Always Sometimes Never

Notes, Examples, and Practice (w/solutions)

Topics include quadrilaterals, triangles, true/false, and more.

Always, Sometimes, Never: Notes and Examples

ASN (Always Sometimes Never) questions are common in math, especially geometry. While they are effective ways to demonstrate reasoning and attention to detail, they can be confusing, (leading to slight mistakes).. At the right, are straight-forward steps that should help you answer ASN questions. Below are several examples.

Strategy to Solve:
1) Find an example that fits
   (This eliminates 'never')
2) Find a counterexample that doesn't
   (This eliminates 'always')
3) Try to make a general expression to validate
4) Beware of unique exceptions
   (e.g. distance $\not< 0$; 0 in the denominator; prime number is 2; 2 + 2 = 2 x 2)

Examples:

1) For $X \neq 0$

$X^2 > 0$

If X = 5, then $X^2 = 25 > 0$
(This eliminates 'never')

If X = -3, then $X^2 = 9 > 0$
(unable to find a counterexample)

ALWAYS

$X^3 > 0$

If X = 3, then $X^3 = 27 > 0$
(This eliminates 'never')

If X = -2, then $X^3 = -8 \not> 0$
(This eliminates 'always')

SOMETIMES

If X is positive, then X X X > 0
and if X is negative, then $(-X)(-X)(-X) \not> 0$

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2) X and Y are integers.

$$\frac{4X + 1}{2} = Y$$

If X = 3, then $\frac{4(3) + 1}{2} = 6\ 1/2 = Y$

Y is not an integer
(This eliminates 'always')

Cannot find an example.

NEVER

General expression:

$$\frac{4X + 1}{2} = Y$$

$2X + 1/2 = Y$

Since X is an integer, then 2X is also an integer.
Therefore, 2X + 1/2 cannot be an integer..

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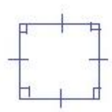
3) ABCD is a quadrilateral.

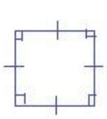
If ABCD is a square, it is equiangular.



ALWAYS true.
(All angles are 90 degrees)

If ABCD is equiangular, it is a square.

NOTE: The <u>order</u> of a statement may determine the outcome!



This shape is equiangular, and it is a square.
(this eliminates 'never')

This shape is equiangular, but it is a rectangle. Counterexample!
(this eliminates 'always')

SOMETIMES the second statement is true..

Always, Sometimes, Never: Notes and Examples
continued...

4) When you add two numbers, you get the same result as multiplying them.

$3 + 6 = 3 \times 6$? $9 = 18$ (this eliminates 'always')

$2 + 2 = 2 \times 2$? $4 = 4$ (this eliminates 'never')
$0 + 0 = 0 \times 0$? $0 = 0$

The answer is SOMETIMES

To validate, let's make a general expression:

$X + Y = XY$

$X = XY - Y$

$X = Y(X - 1)$

$\dfrac{X}{(X - 1)} = Y$

$Y = \dfrac{X}{(X - 1)}$ there are countless possibilities!

0, 0 2, 2 3, 3/2 4, 4/3 etc....

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5) Given Triangle ABC (with sides a, b, c)

i)	ii)	iii)	iv)
$a = 4$ $b = 7$ then, $c = 5$	$a = 6$ $b = 9$ then, $c = 17$	equilateral $a = 8$ $b = 8$ then, $c = 8$	isosceles $a = 6$ $b = 6$ then, $c = 14$
SOMETIMES	NEVER	ALWAYS	NEVER
$3 < c < 11$	$3 < c < 15$	Since it is equilateral, all sides must be 8	Since it is isosceles, 2 sides are the same. But, the third side must be less than 12!

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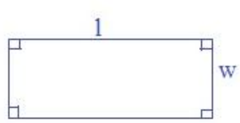
6) Area of a rectangle = Perimeter of a rectangle

We can give a quick counterexample: 3 x 4 rectangle $A = 12$
(this eliminates 'always') $P = 14$

Is it 'sometimes' or 'never'?



