## Fractions

Notes, Examples, and Exercises (with solutions)


Topics include common denominator, mixed numbers, improper fractions, and more.


Example: $\quad \frac{1}{3}+\frac{1}{4}$
Step 1: Change to common denominators
since 3 and 4 are both factors of $12 \ldots$


Step 2: Add terms
$\frac{1}{3}$

$+$


再

Example: $\frac{3}{4}-\frac{2}{5}$
Step 1: Change to common denominators since 4 and 5 are both factors of $20 \ldots$


Example: Which fraction is larger? Convert the fractions (to common denominators)....

$$
\frac{5}{8} \text { or } \frac{3}{5} ? ?
$$

$$
\frac{5}{8}
$$

... then, compare...

$$
\begin{aligned}
& \frac{25}{40}>\frac{24}{40} \\
& \sqrt{V} \\
& \frac{5}{8} \text { is larger... }
\end{aligned}
$$



Math Problems: Fractions

## I. Addition

a) $\frac{3}{8}+\frac{1}{48}=$
b) $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=$
c) $\frac{3}{27}+\frac{2}{6}=$
d) $\frac{10}{25}+\frac{1}{5}=$
e) $3+\frac{2}{7}+\frac{3}{14}=$

## II. Subtraction

a) $\frac{1}{2}-\frac{1}{8}=$
b) $\frac{1}{8}-\frac{1}{2}=$
c) $\frac{5}{8}-\frac{27}{56}=$
d) $\frac{3}{4}-\frac{4}{9}=$
e) $1-\frac{47}{99}=$

Math Problems: Fractions

## III. Multiplication

a) $4 \cdot \frac{7}{8}=$
b) $\frac{-3}{8} \cdot \frac{5}{6}=$
c) $\frac{2}{9} \cdot \frac{3}{28}=$
d) $\frac{14}{17} \cdot \frac{85}{96}=$
e) $\frac{-47}{48} \cdot \frac{96}{94}=$

## IV. Division

a) $3 \div \frac{3}{4}=$
b) $\frac{2}{5} \div \frac{4}{15}=$
c) $0 \div \frac{9}{10}=$
d) $\frac{1}{6} \div \frac{11}{60}=$
e) $\frac{27}{63} \div \frac{1}{7}=$
Hidden Message

Clue: "What fractions can be?"

Solve and answer the thirteen questions below. Then, translate numbers to letters to reveal the hidden message.

## Letter Key:

$\begin{array}{cccccccccc}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 0 \\ \mathrm{~A} & \mathrm{I} & \mathrm{L} & \mathrm{M} & \mathrm{N} & \mathrm{O} & \mathrm{P} & \mathrm{R} & \mathrm{S} & \mathrm{T}\end{array}$

1) Find $X: \frac{3}{X}=\frac{1}{3}$
2) Express $3 \frac{1}{4}$ as an improper fraction.
3) If $\mathrm{m}=\mathrm{n} \neq 0$, then what is $\frac{\mathrm{m}}{\mathrm{n}}$ ?
$\qquad$

$\qquad$

4) $\frac{5}{7}+\frac{7}{5}=\frac{74}{\square 5}$
5) $2 / 3 \times 9 / 2=$
6) Write $\frac{29}{4}$ as a mixed number.
7) $\frac{1}{2}-\frac{1}{3}=\frac{1}{\square}$
8) What is the least common denominator of

$$
1, \frac{1}{2}, \frac{1}{4}, \text { and } \frac{1}{8} ?
$$


9) $\frac{(17-17)}{(1.232+323)}=$
10) $\frac{\text { number of 'a's in the instructions box above }}{\text { number of 'd's in the instructions box above }}=$

$\qquad$

11) What is the reciprocal of $\frac{1}{6}$ ?
12) $\frac{1}{.2}=$
13) $3 \div \frac{1}{3}=$
$\qquad$


