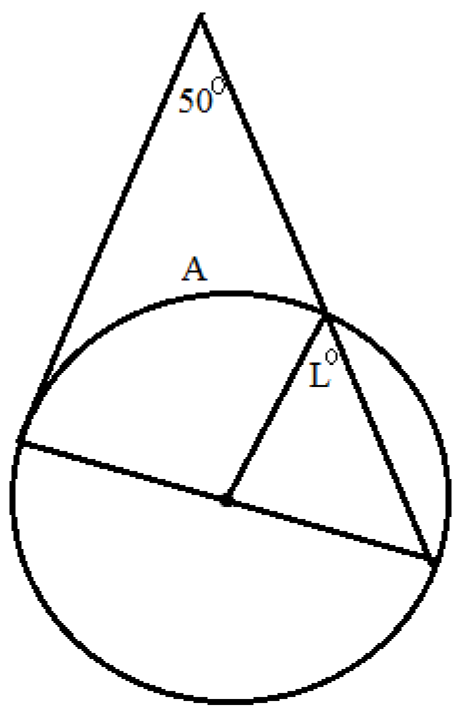
Geometry Circles Review (Honors)

Questions and Answers



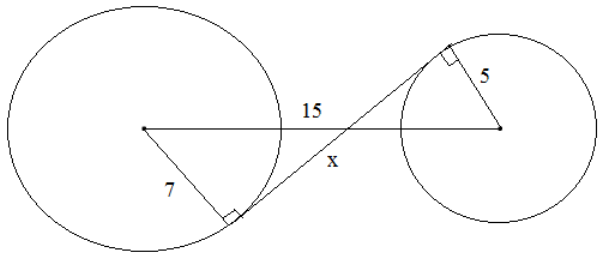
Topics include arc length, common internal tangent length, secant-tangent theorems, chords, area, standard form of circle, and more.

Examples: Two coplaner circles -- with centers 15 units apart -- have radii 7 and 5.

A) Find the length of the common internal tangent.

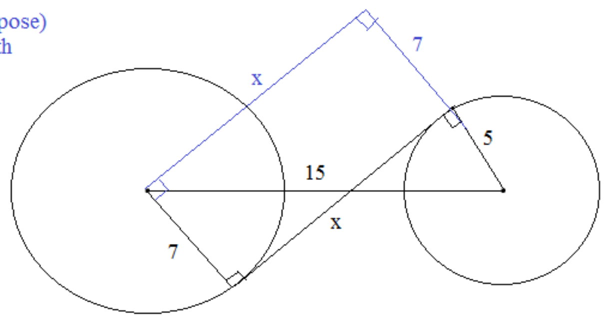
Step 1: Draw a sketch and label

Tangents and radii form right angles...

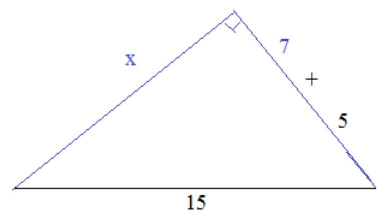


Step 2: Duplicate (transpose) the missing length

By drawing a parallel segment of equal length, a rectangle is created!



Step 3: Extract and solve the right triangle



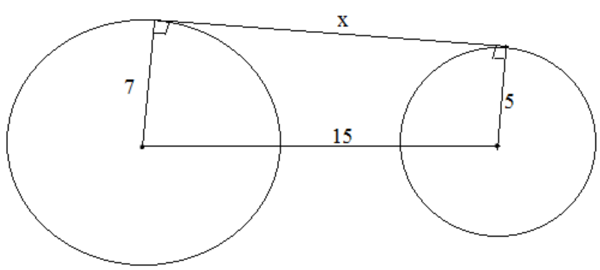
Pythagorean Theorem:

x = 9

B) Find the length of the common external tangent.

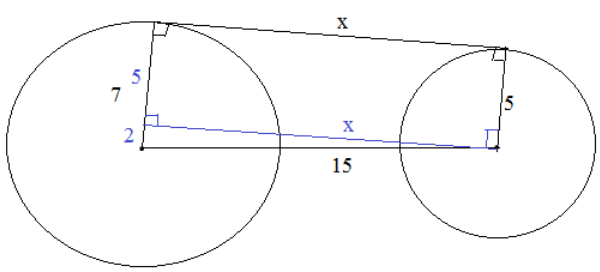
Step 1: Draw a sketch and label

Tangents and radii form right angles...

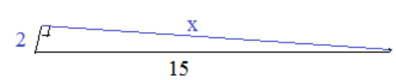


Step 2: Duplicate (transpose) the missing length

The segment creates a rectangle and right triangle...



Step 3: Extract and solve the right triangle



Pythagorean Theorem:

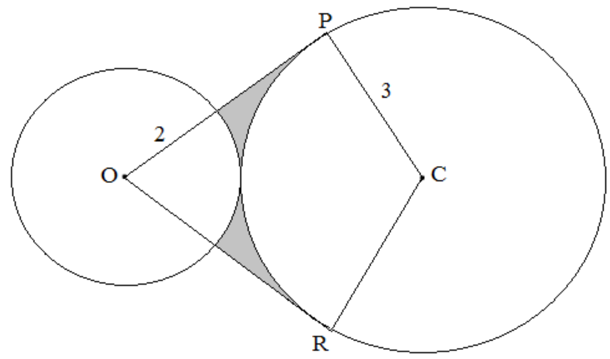
$x = \sqrt{221} = 14.87$

Example: OP and OR are external tangents

a) find $m\angle POR$

$m\angle PCR$

b) find the shaded area

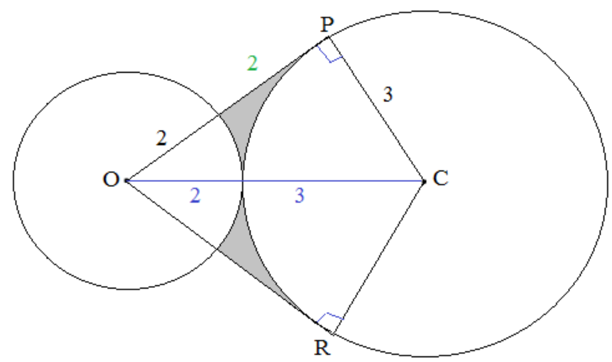


Tangents and radii form right angles...

OP and OR are congruent (because they are external tangents that meet at a common point)

$\triangle POC$ and $\triangle ROC$ are congruent right triangles...

Pythagorean Theorem: 3-4-5 right triangles



a)

$$\sin(\angle POC) = \frac{3}{5}$$

$$\angle POC = 36.87^\circ$$

then, $\angle POR = 2 \times \angle POC = \boxed{73.74^\circ}$

$$\cos(\angle PCO) = \frac{3}{5}$$

$$\angle PCO = 53.13^\circ$$

then, $\angle PCR = 2 \times \angle PCO = \boxed{106.26^\circ}$

b) To find the shaded area:

1) area of each right triangle: $\frac{1}{2}$ (base)(height) $= \frac{1}{2}(4)(3) = 6$

so, area of the triangles is 12

2) area of each sector

Sector in circle O: $\frac{\theta}{360}\pi(\text{radius})^2$

$$\frac{73.74}{360}\pi(2)^2 = 2.57$$

Sector in circle C: $\frac{\theta}{360}\pi(\text{radius})^2$

$$\frac{106.26}{360}\pi(3)^2 = 8.35$$

3) Calculate the shaded area

Shaded = total triangles − sectors

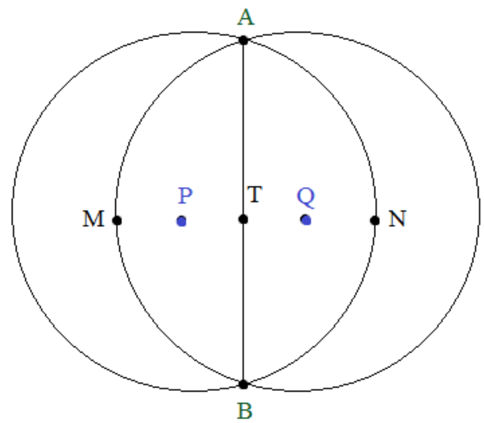
$= (12) - (2.57 + 8.35) = \boxed{1.08 \text{ square units}}$

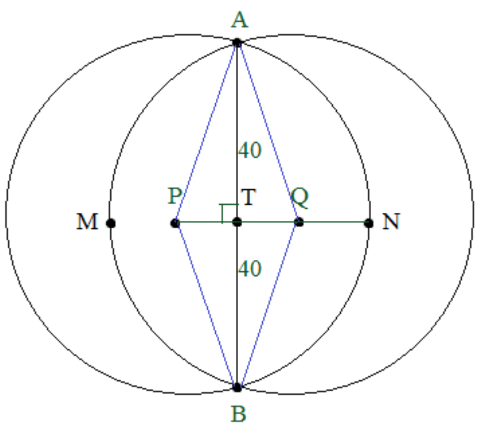
Example: Given: Circle P and Circle Q are congruent

$\overline{MP} = \overline{NQ} = 10$

$\overline{AB} = 80$

What is the distance between P and Q?