$lw = 2l + 2w$

$lw - 2l = 2w$

$l(w - 2) = 2w$

$l = \dfrac{2w}{(w - 2)}$

Let's try $w = 4$:
Then,
$l = \dfrac{2(4)}{((4) - 2)} = 4$

So, if it is a 4 x 4,
$A = 16$ and $P = 16$

(Square)

$\boxed{\text{SOMETIMES}}$

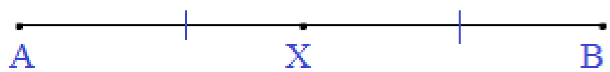
Let's try $w = 10$
then,
$l = \dfrac{2(10)}{((10) - 2)} = \dfrac{20}{8}$

So, if it is a 10 x 2.5,
$A = 25$ and $P = 25$

(Rectangle)

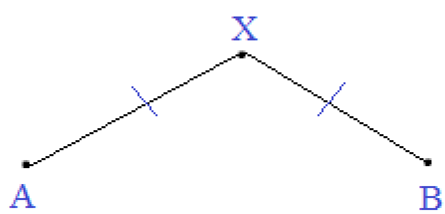
If AX = XB, then A, X, and B are collinear.

Always, Sometimes, or Never?



In this diagram, AX = XB and A, X, and B are collinear
(X lies in line segment AB)

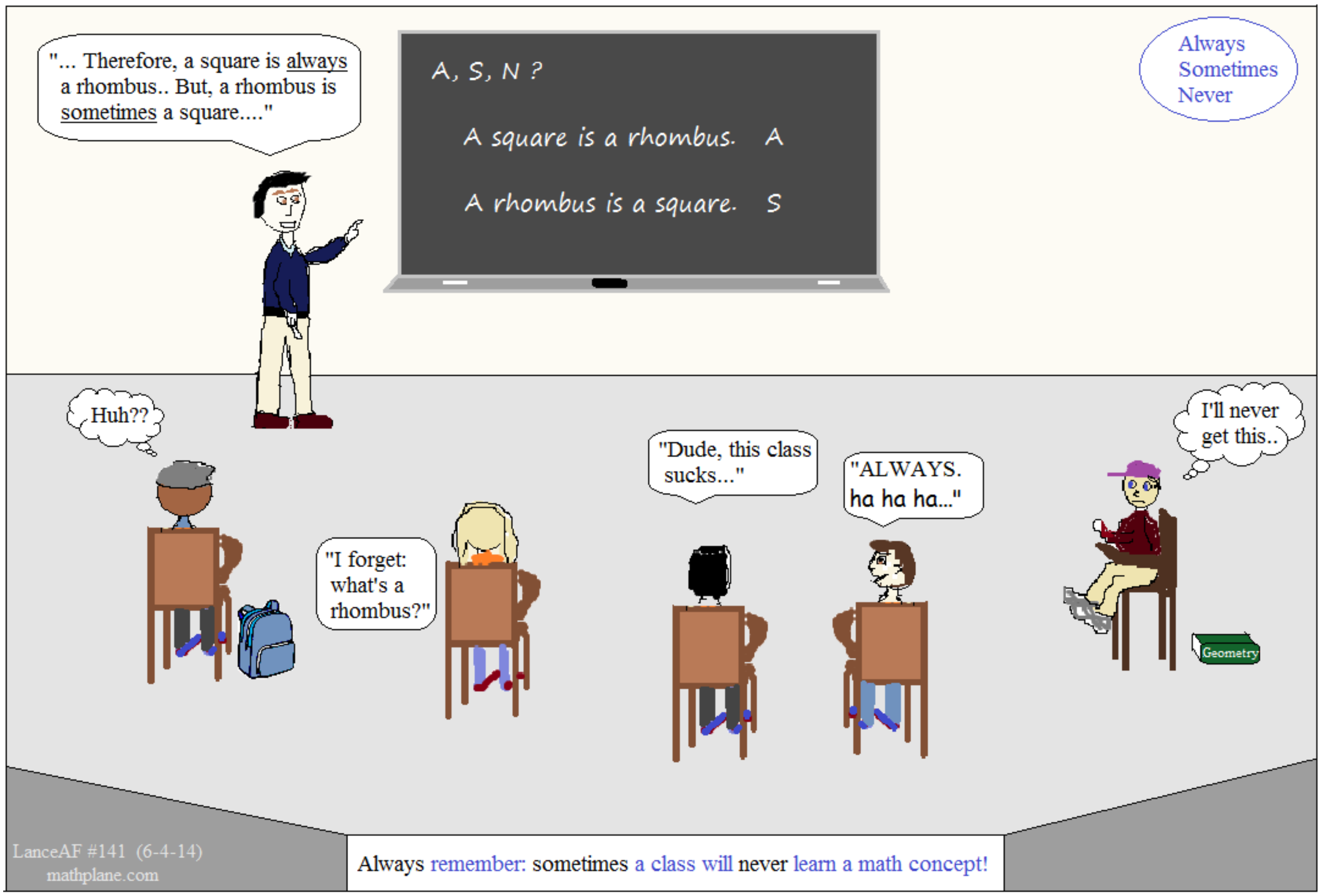
This eliminates "never"!



