$$\$111.95 = \$59.95 + \$4M$$

$$-\$59.95 - \$59.95$$

$$13 \text{ movies}$$

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## More examples and practice-→

#### Solving Word Problems

Basic Strategy:

- 1) "Let Statements" Establish Variables
- 2) Draw a Picture
- 3) Write Relevant Formulas
- 4) Solve (and check solutions)
- 5) Answer the question

Example 1: The product of two consecutive whole numbers is 72. What are the numbers?

- 1) Let X = first whole number Let X + 1 = second whole number
- 3)  $X \cdot (X+1) = 72$
- 4)  $X^2 + X = 72$   $X^2 + X - 72 = 0$  (X + 9)(X - 8) = 0X = -9, 8 Then, X + 1 = -8, 9
- 5) -9 and -8 are <u>not</u> whole numbers × 8 and 9 are consecutive, and 8 x 9 = 72 ×

Example 2: The length of a rectangular backyard is 3 times its width. If the area is 588 square feet, what is the length of the backyard?

Let L = length
 Let W = width

2)

588 square feet

L = 3W

W

- 3) LW = Area of rectangle (area formula) L = 3W (given) Area = 588 sq. feet (given)
- 4) LW = 588 3W(W) = 588  $3W^2 = 588$  $W^2 = 196$

W = -14, 14 (since length/width/area cannot be negative, we eliminated -14)

5) Width is 14 feet Length is 3 x 14 = 42 feet (14 x 42 = 588) Solving Algebra Word Problems (continued)

- Basic Strategy: 1) "Let Statements" Establish Variables
  - 2) Draw a Picture
  - 3) Write Relevant Formulas
  - 4) Solve (and check solutions)
  - 5) Answer the question

Example 3: A school play produced \$28,860 in revenue. Student tickets cost \$30; Adult tickets cost \$45. 736 total tickets were sold. How many adults attended?

- 1) Let S = # of student tickets Let A = # of adult tickets
- 3) Revenue = ticket x price 28,860 = S(30) + A(45)Tickets sold = Adult tickets + Student tickets 736 = A + S
- 4) (use substitution) 28,860 = 30S + 45AS = 736 - A28,860 = 30(736 - A) + 45A28,860 = 22,080 - 30A + 45A6.780 = 15AA = 452 (and, S = 284)

452 adults attended.

 $452 \times $45 = $20,340$  $284 \times $30 = $8,520$ total revenue: \$28,860 total tickets: 736

Questions for you to try!

- 1) Five times a number decreased by six is equal to the number squared. What is the number?
- 2) As I was flying over the farm, I noticed there were only rabbits and chickens. I could only spot 18 heads and 58 feet. How many rabbits were there? How many chickens? (assumption: rabbits have 4 feet and chickens have 2 feet)
- 3) Mr. Planter had a square garden in his backyard. He increased each side by 2 feet. After he enlarged the garden, it had an area of 196 square feet. What was the measure of the sides before he enlarged the garden? How much did the change in length increase the area?

(solutions on next page)

#### Solving Algebra Word Problems (continued)

 Five times a number decreased by six is equal to the number squared. What is the number?

Let X = the number

"5 times a number decreased by six"

5X - 6 $X^2$ 

 $5X - 6 = X^2$ 

$$X^2 - 5X + 6 = 0$$

$$(X-2)(X-3) = 0$$

$$X = 2 \text{ or } 3$$

$$5(3) \cdot 6 = 9 = 3^2$$

As I was flying over the farm, I noticed there were only rabbits and chickens. I could only spot 18 heads and 58 feet. How many rabbits were there? How many chickens? (assumption: rabbits have 4 feet and chickens have 2 feet)

"number squared"

Let R = # of rabbits

4R = # of rabbit feet

C = # of chickens

2C = # of chicken feet

$$R + C = 18$$
 (heads)

$$4R + 2C = 58$$
 (feet)

(use substitution) 
$$R = 18 - C$$

$$4(18 - C) + 2C = 58$$
  
 $72 - 4C + 2C = 58$   
 $14 = 2C$ 

Since R + C = 18,

$$R = 11$$

7 Chickens = 7 heads, 14 feet

11 Rabbits = 11 heads, 44 feet

18 heads; 58 feet

 Mr. Planter had a square garden in his backyard. He increased each side by 2 feet. After he enlarged the garden, it had an area of 196 square feet. What was the measure of the sides before he enlarged the garden? How much did the change in length increase the area?

Let S = side of original garden

$$S + 2 = side$$
 of enlarged garden

Area of square =  $S \times S = S^2$ 

Area of enlarged square =  $196 = (S + 2)^2$ 

$$\sqrt{\left(S+2\right)^2} = \sqrt{196}$$

$$S + 2 = 14$$
 (S cannot be -14, because length of sides cannot be negative)

S = 12

196 sq. feet

S

S+2

Original side is 12 feet

Enlarged side is 14 feet

Original area: 144 sq. ft Enlarged area: 196 sq. ft

Area increased by 52 square feet

#### Word Problems: Practice Quiz

- 1) A video store charges \$3 per rental. The store offers the following "frequent movie plan": \$96 annual fee and movies are \$1 per rental.
  - a) Should a person use the "frequent movie plan" if he rents 2 movies per month?
  - b) What is the "break even point"? (i.e. the cost of rentals w/o plan = cost of rentals with plan)
  - c) The video store is offering the movie plan for \$50. What is the new "break even point"?
- 2) An investor lost 10% last year. What percentage increase must be have this year to end up with his original investment?
- 3) You have 14 feet of wood.
  - a) How many different types of rectangular enclosures could you make? (Each side must be a whole number)
  - b) Which will create the largest area?
  - \*\*Bonus Questions
    - c) How many different types of isosceles triangles could you make? (each side must be a whole number)
    - d) Which will create the largest area?
- 4) Assume you run a 5K race in 20 minutes.
  - a) Express your speed in miles per hour.
  - b) Determine the average rate of each mile.
- 5) The length of a rectangle is twice the width. If the length is decreased by 1, and the width is increased by 3, the area is 72 square meters. Find the original length and width.