Since circles P and Q are congruent,
the radii are all congruent...

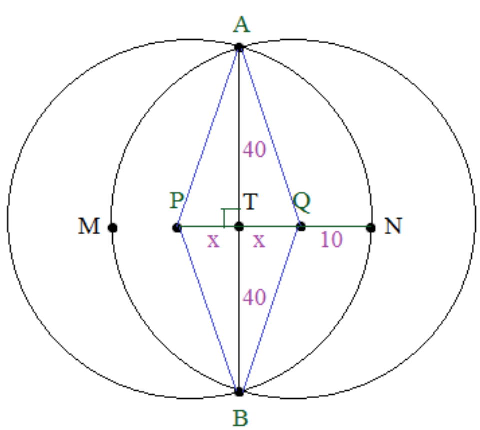
(Angle Bisector Theorem)
We know PQ is a perpendicular bisector of AB..

Therefore, $\overline{AT} = \overline{TB} = 40$
and $\triangle$ PAT is a right triangle

$x^2 + 40^2 = (\overline{PA})^2$

$x + x + 10 = \overline{PN}$

$\overline{PA}$ and $\overline{PN}$ are congruent radii (in circle P)



$x^2 + 1600 = (2x + 10)^2$

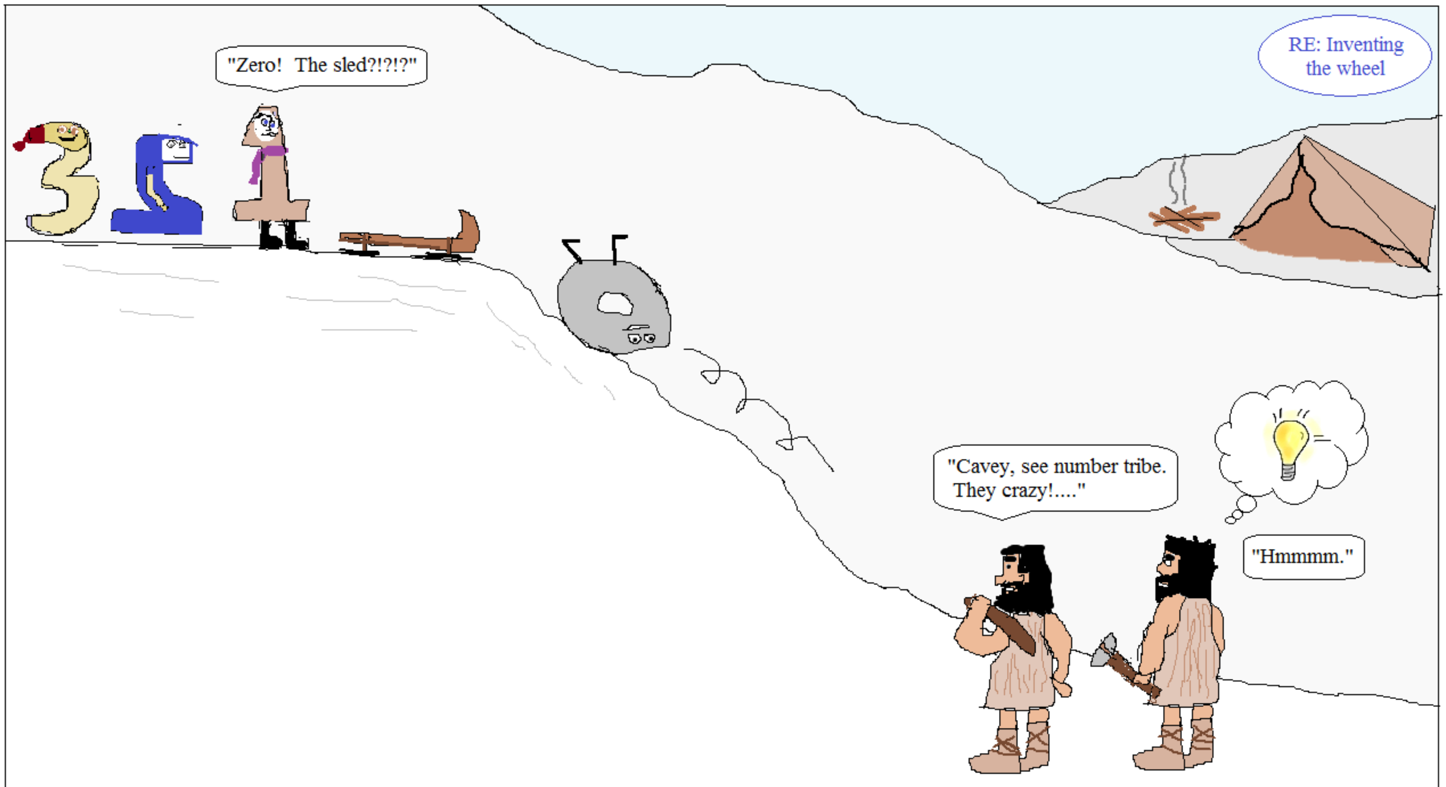
$x^2 + 1600 = 4x^2 + 40x + 100$

$3x^2 + 40x - 1500 = 0$

$(3x - 50)(x + 30) = 0$

since length x cannot be negative, $x = 50/3$

therefore, $\boxed{PQ = 100/3}$



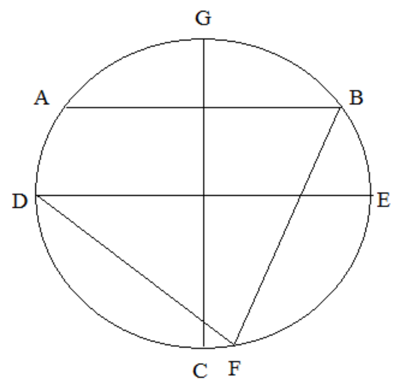
LanceAF #225 (1-28-16)
mathplane.com

Moments before caveman invents the wheel...

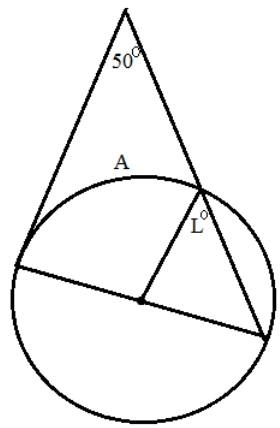
Exercises -→

Circles IV Review (Honors)

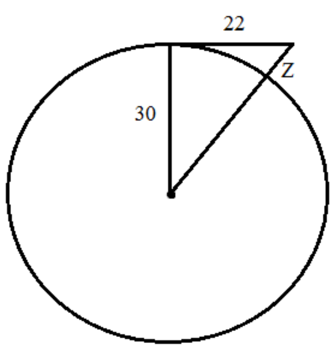
1) Which chords (if any) are congruent?



2) Find L and A

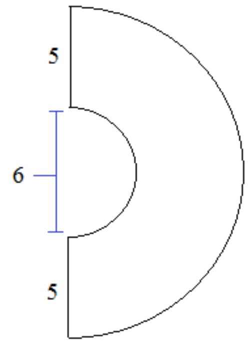


3) Find Z:

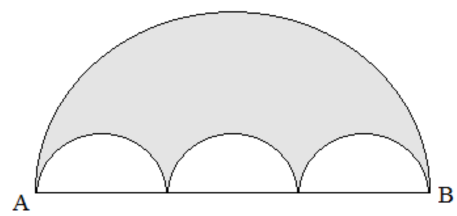


4) What is the equation of a circle containing points (1, 1) (5, 9) and (13, 4)?