In this example, AX = XB but A, X, and B are non-collinear

This eliminates "always"!

The answer is "sometimes"



Practice Exercises-→

Geometry Always/Sometimes/Never

1) A rhombus is a square.

2) A square is a rhombus.

3) If 2 sides and an angle in one triangle are congruent to 2 sides and an angle in another triangle, then the triangles are congruent.

4) If $\triangle ABC \cong \triangle DEF$, then $\overline{AB} \cong \overline{DF}$

5) If $\overline{AB}$ is a perpendicular bisector of $\overline{BC}$, then $\overline{BC}$ is a perpendicular bisector of $\overline{AB}$.

6) A scalene triangle has 2 congruent angles.

7) Two lines that <u>do not</u> intersect are parallel.

8) Diagonals of a rectangle are congruent.

Always, Sometimes, Never Practice Quiz

1) When you multiply two odd numbers, you get an even number.

2) When you multiply a number by itself, it gets bigger.

3) $\frac{X}{Y} = \frac{(X+1)}{(Y+1)}$

4) $X^3 \neq X^5$

5) 4-sided polygons are squares.

6) Squares are 4-sided polygons.

Triangles: Always, Sometimes, Never & True/False

I. Always, Sometimes, or Never (ASN)
Label each statement. Provide examples and counterexamples.

1) A triangle has 3 unique *medians*.

2) *Perpendicular line segments* form 4 congruent right angles.

3) If two triangles have 2 congruent sides and 1 congruent angle, then they are congruent.

4) In an isosceles triangle, the *perpendicular bisector* from the base goes through the vertex.

5) The *altitudes* of a right triangle are concurrent (intersect) at a point inside the triangle.

6) Two angles on a straight line are *supplementary*.

7) If two triangles have 2 congruent angles and 1 congruent side, then they are congruent.

8) An *obtuse triangle* can have 2 obtuse angles.

Triangles: Always, Sometimes, Never & True/False

II. True/False

Answer each statement. If false, provide a counterexample.

1) If one of the base angles of an isosceles triangle is 20 degrees, then the triangle is obtuse.

2) In an equilateral triangle, an *altitude* is a *median*.

3) If the corresponding angles of 2 triangles are congruent, then the triangles must be congruent.

4) If $\overline{AB}$ bisects $\overline{CD}$ through point M, then $\overline{AM} = \overline{MB}$

5) If the length of segment JK = 5 and the length of segment KL = 5, then the length of segment JL must be 10.

6) The *angle bisector* of an obtuse angle produces 2 congruent acute angles.

7) An obtuse triangle can be *scalene*.

8) The third side of an *isosceles* triangle with sides 4 and 9 must be 9.