- A video store charges \$3 per rental. The store offers the following "frequent movie plan": \$96 annual fee and movies are \$1 per rental.
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  - c) The video store is offering the movie plan for \$50. What is the new "break even point"?

Let M = # of movies rented Cost w/o plan = 3MCost with plan = 96 + 1M

a) M = 24 movies per year Cost w/o plan = 3(24) = \$72Cost with plan = 96 + 1(24) = \$120

No, he should not use the movie plan.

b) Break even point: Cost w/o plan = Cost with plan3M = 96 + 1M

$$2M = 96$$

$$M = 48$$

The "break even point" is 48 movies in a year... If he rents 47, he should go w/o the plan. If he rents 49, he would save money with the plan...

Cost of 48 movies w/o plan: 3(48) = \$144Cost of 48 movies with plan: 96 + 1(48) = \$144

c) Cost w/o plan = 3MCost with new plan = 50 + 1M

> Break even point: 3M = 50 + 1M

$$2M = 50$$

$$M = 25 \text{ movies}$$

Cost of 25 movies w/o plan: \$75 Cost of 25 movies with plan: \$75

The "new break even point" is 25 movies. If he rents 24 or less, he should go w/o the new plan. If he rents 26 or more, he would save money using the new plan.

An investor lost 10% last year. What percentage increase must he have this year to end up with his original investment?

> Let X = original investmentThen.

amount after first year is: X - (.10X) = .90X

Now, let r = rate of return for 2nd year..

Therefore, we want X = .90X + r(.90X)

$$X = .90X (1 + r)$$

Example: original investment: \$100 lose 10% year one total: 100 - 10 = \$90gain 11.11% year two total: 90 + 9.99 ≦

\$100 \

$$X = .90X (1 + r)$$

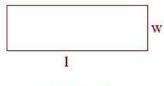
$$\frac{X}{.90X} = 1 + r$$

Therefore, the 2nd year must have an 11.11% return to end up with the original amount...

$$\frac{1}{.90} - 1 = r$$

$$r = .111$$

- 3) You have 14 feet of wood.
  - a) How many different types of rectangular enclosures could you make? (Each side must be a whole number)



$$21 + 2w = 14$$

$$1=1$$
  $w=6$   
 $1=2$   $w=5$   
 $1=3$   $w=4$   
(three types)

Note: 
$$3 \times 4 = 4 \times 3$$
  
 $2 \times 5 = 5 \times 2$   
 $1 \times 6 = 6 \times 1$ 

b) Which will create the largest area?

 $1 \times 6 = 6$  square feet

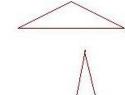
 $2 \times 5 = 10$  square feet

 $3 \times 4 = 12$  square feet

- \*\*Bonus Questions
  - c) How many different types of isosceles triangles could you make? (each side must be a whole number)



$$b + 2s = 14$$



$$s = 1$$
  $b = 12$  no  
 $s = 2$   $b = 10$  no  
 $s = 3$   $b = 8$  no  
 $s = 4$   $b = 6$  yes

$$s=4$$
  $b=6$  yes  
 $s=5$   $b=4$  yes  
 $s=6$   $b=2$  yes

s = 7 b = 0 no

3 different types (with whole numbers)

d) Which will create the largest area?



$$b = 6$$

$$h = \sqrt{16 \cdot 9} = \sqrt{7}$$

Area = 
$$3\sqrt{7}$$

**€**7.93

$$b = 4$$
  
 $h = \sqrt{25 - 4} = \sqrt{21}$ 

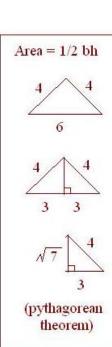
Area = 
$$2\sqrt{21}$$
  
 $\cong 9.16$ 

$$6 \bigwedge_{2}^{6}$$

$$b = 2$$
  
 $h = \sqrt{36 - 1} = \sqrt{35}$ 

Area = 
$$\sqrt{35}$$

$$\cong 5.91$$



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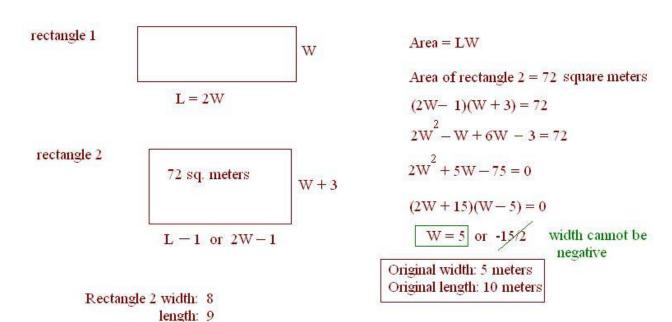
4) Assume you run a 5K race in 20 minutes.