1) $\frac{3}{4} \div \frac{7}{8} \times \frac{1}{5}+\frac{9}{10}$
2) $\left(\frac{1}{3}-\frac{1}{5}\right) \times \frac{1}{8} \div \frac{1}{4}$
3) $\frac{3}{7} \times \frac{8}{9}-\frac{1}{9} \div \frac{7}{10}$ (don't forget order of operations!)
4) $\left(\frac{3}{7}+\frac{5}{8}\right) \times \frac{28}{59} \div \frac{9}{10}$
5) $\left(\frac{1}{2}-\frac{3}{5}\right) \times \frac{7}{8}+\frac{11}{20}$
6) $\frac{4}{3}+\frac{5}{6}-\frac{1}{1.2} \div \frac{2}{9}$
$7 \frac{3}{4}$


## What is the perimeter of the quadrilateral?



## ANSWERS- $\rightarrow$

## I. Addition

a) $\frac{3}{8}+\frac{1}{48}=\frac{18}{48}+\frac{1}{48}=\frac{19}{48}$
b) $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=\frac{3}{6}+\frac{2}{6}+\frac{1}{6}=1$
c) $\frac{3}{27}+\frac{2}{6}=\frac{1}{9}+\frac{1}{3}=\frac{1}{9}+\frac{3}{9}=\frac{4}{9}$
d) $\frac{10}{25}+\frac{1}{5}=\frac{2}{5}+\frac{1}{5}=\frac{3}{5}$
e) $3+\frac{2}{7}+\frac{3}{14}=3+\frac{4}{14}+\frac{3}{14}=3 \frac{1}{2}$
II. Subtraction
a) $\frac{1}{2}-\frac{1}{8}=\frac{4}{8}-\frac{1}{8}=\frac{3}{8}$
b) $\frac{1}{8}-\frac{1}{2}=\frac{1}{8}-\frac{4}{8}=\frac{-3}{8}$
c) $\frac{5}{8}-\frac{27}{56}=\frac{35}{56}-\frac{27}{56}=\frac{8}{56}=\frac{1}{7}$
d) $\frac{3}{4}-\frac{4}{9}=\frac{27}{36}-\frac{16}{36}=\frac{11}{36}$
e) $1-\frac{47}{99}=\frac{99}{99}-\frac{47}{99}=\frac{52}{99}$

## III. Multiplication

a) $4 \cdot \frac{7}{8}=\frac{4}{1} \cdot \frac{7}{8}=\frac{28}{8}=\frac{7}{2}$
b) $\frac{-3}{8} \cdot \frac{5}{6}=\frac{-15}{48}=\frac{-5}{16}$
c) $\frac{1}{9} \cdot \frac{3}{28}=\frac{1}{42}$
d) $\frac{14}{17} \cdot \frac{85^{5}}{96}=\frac{14}{1} \cdot \frac{5}{96}=\frac{35}{48}$
e) $\frac{-17}{48} \cdot \frac{96^{2}}{94}=-1$
IV. Division
(to divide fractions, simply
"invert and multiply" )
a) $3 \div \frac{3}{4}=\frac{3}{1} \cdot \frac{4}{3}=4$
b) $\frac{2}{5} \div \frac{4}{15}=\frac{1}{2} \cdot \frac{15^{3}}{4}=\frac{3}{2}$
c) $0 \div \frac{9}{10}=0$
d) $\frac{1}{6} \div \frac{11}{60}=\frac{1}{6} \cdot \frac{60}{11}=\frac{10}{11}$
e) $\frac{27}{63} \div \frac{1}{7}=\frac{3}{7} \cdot \frac{7}{1}=3$

## Hidden Message

Clue: "What fractions can be?"

Solve and answer the thirteen questions below. Then, transate numbers to letters to reveal the hidden messgge.