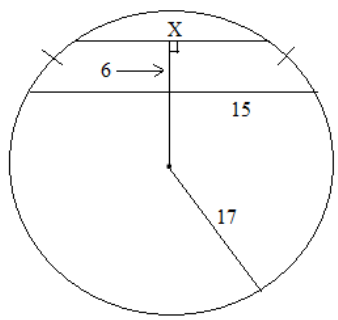
5) Find the perimeter.



6) Length of $\overline{AB}$ is 18.
The 3 semicircles are congruent.
What is the shaded area?

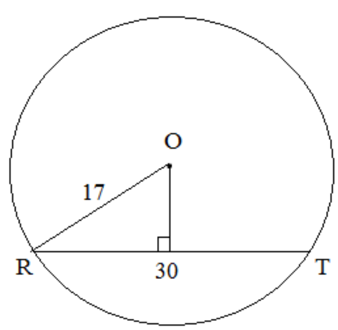


7) Find X:



8) What is $m\overset{\frown}{RT}$?

What is the arc length of $\overset{\frown}{RT}$?



9) Given: $\triangle ABC$ is isosceles with base $\overline{BC}$

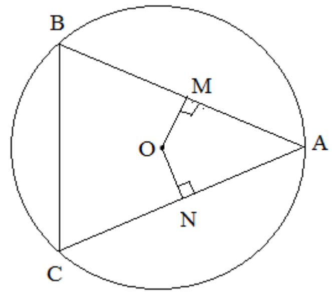
$\overline{OM} \perp \overline{AB}$ $\overline{ON} \perp \overline{AC}$

$\overline{OM} = 7x - 10$

$\overline{AB} = 8x + 2$

$\overline{ON} = 3x + 6$

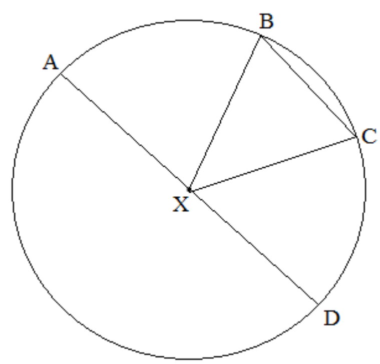
Find: the area of circle O



10) Given: $\angle BXA = \angle CXD$

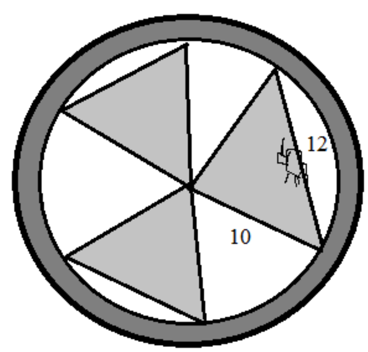
$\angle XBC = 68^{\circ}$

Find: $m\,\widehat{CDA}$



11) A wheel's spokes are 10" long.
And, the chords joining the spokes are 12".

If a bug lands on the middle of a chord, how far will it travel in one spin?



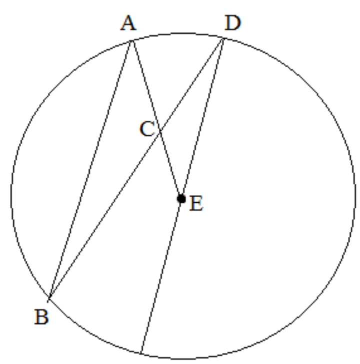
12) A 9 x 12 rectangle is inscribed in a circle.

What is the circumference of the circle?

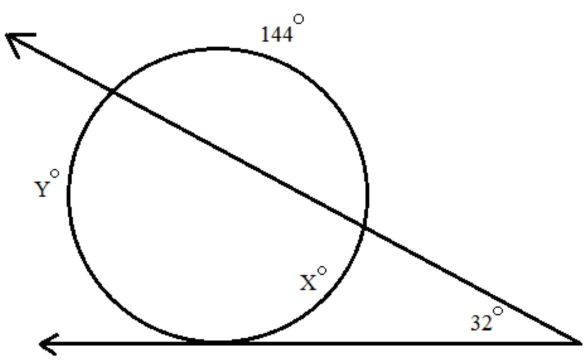
13) What is the center and radius of the circle?

$$x^2 - 8x + y^2 + 14y = -6$$

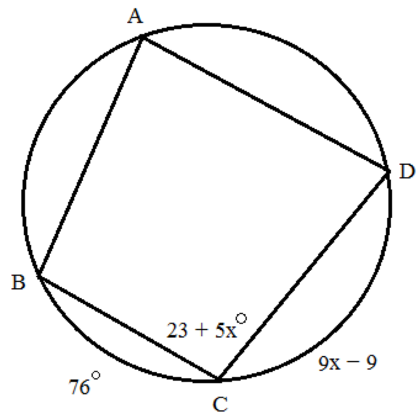
14) Is $\angle AED$ $<$, $>$, or $=$ to $\angle ABD$?



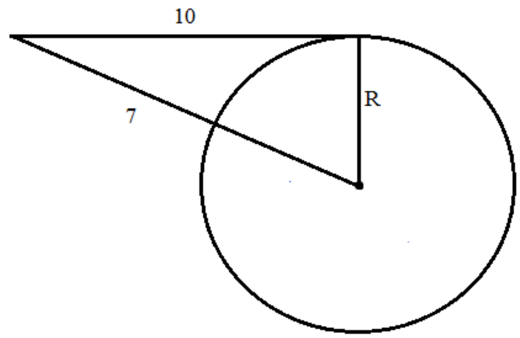
15) Find X and Y



16) Quadrilateral ABCD is inscribed in the circle.
Find $\angle BCD$

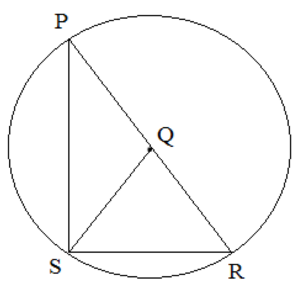


17) What is radius R?



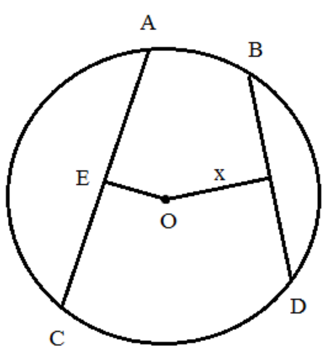
18) Given: Circle Q
$P = 36^{\circ}$
$\overline{PS} \perp \overline{SR}$

Find: a) $\angle PSQ$
b) $\angle R$



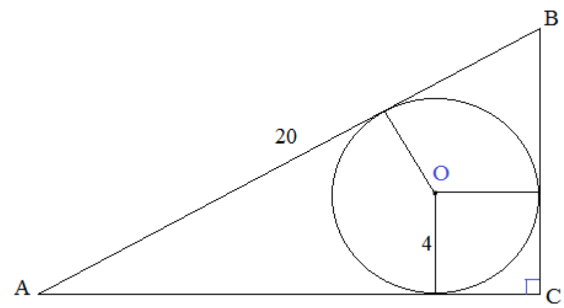
19) $\overline{AC} = 30$
$\overline{BD} = 20$
$\overline{OE} = 8$

What is x?



20) A circle is inscribed in a 21-28-35 right triangle.
What is the radius of the circle?

21) Circle O is inscribed in right triangle ABC.
If the radius is 4 and AB is 20,
what is the perimeter?



22) The radii of two circles are 3 and 8.
If the external tangent is 20, what is the distance between the circles?

23) A circle with diameter of 40 has a chord with endpoints $(-3, 2)$ and $(2, 14)$.
What is the distance of the chord from the center of the circle?