Area = 72 \_\_\_\_

- a) Express your speed in miles per hour.
- b) Determine the average rate of each mile.
- a) 5K in 20 minutes = 15K in 1 hour 1 Kilometer ≅ .62 miles 15K ≅ 15(.62) ≅ 9.3 miles/hour
- Assuming 9.3 miles/hour..
  then, 9.3 miles/60 minutes..
  or,
  1 mile/6.45 minutes

  45 minutes 60 seconds
  1 minute

  average rate: 6 minute 27 second
  per mile
- 5) The length of a rectangle is twice the width. If the length is decreased by 1, and the width is increased by 3, the area is 72 square meters. Find the original length and width.



#### "Convergence Question"

Word Problems: Distance = rate x time

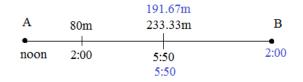
Example: Joe leaves town A at noon, going 40 miles per hour toward town B. Bill leaves town B at 2:00 pm, going 50 miles per hour toward town A.

If town A and town B are 425 miles apart, when and where will Joe and Bill meet?

At 12:00 Joe leaves... At 2:00, Joe has traveled 80 miles.. (d = rt)so, when Bill leaves at 2:00, they are 345 miles apart...

Since Joe is going 40 mph and Bill is going 50 mph, they are gaining 90 mph...

345 miles = 
$$90 \frac{\text{miles}}{\text{hour}} \times \text{(time)}$$
  
time =  $\frac{345}{90} = 3 \frac{5}{6} \text{ hours}$  or 3 hours, 50 minutes



"Chase Question"

Example: At 8:00am, Mike starts on a bike ride going 12 mph. Then, at 10:30am, John starts on the same road going 18 mph. What time will John catch up to Mike?

Approach 1: Matching the distances

distance = rate x time

Mike: d = 12 mph x (time)

John: d = 18 mph x (time -2.5 hours)

(substitution: set d = d)

$$12t = 18(t - 2.5)$$

12t = 18t - 45

-6t = -45

John rode for 5 hours t = 7.5

Mike rode for 7.5 hours

Approach 2: Using related speeds

(Establish Mike's lead)

Since John (the 2nd rider) goes 6 mph faster than Mike, the gap will close at 6mph...

John will spend 5 hours catching Mike... And, Mike will ride for 7.5 hours..

Can you answer this question?

Kelly leaves home at noon going 18 mph. At 4:00, Eric leaves home and rides at 30 mph. What time will Eric catch up to Kelly?

This is a "chase" question...

Kelly leaves 4 hours before Eric...

distance = rate x time

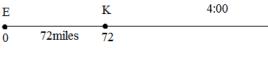
= 
$$18 \frac{\text{miles}}{\text{hour}}$$
 x 4 hours = 72 miles...

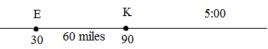
When Eric, leaves, he is 72 miles behind Kelly ....

Since he rides at 30 mph, Eric gains 12 miles per hour...

distance = rate x time 72 miles = 
$$12 \frac{\text{miles}}{\text{hour}}$$
 x time  
time = 6 hours

It takes Eric 6 hours, so he reaches Kelly at 10:00...





36 miles at 7:00

24 miles at 8:00

12 miles at 9:00

0 miles at 10:00

Suppose Charlie and Sam can paint a house together in 6 days. Separately, it takes Sam 5 days longer than Charlie to paint a house. How fast can each paint a house when working alone?

#### Step 1: Establish Variables and Formulas

Let C = Charlie's Rate 
$$S = Sam's Rate$$
  $rate * time = house$   $rate = \frac{1 \ house}{time} = \frac{1 \ house}{x \ days}$ 

#### Step 2: Construct Equations and Solve

C & S rate (together) = 
$$\frac{1 \text{ house}}{6 \text{ days}}$$
  $C = \frac{1 \text{ house}}{x \text{ days}}$   $S = \frac{1 \text{ house}}{(x+5) \text{ days}}$   $C$  (time) + S (time) = (C & S) time

$$\frac{1}{x} t + \frac{1}{(x+5)} t = \frac{1}{6} t$$
 (divide equation by t) 
$$\frac{1}{x} + \frac{1}{(x+5)} = \frac{1}{6}$$
 (find least common denominator on the left side)

$$\frac{(x+5)}{x(x+5)} + \frac{x}{x(x+5)} = \frac{1}{6}$$
 (add and consolidate)

$$\frac{2x+5}{x^2+5x} = \frac{1}{6}$$
 (cross multiply and solve for x)

$$x^2 + 5x = 6(2x + 5)$$

$$x^2 - 7x - 30 = 0$$

$$(x-10)(x+3)=0$$

$$x = 10$$
 or  $-3$   $\longrightarrow$  Charlie can paint a house in 10 days...

\*\*We eliminate the extraneous solution: -3 because Charlie does not paint a house in -3 days (unless he is destroying the house, the rate can't be negative!)

And, if x = 10, then x + 5 = 15 — Sam can paint a house in 15 days...