## Letter Key:

$\begin{array}{cccccccccc}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 0 \\ \mathrm{~A} & \mathrm{I} & \mathrm{L} & \mathrm{M} & \mathrm{N} & \mathrm{O} & \mathrm{P} & \mathrm{R} & \mathrm{S} & \mathrm{T}\end{array}$

## SOLUTIONS

1) Find $X: \frac{3}{X}=\frac{1}{3}$ (cross multiply) $3 \cdot 3=1 \cdot \mathrm{X} \quad \mathrm{X}=9$
2) Express $3 \frac{1}{4}$ as an improper fraction. $(4 \times 3)+1=13 \quad \frac{13}{4}$
3) If $\mathrm{m}=\mathrm{n} \neq 0$, then what is $\frac{\mathrm{m}}{\mathrm{n}}$ ? $\quad \frac{\mathrm{m}}{\mathrm{n}}=1$ if $\mathrm{m}=\mathrm{n}$
4) $\frac{5}{7}+\frac{7}{5}=\frac{74}{35} \quad \frac{25}{35}+\frac{49}{35}=\frac{74}{35}$
5) $2 / 3 \times 9 / 2=\quad \frac{2}{3} \times \frac{9}{2}=9 / 3=3$
6) Write $\frac{29}{4}$ as a mixed number. $\begin{aligned} & 29 \text { divided by } 4 \text { is } 7 \text { with a } \\ & \text { remainder of } 1 \ldots\end{aligned}$

7 and $1 / 4$
7) $\frac{1}{2}-\frac{1}{3}=\frac{1}{6} \quad \frac{3}{6}-\frac{2}{6}=\frac{1}{6}$
8) What is the least common denominator of

$$
1, \frac{1}{2}, \frac{1}{4} \text {, and } \frac{1}{8} ? \quad \begin{array}{|cccc}
8 & \frac{4}{8} & 2 & \frac{1}{8} \\
\hline
\end{array}
$$

9) $\frac{(17-17)}{(1.232+323)}=\frac{0}{?}=0$
10) $\frac{\text { number of 'a's in the instructions box above }}{\text { number of 'd's in the instructions box above }}=\frac{6}{3}=2$
(a's and d's are outlined above)
11) What is the reciprocal of $\frac{1}{6} ? \frac{1}{\frac{1}{6}}=1 \times \frac{6}{1}=6$
12) $\frac{1}{.2}=\quad($ change .2 into $1 / 5)$

$$
1 /(1 / 5)=5
$$

13) $3 \div \frac{1}{3}=$ (invert and multiply)

$$
3 \times \frac{3}{1}=9
$$


$7 \frac{1}{4} \rightarrow \mathrm{P}$



1) $\frac{3}{4} \div \frac{7}{8} \times \frac{1}{5}+\frac{9}{10}$

$$
\frac{3}{4} \times \frac{8^{2}}{7}
$$

$$
\frac{6}{7} \times \frac{1}{5} \leadsto \frac{6}{35}+\frac{9}{10} \longmapsto \frac{12}{70}+\frac{63}{70}=\frac{75}{70}=\frac{15}{14}
$$

2) $\left(\frac{1}{3}-\frac{1}{5}\right) \times \frac{1}{8} \div \frac{1}{4}$

$$
\begin{aligned}
& \frac{5}{15}-\frac{3}{15} \\
& \quad \frac{2}{15} \times \frac{1}{8} \leadsto \frac{1}{60} \times \frac{4}{1}=\frac{1}{15}
\end{aligned}
$$

3) $\frac{3}{7} \times \frac{8}{9}-\frac{1}{9} \div \frac{7}{10}$ (don't forget order of operations!)

$$
\frac{3}{7} \times \frac{8}{9}-\frac{1}{9} \times \frac{10}{7}
$$

$$
\frac{24}{63}-\frac{10}{63}=\frac{14}{63}=\frac{2}{9}
$$

4) $\left(\frac{3}{7}+\frac{5}{8}\right) \times \frac{28}{59} \div \frac{9}{10}$

$$
\frac{24}{56}+\frac{35}{56}
$$

$\frac{1}{29}_{26}^{56} \times \frac{28}{59} 1 \Rightarrow \frac{1}{2} \times \frac{10}{9}=\frac{5}{9}$
5) $\left(\frac{1}{2}-\frac{3}{5}\right) \times \frac{7}{8}+\frac{11}{20}$

$$
\begin{aligned}
& \frac{5}{10}-\frac{6}{10} \\
& \quad\left(-\frac{1}{10}\right) \times \frac{7}{8} \leadsto-\frac{7}{80}+\frac{44}{80}=\frac{37}{80}
\end{aligned}
$$