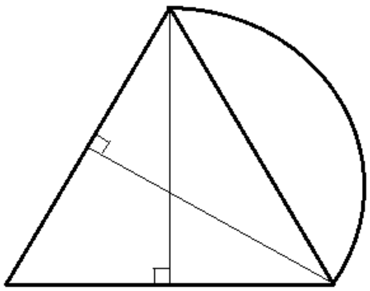
Circles IV Review (Honors)

Below are 3 identical equilateral triangles -- each having side lengths 6.

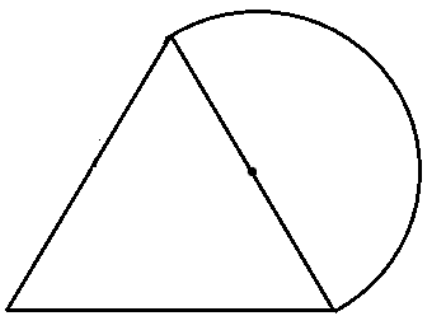
Determine each a) arc angle measure

b) arc length

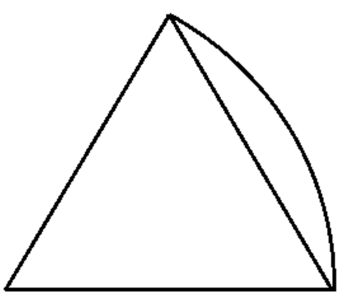
A)

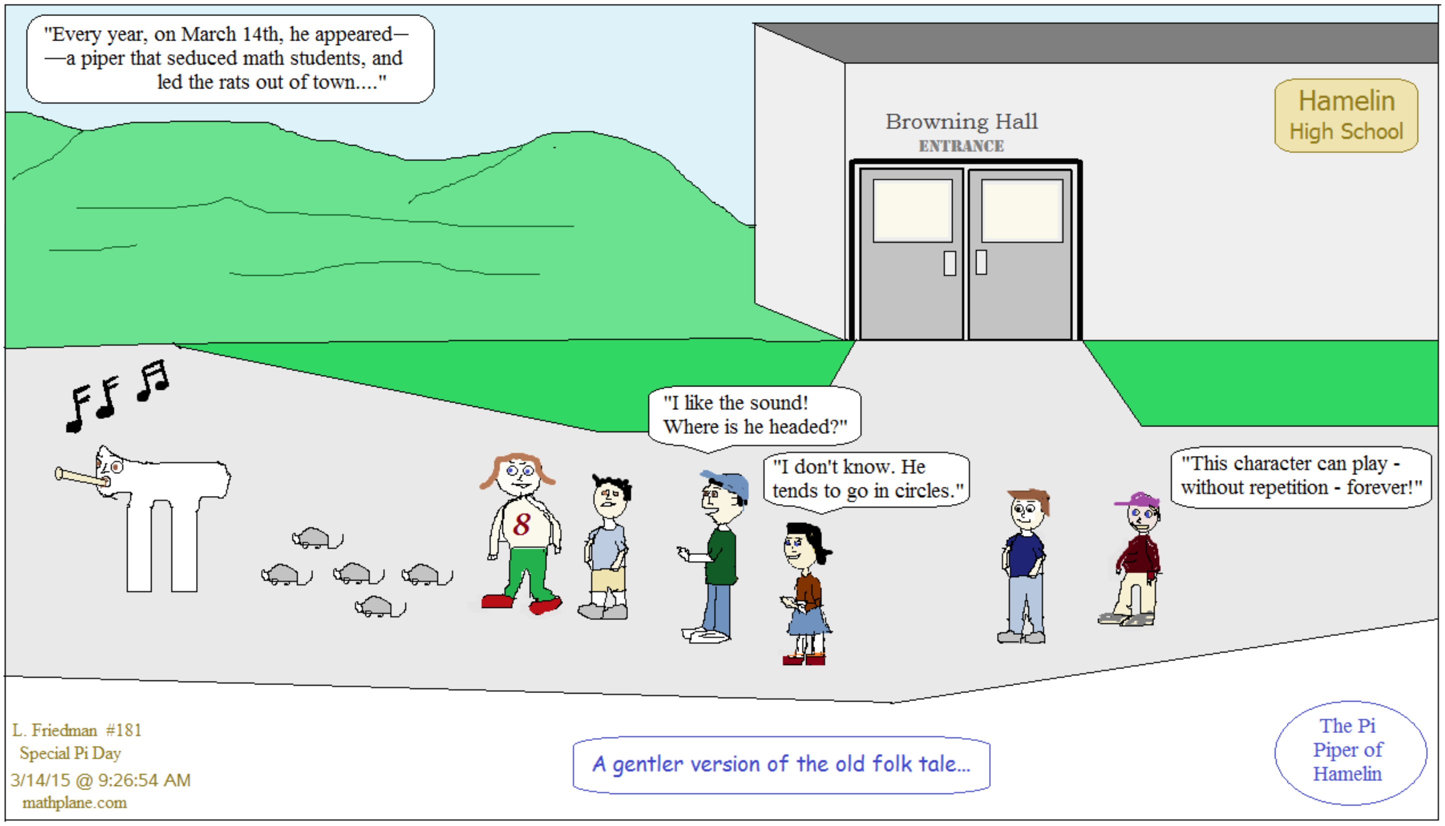


B)



C)



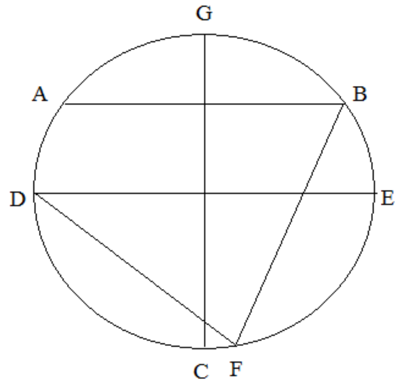

"Every year, on March 14th, he appeared—
—a piper that seduced math students, and
led the rats out of town...."
Browning Hall
ENTRANCE
Hamelin
High School
"I like the sound!
Where is he headed?"
"I don't know. He
tends to go in circles."
"This character can play -
without repetition - forever!"
8
L. Friedman #181
Special Pi Day
3/14/15 @ 9:26:54 AM
mathplane.com
A gentler version of the old folk tale...
The Pi
Piper of
Hamelin

Solutions-→

Circles IV Review (Honors)

SOLUTIONS

1) Which chords are congruent?



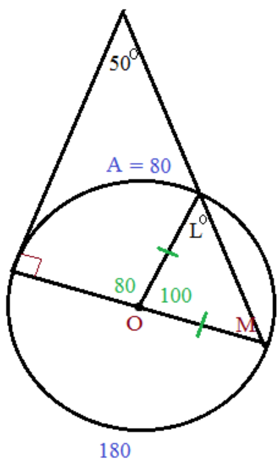
$\boxed{DE = GC}$

the diameters are the same!
(and, a diameter is a chord-- the longest possible chord in a circle)

while the other chords MIGHT be congruent, we don't KNOW...

2) Find L and A

external tangent is perpendicular to radius



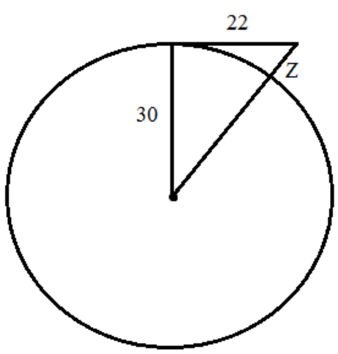
Angle-Arc relationship: $\frac{1}{2}(180 - A) = 50$ $\boxed{A = 80}$

Since A = 80, the central angle is 80, and it's supplement is 100

Triangle LMO is isosceles (because the radii are congruent)... Therefore, L and M are congruent angles... 40 and 40 $\boxed{L = 40}$

Also, the large triangle has angles 40, 50 and 90...
(sum is 180)

3) Find Z:



Since a radius is perpendicular to a tangent line, the diagram is a right triangle:

$22^2 + 30^2 = (\text{radius} + Z)^2$

$484 + 900 = (\text{radius} + Z)^2$

radius + Z = 37.20

Since radius is 30, Z = 7.20

Also, using tangent-secant power theorem:

$(\text{radius} + \text{radius} + Z)(Z) = 22^2$

whole secant | external part | tangent squared

$(30 + 30 + Z)(Z) = 484$

$Z^2 + 60Z - 484 = 0$

(Quadratic formula)

$Z = 7.20$ or -67.20

Z must be positive, so the answer is $\boxed{7.20}$

Quick check:

$22^2 + 30^2 = 37.20^2$

$484 + 900 = 1383.84$

(Pythagorean Theorem)

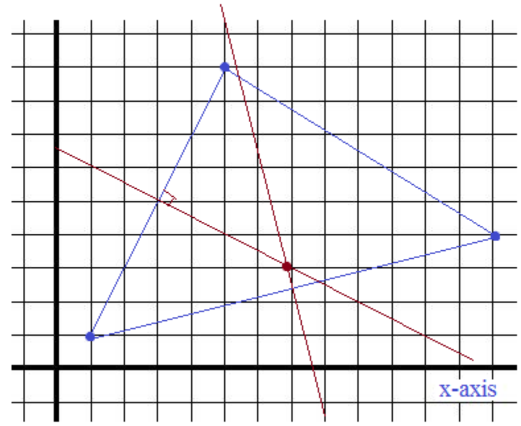
SOLUTIONS

4) What is the equation of a circle containing points (1, 1) (5, 9) and (13, 4)?