#### Step 3: Check your answers!

It takes 6 days for them to paint a house together.. Therefore,

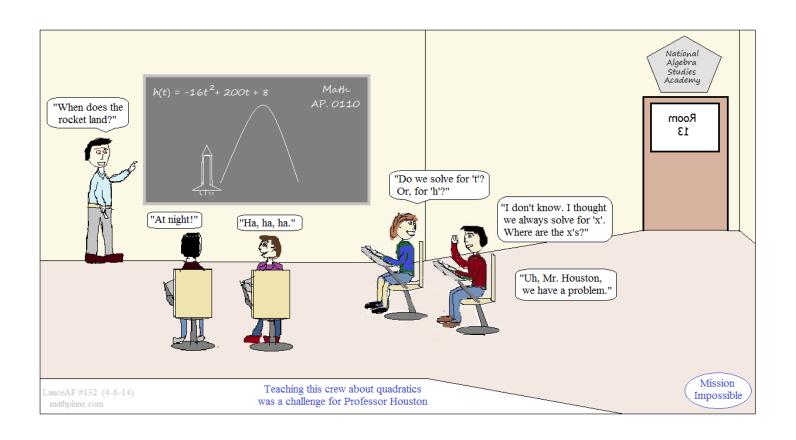
6 days (1 house/10 days) + 6 days (1 house/15 days) =

$$\frac{6 \text{ house}}{10} + \frac{6 \text{ house}}{15} = 1 \text{ house!}$$

## If A, B, C, D, and E are integers, then what are they?

$$A^2 + B^2 + C^2 = D^2 + E^2$$

## Answer on Next Page-→



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$$A^2 + B^2 + C^2 = D^2 + E^2$$

If A, B, C, D, and E are consecutive integers, what are they?

Let A = x

Then, since they are consecutive integers,

$$x^{2}+ (x+1)^{2} + (x+2)^{2} = (x+3)^{2} + (x+4)^{2}$$
  
A B C D E

Expand the terms ("FOIL")

$$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = x^2 + 6x + 9 + x^2 + 8x + 16$$

Collect the 'like' terms

$$x^2 - 8x - 20 = 0$$

Factor

$$(x-10)(x+2)=0$$

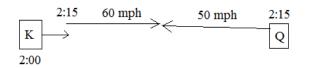
$$x = -2 \text{ or } 10$$

Example: At 2:00, George leaves Kingstown and drives toward Queensland at 60 miles/hour. At 2:15, Mary leaves Queensland and drives toward Kingstown at 50 miles/hour.

Rate problem

If Kingstown and Queensland are 240 miles apart, what time will they meet?

Step 1: Draw a sketch and describe the sequence



From 2:00 to 2:15, George drives 15 miles.

15 minutes • 
$$\frac{1 \text{ hour}}{60 \text{ minutes}}$$
 •  $\frac{60 \text{ miles}}{1 \text{ hour}}$  = 15 miles

#### Step 2: Solve

#### At 2:15, Mary and George are 225 miles apart.

225 miles = 
$$110 \frac{\text{miles}}{\text{hour}}$$
 (time)

 $distance = rate \cdot time$ 

since they are going toward each other, the rates are added. (i.e. they are approaching each other at 110 miles/hour)

time =  $2.045\overline{45}$  hours

= 2 hours 2 minutes 44 seconds

#### Step 3: Answer the question

At 2:00, George leaves and goes 15 miles. Then, at 2:15, Mary begins driving... Then, 2 hrs, 2 minutes, 44 seconds later, they meet.

4:17

Example: At the Nuthouse, nuts cost \$2.40 per pound and raisins cost \$3.20 per pound If you spend \$129.60 for 50 pounds of a mixture, how many pounds of each did you buy?

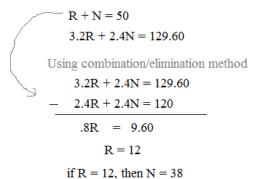
Mixture Problem

#### Step 1: Establish variables and relevant equations

Let 
$$R = \#$$
 of raisins (pounds)  
 $N = \#$  of nuts (pounds)
$$R + N = 50$$
then,  $3.2R = cost per pound of raisins (dollars)
 $2.4N = cost per pound of nuts (dollars)$ 

$$3.2R + 2.4N = 129.60$$$ 

#### Step 2: Solve



#### Step 3: Answer question and check

The 50 pound mixture consisted of 12 pounds of Raisins and 38 pounds of Nuts

12 pounds of raisins cost 12 x 3.2 = 38.40 38 pounds of nuts cost 38 x 2.4 = 91.20 total cost: 129.60 V

#### Algebra Word Problems

Example: A speeding car goes 80 miles per hour for 1 hour and 12 minutes.

Then, it exits the highway, and drives the last fifteen minutes at 30 miles per hour.

What was the average speed for the entire trip?

```
distance = (rate)(time)

80 mph x 1.2 hours = 96 miles

30 mph x .25 hours = 7.5 miles

total distance = distance at 80 mph + distance at 30 mph

total time = 1.2 hours + .25 hours

103.5 miles = (total rate)(1.45 hours)

approx. 71.38 miles per hour
```

Example: At the gas station, I paid \$42.51 to fill my car's gas tank.

The gas station charged \$2.63 per gallon.

Two days later, the price of gas fell to \$2.52 per gallon.

If I had waited (to fill up the gas tank), how much could I have saved?