6) $\frac{4}{3}+\frac{5}{6}-\frac{1}{1.2} \div \frac{2}{9}$

ORDER OF OPERATIONS!!
$4^{\frac{1}{12}} \times \frac{\phi}{2}^{3}$
$\frac{4}{3}+\frac{5}{6}-\frac{3}{8} \rightleftharpoons \frac{32}{24}+\frac{20}{24}-\frac{9}{24}=\frac{43}{24}$

Mixed Number (or, Mixed Fraction)

What is it? A number written as a whole number combined with a proper fraction

Examples:
$5 \frac{2}{3}$
Yes.
5 is whole number $\frac{2}{3}$ is a proper fraction
$6 \frac{7}{4}$
No
6 is a whole number, but $7 / 4$ is not a proper fraction...
$-3 \frac{23}{24}$
Not exactly
-3 is not a whole number..
But, written as
$(-1) \cdot 3 \frac{23}{24} \quad 3$ is a whole number and $\frac{23}{24}$
is a proper fraction...

The value of a mixed number $=$ whole + fraction

Examples: $\quad 3 \frac{5}{8}=3+\frac{5}{8}$

$$
\int_{\text {hole }}^{\substack{\text { proper } \\ \text { fraction }}}
$$

## Improper Fraction

What is it? A fraction where the numerator is greater than the denominator.
Examples:


No
$5 \ngtr 9$
(this is a proper fraction)

????
rewritten as $(-1) \frac{8}{5}$
$8>5$, so this is an improper fraction

Converting Improper Fraction into Mixed Number

Example: $\frac{23}{4}$

How many times can 4 go into 23 ?
$23 \div 4=5$ with remainder 3

Notice, converting improper fraction into mixed number is simply "the numerator divided by the denominator", where the remainder is expressed as a fraction...

## Converting Mixed Number into Improper Fraction

Example: $7 \frac{2}{5}$

$$
\text { Remember, } 7 \frac{2}{5}=7+\frac{2}{5} \underset{\substack{\text { common } \\ \text { denominator }}}{\sim} \frac{35}{5}+\frac{2}{5}=\frac{37}{5}
$$

***SHORTCUT: "Multiply, then add.. Over the denominator.." Or, "Denominator times whole plus numerator, over the denominator"

$$
\begin{gathered}
7 \\
35+2=37 \\
x^{2} \\
\\
3 \times 5=35 \\
\frac{2}{5}
\end{gathered} \quad \text { whole }-7 \frac{2}{5} \quad \frac{5 \times 7+2}{5}=\frac{37}{5}
$$

Mixed Number Operations
Multiplication: "Convert to improper fractions" then, "multiply and simplify"


Division: "Convert to improper fractions" then, "invert \& multiply and simplify"

$$
\begin{array}{cl}
\text { Examples: } & 7 \frac{2}{3} \div 2 \frac{1}{4} \\
& \frac{23}{3} \div \frac{9}{4} \\
& \frac{23}{3} \times \frac{4}{9} \\
& \text { or } 3 \frac{11}{27}
\end{array}
$$

$$
4 \frac{1}{3} \div 9 \frac{3}{4}
$$

$$
\frac{13}{3} \div \frac{39}{4}
$$

$$
\frac{13}{3} \times \frac{4}{39} \text { invert and multiply (cross cancel) }
$$

$$
\frac{1}{3} \times \frac{4}{3}=\frac{4}{9}
$$

Addition: "Add the individual parts" then, "simplify"
OR "Convert to improper fractions" then, "add and simplify"

$$
\begin{array}{lll}
\text { Example: } & 6 \frac{2}{7}+5 \frac{5}{6} & 6 \frac{2}{7}+5 \frac{5}{6} \\
6+\frac{2}{7}+5+\frac{5}{6} & \frac{44}{7}+\frac{35}{6} \\
11+\frac{47}{42}=12 \frac{5}{42} & \frac{264}{42}+\frac{245}{42}=\frac{509}{42}=12 \frac{5}{42} \\
/ &
\end{array}
$$