Since we cannot assume that 2 of these points are endpoints of a diameter, we must solve the system..

Standard form of a circle: $(x-h)^2 + (y-k)^2 = r^2$

Using Geometry: the perpendicular bisectors will intersect at the circumcenter (i.e the center of a circle that circumscribes the triangle)



The perpendicular bisector of (1, 1) and (5, 9)

midpoint: (3, 5)

slope of line: 2 slope of perpendicular bisector: $-1/2$

$$y - 5 = (-1/2)(x - 3)$$

$$y = \frac{-1}{2}x + \frac{13}{2}$$

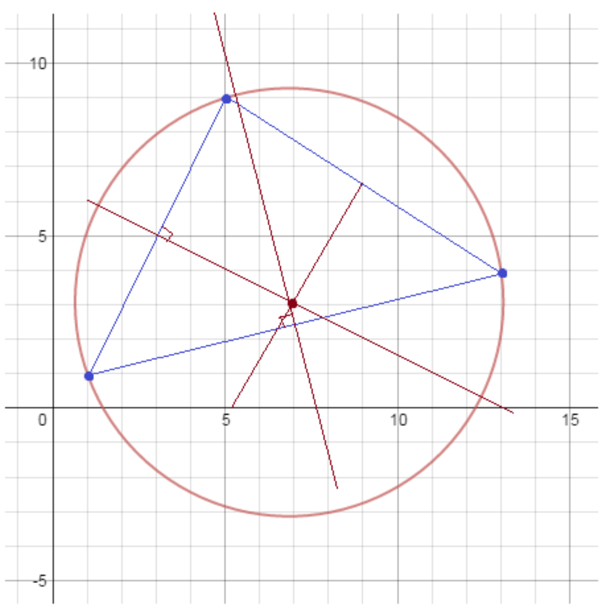
The perpendicular bisector of (1, 1) and (13, 4)

midpoint: (7, 5/2)

slope of line: 1/4 slope of perpendicular bisector: -4

$$y - 5/2 = (-4)(x - 7)$$

$$y = -4x + 30.5$$



the intersection of the perpendicular bisectors:

$$\frac{-1}{2}x + \frac{13}{2} = -4x + 30.5$$

$3.5x = 24$

then, $y = 3.07$

$x = 6.86$

center: (6.86, 3.07)

Finally, what is the distance from (6.86, 3.07) to each of the 3 points?

$$\text{distance} = \sqrt{(6.86 - 1)^2 + (3.07 - 1)^2} = 6.21$$

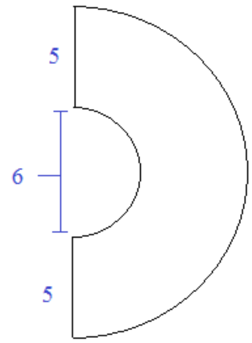
radius

$$\boxed{\text{circle: } (x - 6.86)^2 + (y - 3.07)^2 = 38.56}$$

SOLUTIONS

Circles IV Review (Honors)

5) Find the perimeter.



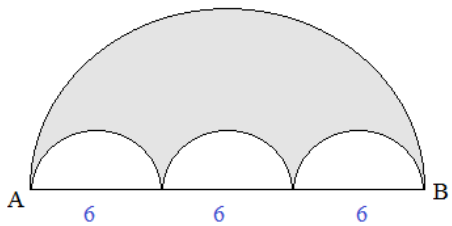
arc length of semicircle: (1/2)π (diameter)

diameter of inner semicircle is 6
diameter of outer semicircle is 16

$$\left(\frac{1}{2}\right)16\pi + \left(\frac{1}{2}\right)6\pi + 5 + 5$$

$$\boxed{11\pi + 10}$$

6) Length of $\overline{AB}$ is 18.
The 3 semicircles are congruent.
What is the shaded area?



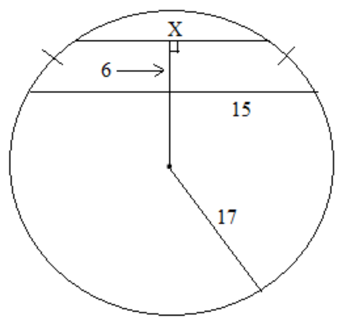
Area of semicircle = (1/2) π (radius)^2

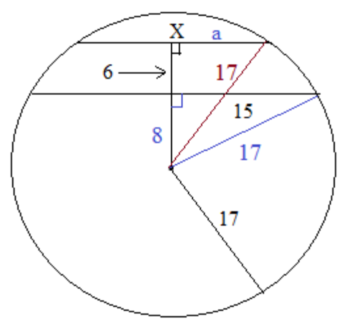
Large semicircle: $\left(\frac{1}{2}\right)81\pi = 40.5\pi$

Small semicircle: $\left(\frac{1}{2}\right)9\pi = 4.5\pi$

Shaded area: $40.5\pi - 3\left(4.5\pi\right) = \boxed{27\pi}$

7) Find X:





all radii are congruent...
Pythagorean Theorem..

$$(a)^2 + (6+8)^2 = 17^2$$

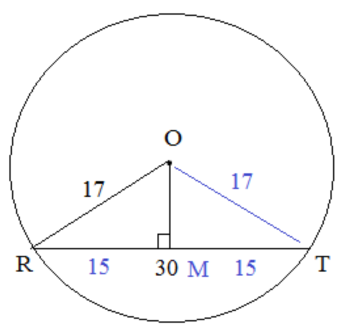
$$a^2 = 93$$

$$a = \sqrt{93}$$

$$\boxed{X = 2\sqrt{93}}$$

8) What is m$\overset{\frown}{RT}$?

What is the arc length of $\overset{\frown}{RT}$?



Using trig ratios, determine central angle O...

$\text{Sin}(O) = \frac{15}{17}$ angle O = 61.9°

Then, since OM is a perpendicular bisector,

angle ROT is $2 \times 61.9 = \boxed{123.8^\circ}$

Then, the arc length is $\frac{123.8}{360} \cdot (34\pi) = \boxed{36.7}$

Circles IV Review (Honors)

SOLUTIONS

9) Given: $\triangle$ ABC is isosceles with base $\overline{BC}$

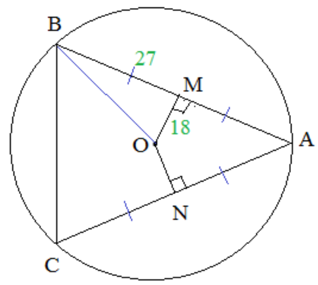
$\overline{OM} \perp \overline{AB}$ $\quad$ $\overline{ON} \perp \overline{AC}$

$\overline{OM} = 7x - 10$

$\overline{AB} = 8x + 22$

$\overline{ON} = 3x + 6$

Find: the area of circle O



Step 1: Find x

Since ABC is isosceles, AB $\cong$ AC...
If 2 chords are congruent, then the distance to the center is the same!

Therefore, $\overline{OM} = \overline{ON}$

$7x - 10 = 3x + 6$ $\qquad$ $x = 4$

OM = ON = 18

Step 2: Find radius

Since x = 4, AB = 8(4) + 22 = 54
Therefore, BM is 27
(note: the radius to chord is a perpendicular bisector.)