First, let's find out the size of the gas tank.

$$42.51 = (tank) \frac{2.63}{1 \text{ gallon}}$$

tank = 16.16 gallons

Second, find the cost of the cheaper fill-up.

total = 
$$(16.16 \text{ gallon}) \frac{\$2.52}{1 \text{ gallon}}$$
  
total =  $\$40.72$ 

Finally, compare the costs...

interest - mixture problem

There are 2 components: percentage and amount that form the total portions/mixtures.

$$\begin{vmatrix} 4000 \\ 3.5\% \end{vmatrix} + \begin{vmatrix} x \\ 8\% \end{vmatrix} = \begin{vmatrix} 4000 + x \\ 5\% \end{vmatrix}$$

$$\begin{vmatrix} interest \\ earned \\ at 3.5\% \end{vmatrix} + \begin{vmatrix} interest \\ earned \\ at 8\% \end{vmatrix} = \begin{vmatrix} total \\ interest \\ earned \end{vmatrix}$$

$$4000(.035) + x(.08) = (4000 + x)(.05)$$

$$mixture 1 \qquad mixture 2 \qquad total together$$

$$140 + .08x = 200 + .05x$$

$$.03x = 60$$

$$x = 2000$$

Quick check: If he earns 8% on 2000 dollars, Albert gets 160 dollars in interest.

And, he earns 140 dollars from the 3.5% investment...

nent...

Total invested: \$6000 Total interest: \$300

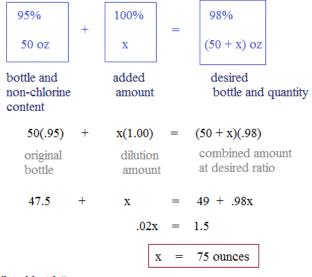
Investment rate: 5% \

Dilution Mixture problem

Example: A 50 ounce bottle of detergent contains 5% chlorine.

If you want to wash your clothes with a 2% chlorine content,
how much non-chlorine solution should you add?

There are 2 components: percentage and amount that form the portions/mixtures



Quick check: "diluted bottle"

amount: 50 + 75 = 125 ounces chlorine amount:  $.02 \times 125 = 2.5$  ounces...

(and, 5% of a 50 ounce bottle is  $.05 \times 50 = 2.5$  ounces...)

Example: A homeowner wishes to enclose/fence in 3 plots of land, 80 square feet each. If he has 88 feet of fence, what are the dimesions of each lot?

Area/Perimeter Problem

#### Step 1: Draw a picture and label parts

#### Step 2: Develop the equations

Let 
$$x = width$$
  
Let  $y = length$ 

There is 88 feet of fence, so

$$88 \text{ feet} = 4x + 2y$$

Each plot is 80 sq. feet, so

80 sq. feet = 
$$x \cdot (1/3)y$$

#### Step 3: Solve

Since we have 2 equations and 2 unknowns, we can solve!

88 feet = 
$$4x + 2y$$
 44 ft =  $2x + y$   
80 sq. feet =  $x \cdot (1/3)y$   $y = 44$  ft -  $2x$   
80 sq. ft =  $x \cdot (1/3)(44$  ft -  $2x$ )  
240 sq. ft =  $x(44$  ft -  $2x)$ 

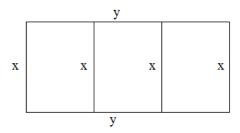
$$2x^{2} - 44x + 240 = 0$$
$$2(x - 10)(x - 12) = 0$$

$$x = 10 \text{ or } 12$$

#### length

width

|--|



Step 4: Answer the question and check

If 
$$x = 10$$
, then  $y = 24$  and  $(1/3)y = 8$ 

#### 10 x 8

area of each plot: 80 total fencing: 88

If 
$$x = 12$$
, then  $y = 20$   
 $12 \times 20/3$  and  $(1/3) = 20/3$ 

area of each plot:  $12 \times (20/3) = 80$  total fencing: 4(12) + 2(20) = 88

#### Algebra Word Problems: "Work Problem Example"

Example: During a one-hour workout at the gym, a woman cycles 4 miles and runs 2 1/2 miles. If she cycles 8 miles/hour faster than she runs, how fast is her running rate?

SOLUTION:

distance = (rate) x (time) 
$$r = \frac{d}{t}$$
  $t = \frac{d}{r}$ 

Step 1: Establish variables and formulas

running rate: 
$$r_{running} = \frac{x \text{ miles}}{\text{hour}}$$

cycling rate:  $r_{cycling} = \frac{(x+8) \text{ miles}}{\text{hour}}$   $d_{cycling} = 4 \text{ miles}$ 

Step 2: Set up equation and solve

substitution 
$$\frac{d_{running}}{r_{running}} + t_{cycling} = 1 \text{ hour}$$

$$\frac{d_{running}}{r_{running}} + \frac{d_{cycling}}{r_{cycling}} = 1 \text{ hour}$$

$$\frac{2.5}{x} + \frac{4}{x+8} = 1$$

$$2.5(x+8) + 4(x) = 1(x)(x+8)$$

$$2.5x + 20 + 4x = x^2 + 8x$$
simplify units
(the miles and hours will cancel) 
$$\frac{x \text{ miles}}{hour} + \frac{4 \text{ miles}}{(x+8) \text{ miles}} = 1 \text{ hour}$$

$$\frac{2.5 \text{ miles}}{x \text{ cycling}} + \frac{4 \text{ miles}}{(x+8) \text{ miles}} = 1 \text{ hour}$$

$$\frac{x^2 + 1.5x - 20 = 0}{use \text{ quadratic formula}}$$
\*\*since miles can't be negative,
-5.28 is extraneous and eliminated

time:  $t_{running} + t_{cycling} = 1 \text{ hour}$ 

distance:  $d_{running} = 2 \frac{1}{2}$  miles

Step 3: Answer question and check

A math center charges \$400 for a course, and they get 750 students. For every \$25 increase in price, they lose 30 students. What price would maximize revenue? What is the domain and range?