9) If $\triangle ABC \cong \triangle DEF$, then $\angle CAB \cong \angle EDF$

10) If all corresponding angles are congruent, then 2 triangles are necessarily congruent.

11) If all corresponding sides are congruent, then two triangles are congruent.

Geometry Always/Sometimes/Never

Answers

1) A rhombus is a square.

SOMETIMES

Definition of a rhombus: quadrilateral with 4 congruent sides
Definition of a square: quadrilateral with 4 congruent sides and angles.

2) A square is a rhombus.

ALWAYS

3) If 2 sides and an angle in one triangle are congruent to 2 sides and an angle in another triangle, then the triangles are congruent.

SOMETIMES

If 2 sides and included angle are congruent, then triangles are always congruent (SAS); but, 2 sides and extra angle may or may not be congruent.

4) If $\triangle ABC \cong \triangle DEF$, then $\overline{AB} \cong \overline{DF}$

SOMETIMES

$\overline{AB} = \overline{DE}$, $\overline{BC} = \overline{EF}$, and $\overline{CA} = \overline{FD}$ are always congruent

5) If $\overline{AB}$ is a perpendicular bisector of $\overline{BC}$, then $\overline{BC}$ is a perpendicular bisector of $\overline{AB}$.

SOMETIMES

examples: diagonals of a square: both perpendicular bisectors
diagonals of a kite: one is a perpendicular bisector; the other is not

6) A scalene triangle has 2 congruent angles.

NEVER

Scalene triangle: 3 sides of different lengths
(therefore, opposite angles must be different measures)

7) Two lines that <u>do not</u> intersect are parallel.

SOMETIMES

If the lines are in the same plane, then they are parallel.
but, in a 3-d space, if they are *skew* lines, they don't intersect.

8) Diagonals of a rectangle are congruent.

ALWAYS

Always, Sometimes, Never Practice Quiz

SOLUTIONS

1) When you multiply two odd numbers, you get an even number.

$5 \times 7 = 35$ (this eliminates 'always')
Is it 'never' or sometimes?

Assume 2 odd numbers: $2n + 1$ and $2m + 1$

$(2n + 1)(2m + 1) = 4mn + 2m + 2n + 1 = 2(2mn + m + n) + 1$

$2(\ )$ is even.. then, $2(\ ) + 1$ is odd

NEVER

2) When you multiply a number by itself, it gets bigger.

Try $5 \times 5 = 25$ It gets bigger (eliminates 'never')

Then, try $1/2 \cdot 1/2 = 1/4$ It gets smaller (eliminates 'always')

SOMETIMES

General Expression: If $n > 1$, then $n^2 > n$

$n < 1$, then $n^2 < n$

3) $\frac{X}{Y} = \frac{(X + 1)}{(Y + 1)}$

Try $\frac{3}{4} \neq \frac{4}{5}$ and $\frac{-3}{-4} \neq \frac{-2}{-3}$ positive and negative numbers didn't work (eliminates 'always')

Now, suppose $X = Y$.. $\frac{3}{3} = \frac{4}{4}$ This eliminates 'never'

SOMETIMES

4) $X^3 \neq X^5$

If $X = 2$, then $2^3 = 8 \neq 32 = 2^5$ (This eliminates 'never')

Is there a counterexample?

$X^3 = X^5$ $\quad X^5 - X^3 = 0$

$X^3(X^2 - 1) = 0$

Three counterexamples
(this eliminates 'always')

SOMETIMES

$(X^2 - 1) = 0$ $\quad X = 1, -1$

$X^3 = 0$ $\quad X = 0$

5) 4-sided polygons are squares.

SOMETIMES

Squares are 4-sided..
Trapezoid, Rhomus, and Rectangles are counterexamples.

6) Squares are 4-sided polygons.

ALWAYS

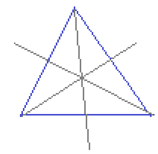
Triangles: Always, Sometimes, Never & True/False

ANSWERS

I. Always, Sometimes, or Never (ASN)

Label each statement. Provide examples and counterexamples.