Pythagorean Theorem: $18^2 + 27^2 = OB^2$

OB = 32.45

Step 3: Find area

$\pi(\text{radius})^2 = \boxed{1053\pi}$

10) Given: $\angle BXA = \angle CXD$

$\angle XBC = 68^\circ$

Find: $m\,\widehat{CDA}$

Step 1: Label the diagram

Since all radii are congruent, BX $\cong$ CX....
Therefore, angle B = angle C
(If $\cong$ sides, then $\cong$ angles)

Step 2: Find angles and arcs

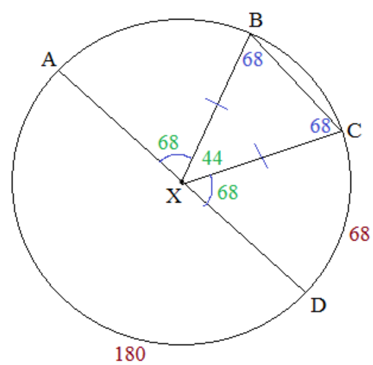
If B and C are 68, then
$\angle BXC = 44$

and, angles BXA and CXD are 68° (because sum of angles between A and D must be 180)

Step 3: calculate major arc

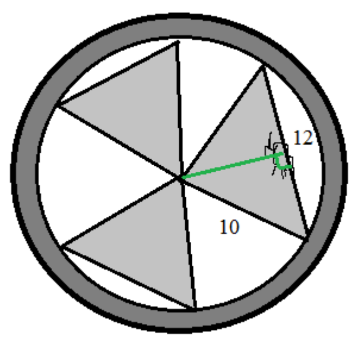
$\widehat{CD} + \widehat{AD} = \widehat{CDA}$

$68 + 180 = \boxed{248}$



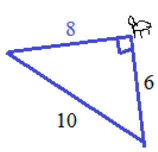
11) A wheel's spokes are 10" long.
And, the chords joining the spokes are 12".

If a bug lands on the middle of a chord, how far will it travel in one spin?



To find the distance the bug travels, we need to find the circumference of the "radius" from the bug to the center of the wheel...

The chord is 12", so the semi-chord is 6"...
And, the spokes are 10"



Using Pythagorean Theorem, we find the distance of the chord (ie bug) to the center is 8" --- radius = 8"

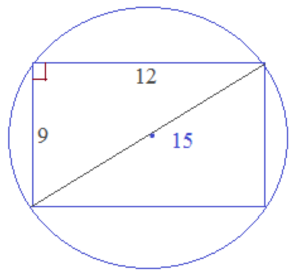
Therefore, the bug will travel $2\pi(\text{radius})$ $\boxed{16\pi}$

SOLUTIONS

12) A 9 x 12 rectangle is inscribed in a circle.

What is the circumference of the circle?

$\boxed{\text{Circumference} = 15\pi}$



Note: the inscribed rectangle is divided into 2 right triangles, where the diameter is the hypotenuse...

Using Pythagorean Theorem, the diameter is 15

13) What is the center and radius of the circle?

$x^2 - 8x + y^2 + 14y = -6$

Complete the square to change equation into *standard form*

$$x^2 - 8x + 16 + y^2 + 14y + 49 = -6 + 16 + 49$$

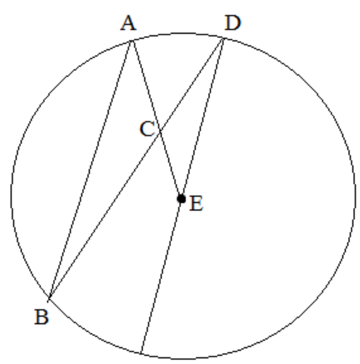
$$(x-4)^2 + (y+7)^2 = 59$$

center: (4, -7)

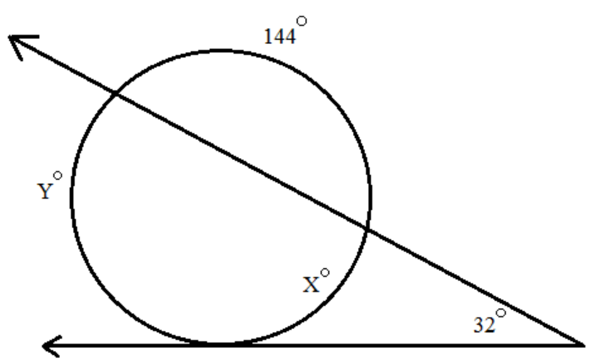
radius: $\sqrt{59}$

14) Is $\angle AED$ <, >, or = to $\angle ABD$?

Since ABD is inscribed angle that intercepts arc AD and AED is a central angle that intercepts arc AD, then ABD must be 1/2 of AED!



15) Find X and Y



$X + Y + 144 = 360$ — Sum of arcs in a circle is 360

$\rightarrow X + Y = 216$

$\frac{1}{2}(Y - X) = 32$ — "Tangent-Secant" Power Theorem

$\rightarrow Y - X = 64$

Then, solve the system of equations:

$X + Y = 216$

$Y - X = 64$

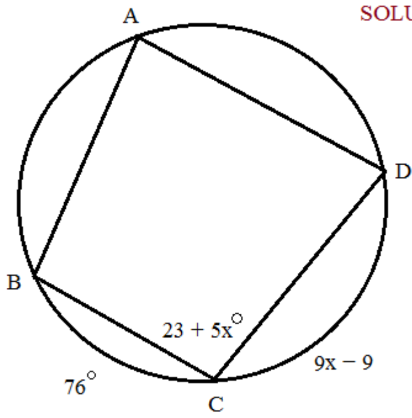
$2Y = 280$

$\boxed{Y = 140 \text{ and } X = 76}$

16) Quadrilateral ABCD is inscribed in the circle.
Find $\angle BCD$

$\overset{\frown}{AB} + \overset{\frown}{AD} + \overset{\frown}{BC} + \overset{\frown}{CD} = 360$

And arc $\overset{\frown}{BAD} = 2(23 + 5x)$
(inscribed angle)



$76 + (9x - 9) + 2(23 + 5x) = 360$

$76 + 9x - 9 + 46 + 10x = 360$

$19x = 247$

$x = 13$

$\boxed{\angle BCD = 23 + 5(13) = 88 \text{ degrees..}}$

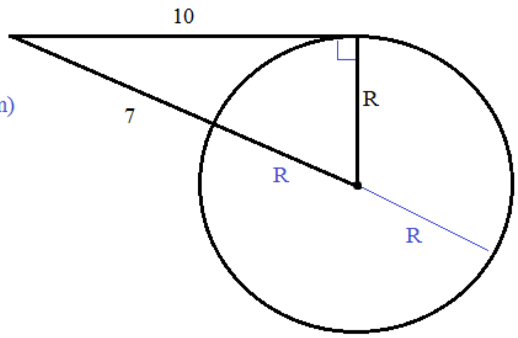
17) What is radius R?

(Using Secant-Tangent Theorem)

$(10)^2 = (7)(R + R + 7)$

$100 = 14R + 49$

$\boxed{R = 51/14}$



(Using Pythagorean Theorem)

$R^2 + (10)^2 = (R + 7)^2$

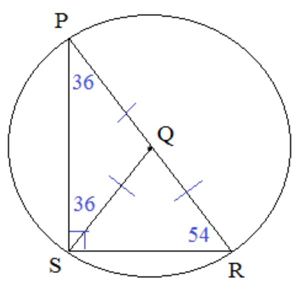
$R^2 + 100 = R^2 + 14R + 49$

$51 = 14R$

$\boxed{R = 51/14}$

18) Given: Circle Q
$P = 36°$
$\overline{PS} \perp \overline{SR}$

Find: a) $\angle PSQ$ 36°
b) $\angle R$ 54°



we know that "all radii are congruent"...