NOTE: this second approach involves larger numbers, so it can be a bit more difficult..

Subtraction and "borrowing" (Sometimes the mixed number being subtracted has a larger proper fraction ... )
Example: $10 \frac{1}{8}-7 \frac{1}{4}$

Approach 1: subtract whole numbers,
subtract fractions, then add results...
$10-7=3 \quad$ difference of whole numbers
$\frac{1}{8}-\frac{1}{4}=-\frac{1}{8} \quad \begin{aligned} & \text { difference of the } \\ & \text { fractions }\end{aligned}$

$$
3+\left(-\frac{1}{8}\right)=2 \frac{7}{8}
$$

Approach 2: Convert to improper fractions

$$
\begin{aligned}
\frac{81}{8}-\frac{29}{4} & = \\
\frac{81}{8}-\frac{58}{8} & =\frac{23}{8} \\
& =2 \frac{7}{8}
\end{aligned}
$$

## Approach 3: "Borrowing"

$$
10 \frac{1}{8}-7 \frac{2}{8}
$$

since $2 / 8>1 / 8$, we'll 'borrow' from the whole number 10 .. (i.e. change 10 into $9+8 / 8$ )

$$
9 \frac{9}{8}-7 \frac{2}{8}
$$

$$
\begin{aligned}
& \text { subtract } \\
& \text { whole numbers }
\end{aligned}
$$

$$
\begin{aligned}
9-7=2 & \quad \frac{9}{8}-\frac{2}{8}=\frac{7}{8} \\
& =2 \frac{7}{8}
\end{aligned}
$$

Example: $\quad 3 \frac{2}{3}+5 \frac{3}{4}=$

Method 1: Add whole numbers Add fractions Combine

Step 1: Add the whole numbers

Step 2: Common Denominators?

Step 3: Add the fractions

Step 4: Change improper fraction?

Step 5: Combine

$$
3+5=8
$$

$$
\frac{2}{3} \quad \frac{3}{4}
$$

$$
\downarrow \quad \downarrow \quad 12 \text { is the least common denominator }
$$

$$
\begin{array}{rr}
\frac{8}{12}+\frac{9}{12}=\frac{17}{12} & 1 \mathrm{R} 5 \\
& \downarrow
\end{array}
$$

$$
8+1 \frac{5}{12}
$$

$$
=9 \frac{5}{12}
$$

Example: $\quad 5 \frac{1}{7}-3 \frac{1}{5}=$

Method 2:
Convert to improper fractions Add/Subtract Convert back to mixed fraction

Step 1: Convert to improper fractions

$$
5 \frac{1}{7}-3 \frac{1}{5}=
$$

$$
7 \times 5+1=36 \quad 5 \times 3+1=16
$$

Step 2: Common Denominators?

Step 3: Subtract

$$
\begin{aligned}
& \frac{36}{7}-\frac{16}{5}= \\
& \downarrow
\end{aligned}
$$

$$
\frac{180}{35}-\frac{112}{35}=\frac{68}{35}
$$

Step 4: Change improper fraction

$$
1 \frac{33}{35}
$$

$$
\begin{gathered}
35{ }^{1} \begin{array}{c}
1 \mathrm{R} 33 \\
-\frac{35}{33}
\end{array}
\end{gathered}
$$