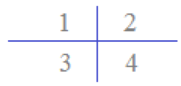
1) A triangle has 3 unique *medians*.

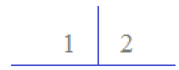


ALWAYS... (a triangle always has 3 vertex/3 sides.. And each median is drawn from each vertex)

2) *Perpendicular line segments* form 4 congruent right angles.

SOMETIMES...

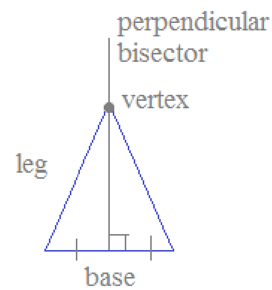




3) If two triangles have 2 congruent sides and 1 congruent angle, then they are congruent.

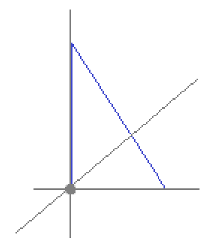
SOMETIMES... If 2 sides and the included angle are congruent, then triangles are congruent (Side-Angle-Side)

4) In an isosceles triangle, the *perpendicular bisector* from the base goes through the vertex.



ALWAYS

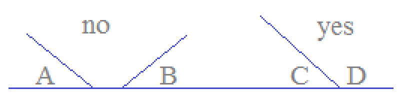
5) The *altitudes* of a right triangle are concurrent (intersect) at a point inside the triangle.



NEVER In a right triangle, the altitudes meet at the 90 degree vertex... (on the triangle, not inside)

6) Two angles on a straight line are *supplementary*.

SOMETIMES



7) If two triangles have 2 congruent angles and 1 congruent side, then they are congruent.

ALWAYS 2 congruent angles guarantee the 3rd pair of angles is congruent ("no choice theorem") Therefore, the 2 triangles are congruent by (Angle-Side-Angle)

8) An *obtuse triangle* can have 2 obtuse angles.

NEVER The sum of two obtuse angles would be greater than 180. Impossible.

Triangles: Always, Sometimes, Never & True/False

ANSWERS

II. True/False

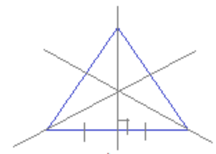
Answer each statement. If false, provide a counterexample.

1) If one of the base angles of an isosceles triangle is 20 degrees, then the triangle is obtuse.

TRUE Since one base angle is 20, the other is 20...
then, the third angle must be 140 degrees... (sum is 180) obtuse

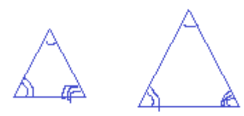
2) In an equilateral triangle, an *altitude* is a *median*.

TRUE



3) If the corresponding angles of 2 triangles are congruent, then the triangles must be congruent.

FALSE The triangles must be <u>similar</u>....
They *might* be congruent...



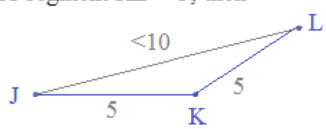
congruent angles
but,
not congruent
triangles

4) If $\overline{AB}$ bisects $\overline{CD}$ through point M, then $\overline{AM} = \overline{MB}$

FALSE $\overline{CM} = \overline{MD}$

5) If the length of segment JK = 5 and the length of segment KL = 5, then the length of segment JL must be 10.

FALSE



6) The *angle bisector* of an obtuse angle produces 2 congruent acute angles.

TRUE obtuse angle x : $90 < x < 180$

divide by 2: $45 < x/2 < 90$

7) An obtuse triangle can be *scalene*.

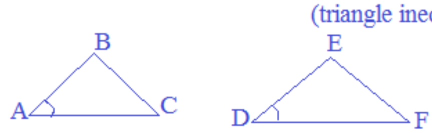
TRUE (and, it can be isosceles)

8) The third side of an *isosceles* triangle with sides 4 and 9 must be 9.