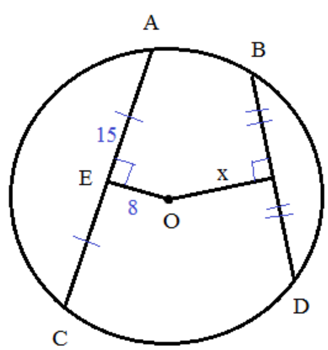
then, "if congruent sides, then congruent angles" for each isosceles triangle

19) $\overline{AC} = 30$

$\overline{BD} = 20$

$\overline{OE} = 8$

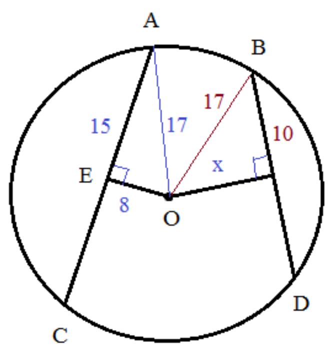
What is x?



$289 - 100 = 189$

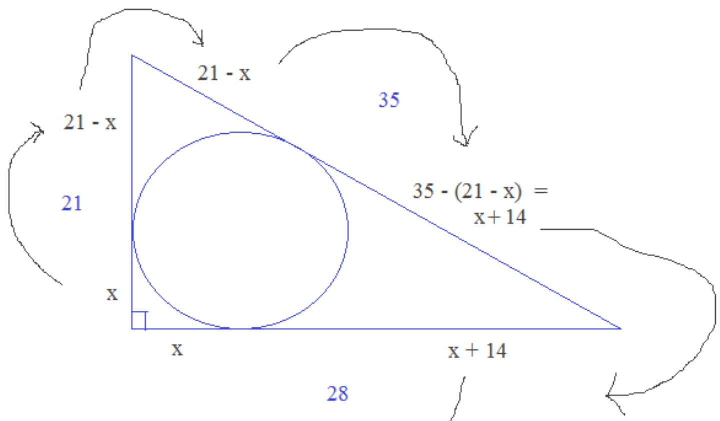
$\boxed{x = \sqrt{189} \text{ or } 13.75 \text{ (approx)}}$

Important: since $BD \neq AC$, the distances to the center are not equal



SOLUTIONS

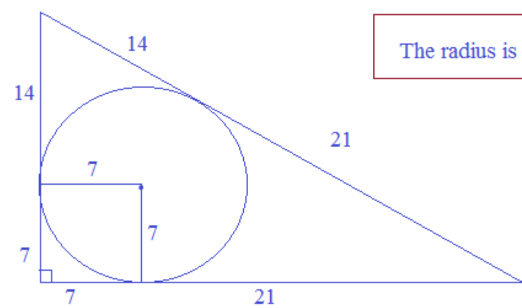
20) A circle is inscribed in a 21-28-35 right triangle.
What is the radius of the circle?



$x + (x + 14) = 28$

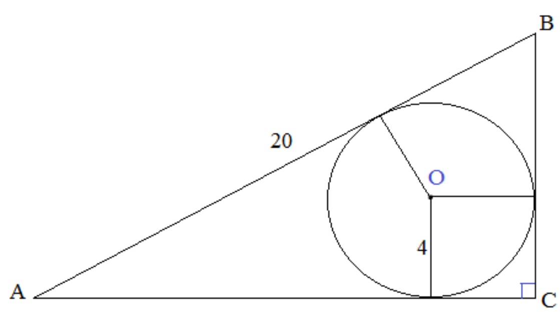
$2x + 14 = 28 \qquad x = 7$

"Walk-around" or "Travel" problems...



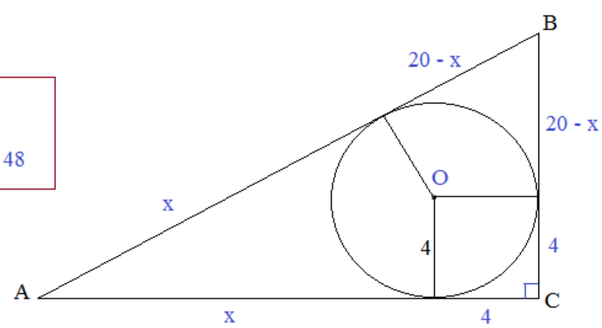
The radius is 7....

21) Circle O is inscribed in right triangle ABC.
If the radius is 4 and AB is 20,
what is the perimeter?



perimeter is

$12 + 16 + 20 = 48$



$(x + 4)^2 + (24 - x)^2 = 20^2$

$x^2 + 8x + 16 + 576 - 48x + x^2 = 400$

$2x^2 - 40x + 192 = 0$

$2(x^2 - 20x + 96) = 0$

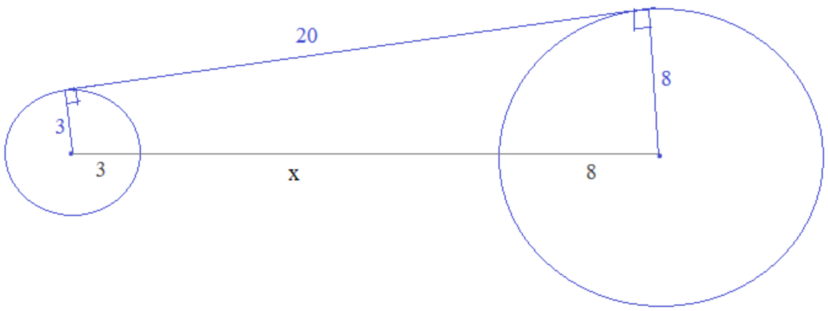
$(x - 8)(x - 12) = 0$

$x = 8 \text{ or } 12$

Circles IV Review (Honors)

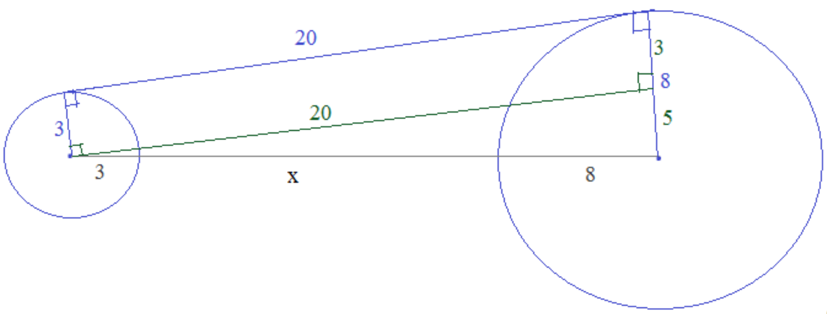
SOLUTIONS

22) The radii of two circles are 3 and 8.
If the external tangent is 20, what is the distance between the circles?

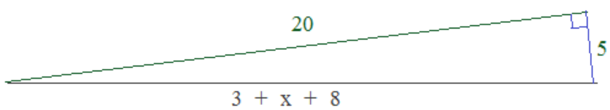


All radii are congruent...

Tangents are perpendicular to radii and point of tangency..



"transpose" the external tangent to create a rectangle and right triangle...



Solve the right triangle to answer the question!

$$(20)^2 + (5)^2 = (3 + x + 8)^2$$

$$425 = (3 + x + 8)^2$$

$$20.6 = 11 + x$$

$$x = 9.6$$

23) A circle with diameter of 40 has a chord with endpoints $(-3, 2)$ and $(2, 14)$.
What is the distance of the chord from the center of the circle?

Using the distance formula, we find the length of the chord:

$$\text{distance} = \sqrt{(-3 - 2)^2 + (2 - 14)^2}$$

$$= \sqrt{25 + 144} = 13$$

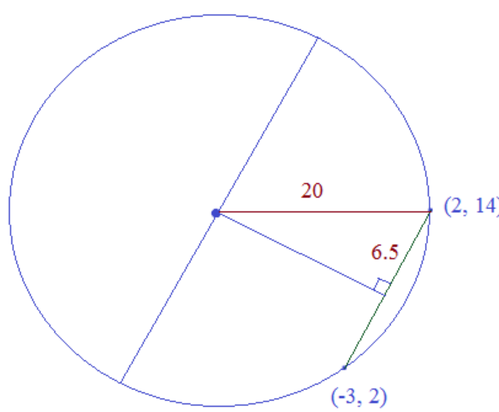
And, the distance from chord to the center is a perpendicular bisector...

So, we know the radius is 20 and the 1/2 chord is 6.5...