1) Circle the mixed numbers. Underline the improper fractions.
$\frac{7}{2} \quad 5 \frac{6}{53} \quad 5 \frac{10}{7} \quad \frac{3}{11} \quad-4 \frac{11}{12}$
"fourteen fourths" "seven and five sixths"
"five sevenths"
2) Convert to mixed numbers

$-\frac{71}{6}=$
$\frac{100}{8}=$
"sixteen fifths" equals
3) Convert to improper fractions
$3 \frac{1}{3}=$
$-7 \frac{5}{8}=$
$2 \frac{21}{23}=$
"four and five ninths" equals
"eight and a third" equals
4) Addition/Subtraction
$6 \frac{1}{2}+11 \frac{1}{2}=$
$5 \frac{1}{6}+8 \frac{1}{3}=$
"fourteen and a half plus six and an eighth" equals
$10 \frac{1}{8}-7 \frac{1}{4}=$
5) Multiplication/Division

$$
2 \frac{1}{5} \times 5 \frac{1}{2}=\quad 7 \frac{2}{9} \cdot 3 \frac{3}{5}=\quad 9 \frac{1}{7} \div 4 \frac{2}{3}=\quad \text { "four and a half times seven and a fourth" }
$$

1) Circle the mixed numbers. Underline the improper fractions.
$\frac{7}{2} \quad 5 \frac{6}{53}$
(not exactly)

Convert to mixed numbers

$$
\frac{7}{4}=1 \frac{3}{4} \quad \frac{21}{5}=4 \frac{1}{5}
$$

$$
-\frac{71}{6}=-11 \frac{5}{6}
$$



100 divided by 8 equals 12 with a remainder of 4
3) Convert to improper fractions

4) Addition/Subtraction


$$
\begin{aligned}
& 2 \frac{1}{5} \times 5 \frac{1}{2}= \\
& \frac{11}{5} \times \frac{11}{2}=\frac{121}{10} \\
& \text { or } \\
& 12 \frac{1}{10}
\end{aligned}
$$

$5 \frac{1}{6}+8 \frac{1}{3}=$
$5+8=13$
$\frac{1}{6}+\frac{1}{3}=\frac{3}{6}=\frac{1}{2}$

OR, using improper fractions....
$\frac{31}{6}+\frac{25}{3}=\frac{81}{6}$

$$
=13 \frac{3}{6}=13 \frac{1}{2}
$$

$$
7 \frac{2}{9} \cdot 3 \frac{3}{5}=
$$

$7 \frac{2}{9} \cdot 3 \frac{3}{5}=$

$$
9 \frac{1}{7} \div 4 \frac{2}{3}=
$$

$\frac{65}{9} \cdot \frac{18}{5}=$
${ }_{1}^{13} \frac{65}{9} \cdot \frac{18}{5}{ }_{1}^{2}=26$
("cross cancelling")
$2.2 \times 5.5=12.1$
$2.2 \times 5.5=12.1$

$$
\begin{cases}\frac{64}{7} \div \frac{14}{3} & 4 \frac{1}{2} \cdot 7 \frac{1}{4} \\ \frac{64}{7} \times \frac{3}{14} & \frac{9}{2} \cdot \frac{29}{4}=\frac{261}{8}\end{cases}
$$

$$
\frac{32}{7} \times \frac{3}{7}=\frac{96}{49}
$$

$$
32 \frac{5}{8}
$$

$1 \frac{47}{49}$
"four and a half times seven and a fourth"

## Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, then let us know Cheers


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## What is the perimeter of the quadrilateral?

a) $26 \frac{1}{4}$
b) $26 \frac{7}{17}$
c) $26 \frac{7}{12}$
d) 27
e) $28 \frac{1}{4}$
$7 \frac{3}{4}$


$$
\begin{aligned}
& 5+7+6+8=26 \\
& \frac{6}{12}+\frac{9}{12}+\frac{8}{12}+\frac{4}{12}=\frac{27}{12}=2 \frac{3}{12} \\
& 26+2 \frac{3}{12}=28 \frac{3}{12}
\end{aligned}
$$