Quadratic vertex example (finding maximum)

(400 + 25x)(750 - 30x) = y

price quantity revenue

(where x is the number of \$25 increases)

method 1: use midpoint of zeros

$$400 + 25x = 0$$
  $x = -16$ 

$$750 - 30x = 0$$
  $x = 25$ 

axis of symmetry of 4.5

method 2: -b/2a

change to standard form and find (-b/2a, f(-b/2a))

$$300000 + 6750x - 750x^2$$

$$\frac{-6750}{2(-750)} = 4.5$$

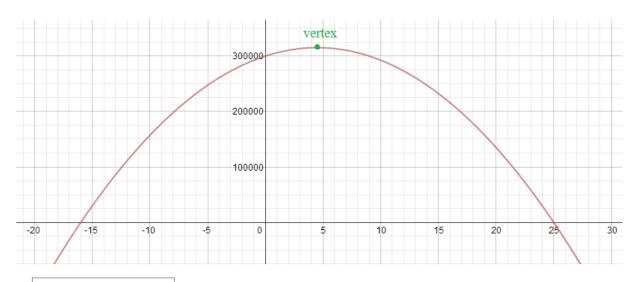
4.5 increases would lead to a price of

$$(400 + 25(4.5)) = 512.50$$

4.5 increases would lead to a quantity of

$$(750 + 30(4.5) = 615$$

revenue = 
$$$315,187.5$$



domain: -16 < x < 25

After 16 price decreases, the price would be 0... (Free items don't have revenue!) After 25 price increases, there will be no sales...

range:  $0 \le y \le 315,187.5$ 

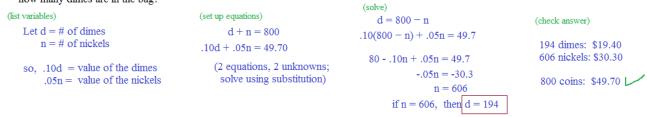
## Word Problems Practice Quiz 2

1)	A bag containing only dimes and nickels holds 800 coins. If the bag contains \$49.70, how many dimes are in the bag?
2)	The tortoise leaves home at 1:00pm, traveling east at 2 miles per hour. The hare leaves 10 hours later and travels east at 9 miles per hour. What time does the hare catch the toroise?
3)	Tom can paint a fence in 5 hours. Huck can paint a fence in 8 hours. If they work together, how long would it take for them to paint <i>three</i> fences?
4)	Caramel popcom costs \$1.50 per pound. Butter popcom costs \$1.10 per pound. A customer purchases 20 pounds of a mixture, paying \$27.70 How much of each flavor of popcom did he buy?
5	) Sal has 20 ounces of a 25% salt water solution. How many ounces of water must he add to dilute the solution to a 15% salt water solution?

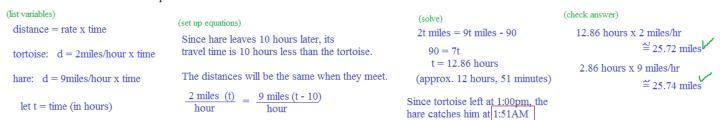
#### Word Problems Practice Quiz 2

#### SOLUTIONS

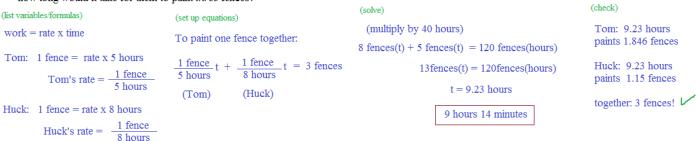
 A bag containing only dimes and nickels holds 800 coins. If the bag contains \$49.70, how many dimes are in the bag?



2) The tortoise leaves home at 1:00pm, traveling east at 2 miles per hour. The hare leaves 10 hours later and travels east at 9 miles per hour. What time does the hare catch the toroise?



3) Tom can paint a fence in 5 hours. Huck can paint a fence in 8 hours. If they work together, how long would it take for them to paint three fences?



4) Caramel popcorn costs \$1.50 per pound. Butter popcorn costs \$1.10 per pound. A customer purchases 20 pounds of a mixture, paying \$27.70 How much of each flavor of popcorn did he buy?

	rate	(pounds) amount	cost	The mixture rate = $\frac{\$27.70}{20}$ = \\$1.385 per pound	caramel: 14.25 pounds \$21.3
caramel	\$1.50	x	\$1.50x	(cost)	butter: 5.75 pounds
butter	\$1.10	(20 - x)	\$1.10(20 - x)	\$1.50x + \$1.10(20 - x) = \$27.70	\$6.325
mixture	?	20	\$27.70	\$1.50x + \$22 - \$1.10x = \$27.70 \$0.40x = \$5.70	20 total pounds; \$27.70 اسا
				x = 14.25 pounds	20 total pounds, \$27.70

5) Sal has 20 ounces of a 25% salt water solution. How many ounces of water must he add to dilute the solution to a 15% salt water solution?