Then, Pythagorean Theorem will get the distance from chord to center...

$$d^2 = (20)^2 - (6.5)^2$$

$$\boxed{d = \sqrt{357.75} \quad \text{approx. } 18.91}$$



Circles IV Review (Honors)

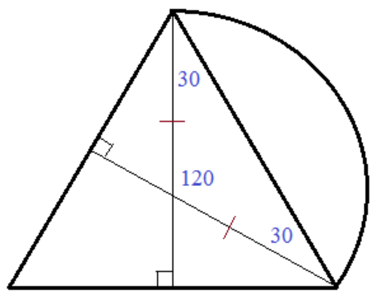
Below are 3 identical equilateral triangles -- each having side lengths 6.

SOLUTIONS

Determine each a) arc angle measure

b) arc length

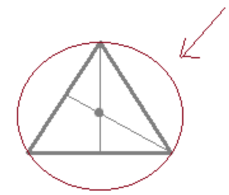
A)

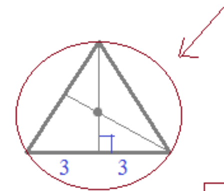


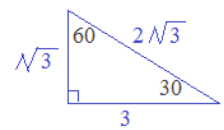
The altitudes are also angle bisectors (and medians...)

then, the congruent sides form the radii of a full circle...

$\boxed{\text{120 degrees}}$





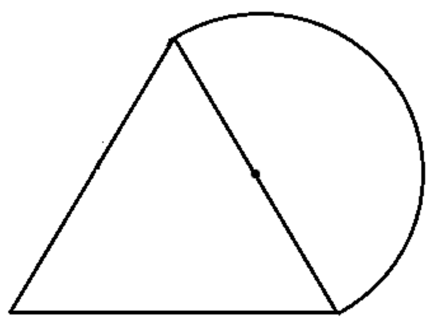


$(1/3) \cdot 4\sqrt{3}\,\pi$

diameter of circle

$\boxed{\frac{4}{3}\sqrt{3}\,\pi}$

B)



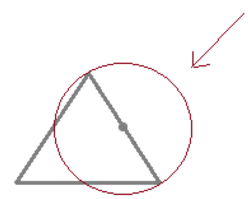
This arc is a semicircle where the diameter is the side of the triangle...

$\boxed{\text{180 degrees}}$

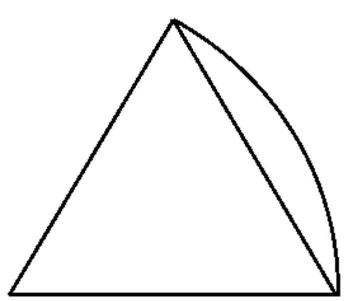
arc length of semicircle:

$\frac{1}{2} \cdot 2\pi(3) = \boxed{3\pi}$

radius



C)



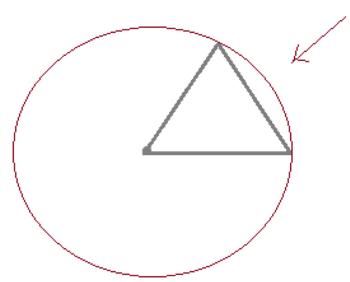
In this case, 2 sides of the triangle are also the radii of the "full circle"

$\boxed{\text{60 degrees}}$

circumference of full circle is 12π

And, this arc is 1/6 of the circle..

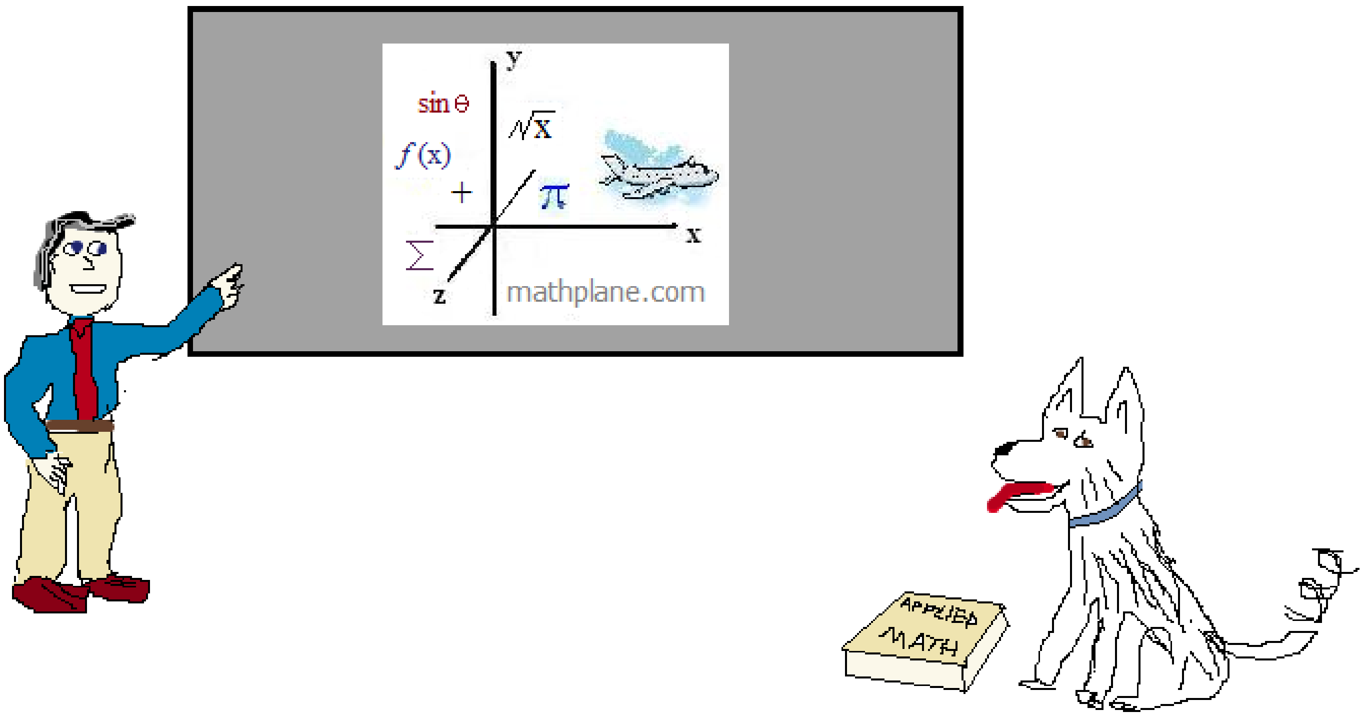
$\boxed{2\pi}$



Thanks for visiting. Hope it helps!

If you have questions, suggestions, or requests, let us know.

Cheers.



Also, at TeachersPayTeachers, Facebook, TES, Google +, and Pinterest

One more challenge question:

Given: 2 concentric circles

The chord of the outer circle is tangent to the inner circle

If the length of the chord is 70, what is the area between the circles (i.e. the ring)?

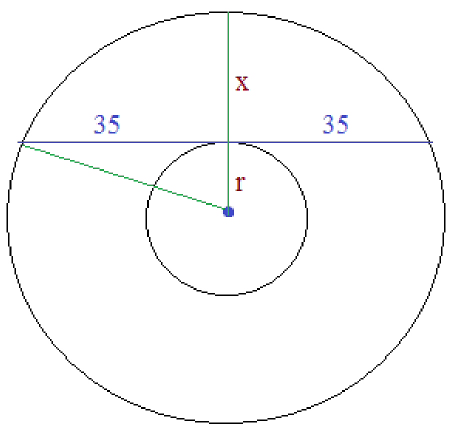
SOLUTION-→

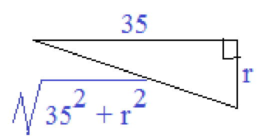
Area of circle = $\pi\,(\text{radius})^2$

Note: All radii are congruent

tangent line is perpendicular to radius

Pythagorean Theorem





$$r + x = \sqrt{35^2 + r^2}$$

$$(r + x)^2 = 35^2 + r^2$$

Area of big circle − Area of small circle

$$\pi\,(r + x)^2 \quad - \quad \pi\,(r)^2$$

$$\pi\,(35^2 + r^2) - \pi\,(r)^2$$

$$1225\pi + \pi r^2 - \pi\,(r)^2$$

$$\boxed{1225\pi}$$