TRUE since the sides are 4 and 9, the third side could be 4 or 9... HOWEVER, if the other side were 4, then it would be a 4-4-9 triangle... IMPOSSIBLE
(triangle inequality theorem)

9) If $\triangle ABC \cong \triangle DEF$, then $\angle CAB \cong \angle EDF$

TRUE



10) If all corresponding angles are congruent, then 2 triangles are necessarily congruent.

FALSE (could be.. but, not necessarily)

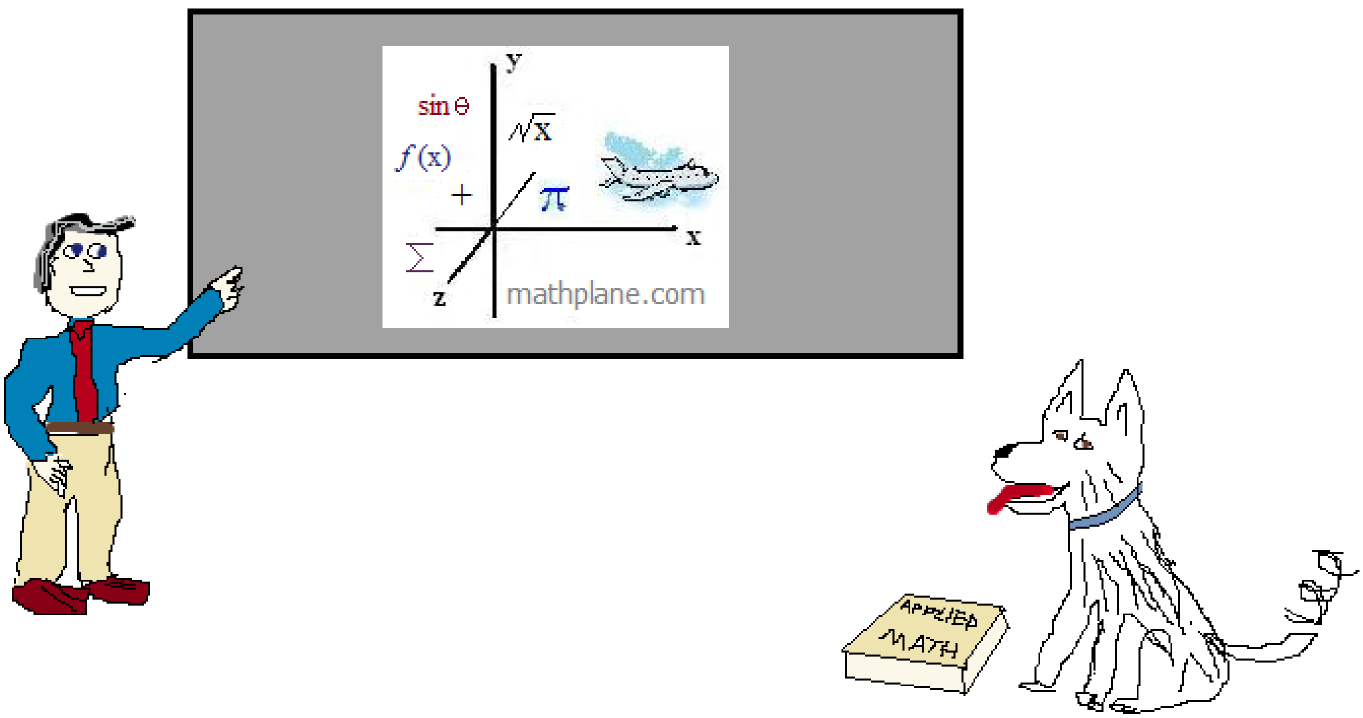
11) If all corresponding sides are congruent, then two triangles are congruent.

TRUE (side-side-side theorem)

Thanks for visiting! Hope the notes helped a bit.

If you have questions, suggestions, or feedback, let us know.

Best always,



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