	(ounces) amount	rate	salt
salt 25%	20	.25/ounce	5
water	X	0/ounce	0
salt 15%	20 + x	.15/ounce	3 + .15x

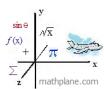
salt amount: 
$$5 + 0 = 3 + .15x$$

$$(25\% + \text{water} = 15\%)$$

$$2 = .15x$$

$$x = 13.333 \text{ ounces of water}$$

33.33 ounces of mixture --15% of 33.333 is
5 ounces of salt



The volume of a metal box is 30 cubic feet. If the length is 5 feet greater than the height and the width is 2 feet less than the height, what are the dimensions of the box?

#### Volume = length x width x height

30 ft 
$$^3 = (h + 5)$$
 feet •  $(h - 2)$  feet •  $(h)$  feet

$$h(h + 5)(h - 2) = 30$$

combine 1st and 2nd terms

$$(h^2 + 5h)(h - 2) = 30$$

FOIL

$$h^3 + 3h^2 - 10h = 30$$

Set equal to zero

$$h^3 + 3h^2 - 10h - 30 = 0$$

solve for h (by grouping)

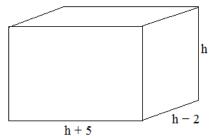
$$h^2(h+3) - 10(h+3) = 0$$

$$h + 3 = 0$$
  $h = -3$ 

$$(h+3)(h^2-10)=0$$

$$h^2 - 10 = 0$$
  $h = \sqrt{10}$ 

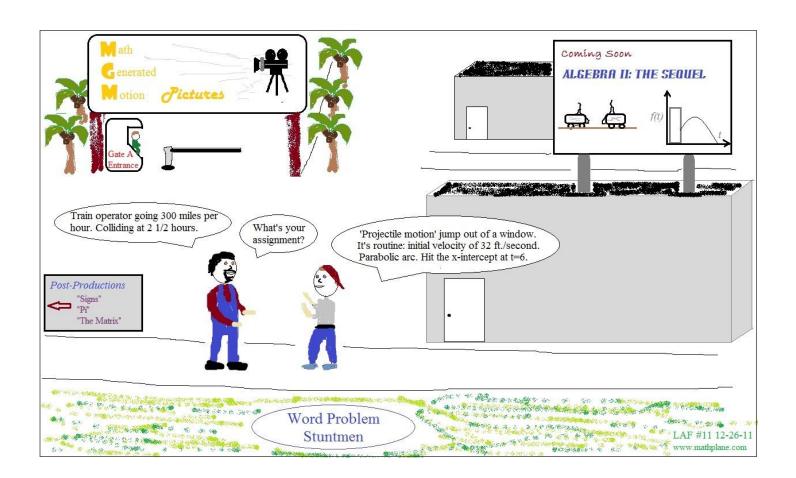
$$h = -\sqrt{10}$$



Since height cannot be negative, our solution is h =

the solution is  $h = \sqrt{10}$ (approximately 3.16)

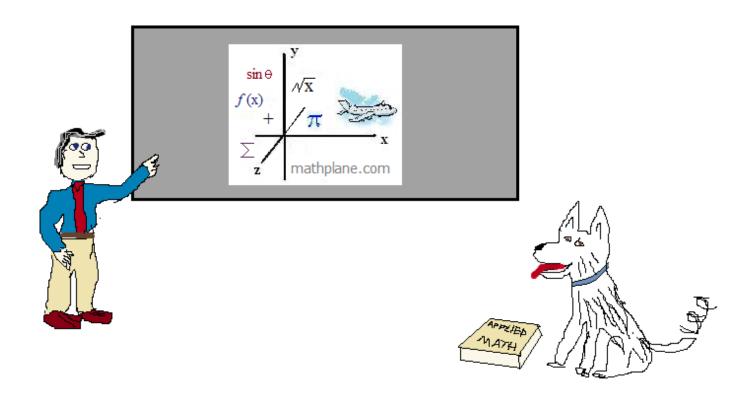
so, the dimensions of the box are approximately 8.16 x 1.16 x 3.16



### Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

#### Cheers



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Hidden Message

Answer the ten word problems below. Then, convert numbers to letters to reveal the hidden answer!

10) A farmer is raising chickens and cows. If he has 23 total animals which have a total of 74 legs,

Number/Letter Key

1 2 3 4 5 6 7 8 9 0 A C D E M O P R S W

Clue: Where you may find x and y in 2 squares?

how many chickens does he have?

1)	The sum of 2 consecutive even integers is 86. What is the difference of the <i>digits</i> of the smaller integer?	$\qquad \qquad \longrightarrow$
2)	If the area of a square backyard is 64 feet, how long is each side?	
3)	A tortoise leaves home at 1:00 and travels east at 4mph A hare leaves the same home at 4:00 and travels east at 6mph. How many hours will it take the hare to catch the tortoise?	$\longrightarrow$
4)	Find the number of a's and o's in the (green) directions above.	$\longrightarrow$
5)	Liz has an appointment at 3:15pm.  If she appears at 2:36pm, how many minutes early is she?	3
6)	Mark drinks a gallon of milk each week.  How many quarts does he consume each year?	$2 \boxed{8} \rightarrow$
7)	The lengths of a rectangle are twice the widths.  If the perimeter is 372 feet, what is the area of the rectangle?	88 ->
8	How many 175 pound people can fit into an elevator with capacity of 3/4 ton?	
9	A taxi cost \$1.25 per person plus \$0.20 per quarter mile.  If it cost you and your friend \$4.90 for a cab ride downtown, how many miles did you travel?	

Answer the ten word problems below. Then, convert numbers to letters to reveal the hidden answer!

Clue: Where you may find x and y in 2 squares? SOLUTIONS

Number/Letter Key

1 2 3 4 5 6 7 8 9 0 A C D E M O P R S W

1) The sum of 2 consecutive even integers is 86.

What is the difference of the digits of the smaller integer?

let 
$$x = 1$$
st integer  
  $x + 2 = 2$ nd integer

$$x + (x + 2) = 86$$
  
 $2x + 2 = 86$ 

$$x = 42$$
$$x + 2 = 44$$

the digits of the smaller integer are 4 and 2. Their difference is 2.



R

2) If the area of a square backyard is 64 feet, how long is each side?

$$\sqrt{64} = 8$$

3) A tortoise leaves home at 1:00 and travels east at 4mph... A hare leaves the same home at 4:00 and travels east at 6mph. How many hours will it take the hare to catch the tortoise?

When the hare leaves home, the tortoise will have traveled 12 miles (4mph x 3 hours).. At 6mph vs. 4mph, the hare will gain 2 miles/hour on the tortoise.. so, it will take 6 hours to catch

4) Find the number of a's and o's in the (green) directions above.

the tortoise.  

$$4mph(t + 3) = 6mph(t)$$

39 total minutes

early

9 total. Sas and oos (see above)

5) Liz has an appointment at 3:15pm. If she appears at 2:36pm, how many minutes early is she?

2:36 is 24 minutes before 3:00... And, 3:15 is 15 minutes after 3:00..

6) Mark drinks a gallon of milk each week. How many quarts does he consume each year?

Mark drinks 1 gallon/week... 1 gallon = 4 quarts, so he drinks 4 quarts/week...

Since there are 52 weeks/year, he drinks 208 lengths of a rectangle are twice the widths

7) The lengths of a rectangle are twice the widths.

If the perimeter is 372 feet, what is the area of the rectangle?

$$2w + 2w + w + w = 372 \text{ ft}$$
  
 $w = 62 \text{ ft}$   
 $2w = 1 = 124 \text{ ft}$ 

Area = 1w = 124 x 62 = 7688

8) How many 175 pound people can fit into an elevator with capacity of 3/4 ton?

$$3/4 \text{ ton} = 1500 \text{ pounds}$$

$$1500 \div 175 = 8.57$$

8 people can fit... (the 9th would be too much)

175p < 1500

9) A taxi cost \$1.25 per person plus \$0.20 per quarter mile.

If it cost you and your friend \$4.90 for a cab ride downtown, how many miles did you travel?

If you and your friend take a cab, the fixed cost is 2.50... therefore, you spent 2.40 for x quarter miles..

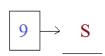
At \$0.20 per quarter mile, you traveled 12 quarter miles...

12 quarter miles = 3 miles...

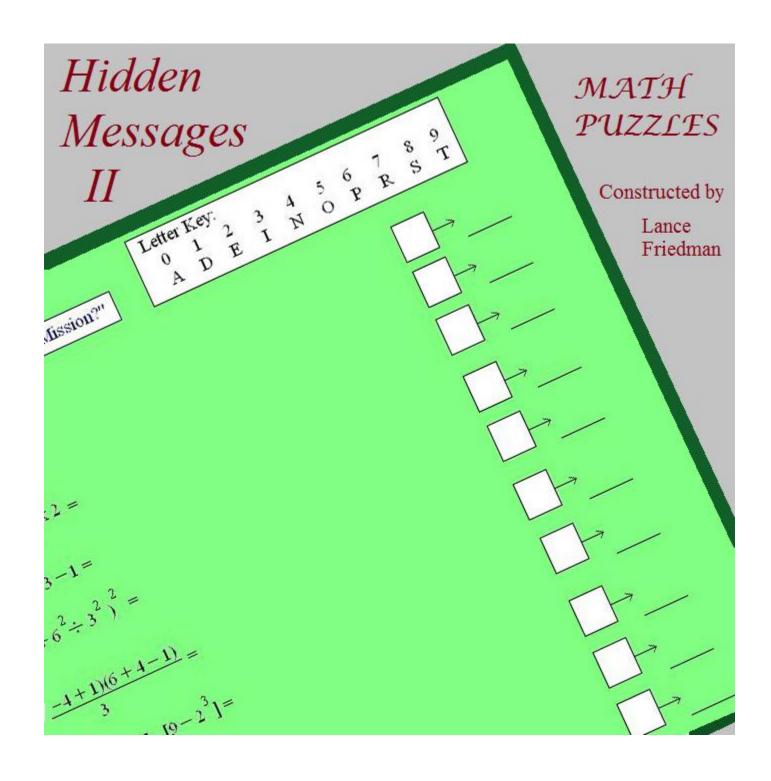
 $3 \rightarrow D$ 

R

10) A farmer is raising chickens and cows. If he has 23 total animals which have a total of 74 legs, how many chickens does he have?



You may find x and y in the square boxes of a crossword puzzle!



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