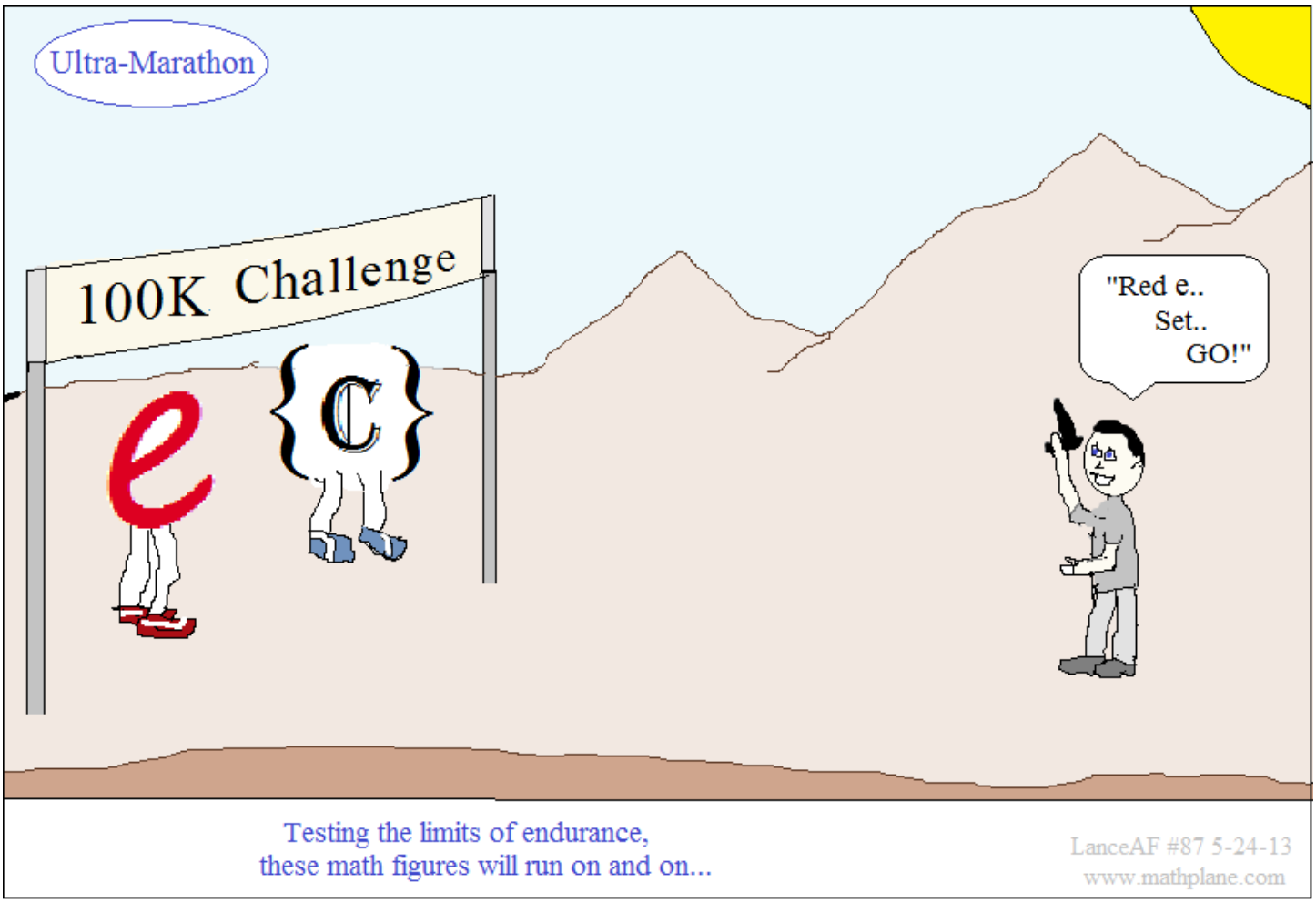
Algebra II (Honors) Review

21+ questions (and answers)



Topics include logarithms, conics, polynomials, factoring, graphing rational expressions, complex numbers, and more.

Algebra II Review 004

1) What is the domain of $\sqrt{6x - x^2}$

2) Simplify $\dfrac{5 - 3i}{3 + 4i}$

3) Find the average rate of change of $f(t) = t^3 + 2t + 4$

between $t = 1$ and $t = 3$

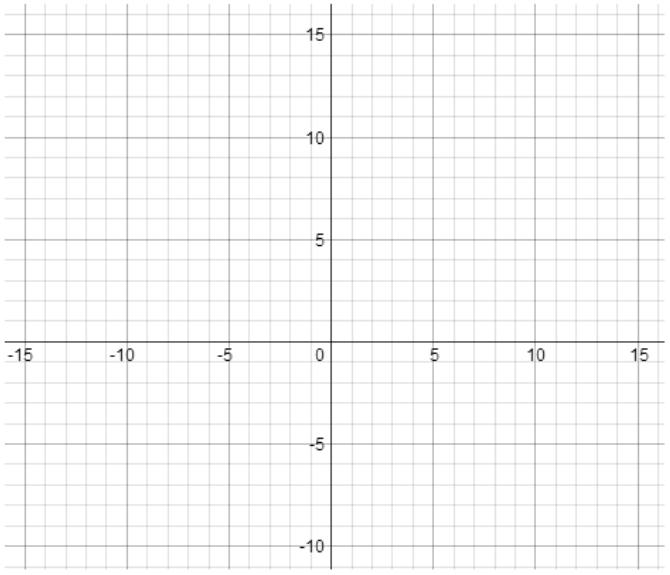
4) For $y = 3(x + 2)^2 + 6$, determine any x and y-intercepts.

5) $\sqrt{7x + 4} = x + 2$

Algebra II Review 004

6) For the function $f(x) = \dfrac{6x^2 - 96}{2x^2 + 5x - 12}$

Identify the intercepts, asymptotes, holes (if any), and graph.



7) $3|x - 2| + 6 < 12$

Solve and graph:

8) $A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & -4 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 6 & 3 \\ 2 & -5 \end{bmatrix}$

Find AB

Find BA

9) $\dfrac{x^2 - 2x - 15}{x^2 - 6x + 5} \div \dfrac{x^2 - x - 12}{x^2 - 1}$

Algebra II Review 004

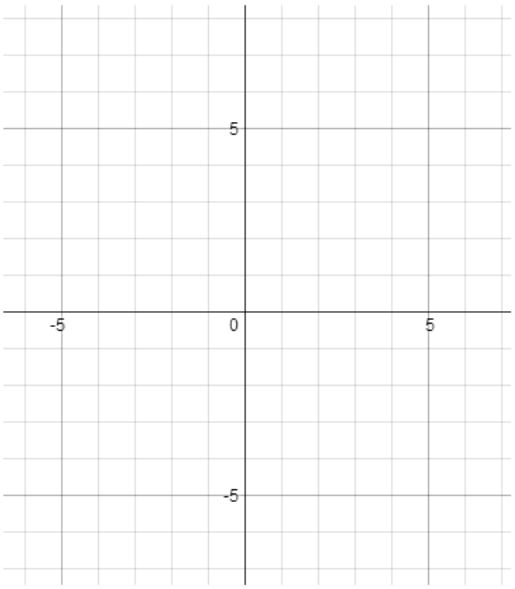
10) How long will it take for a $1000 deposit to increase to $1800 if the bank offers 7% interest compounded annually?
How long if the bank compounds the deposit *continuously*?

11) What is the equation of the ellipse?

Vertices: $(0, \sqrt{26})$ $(0, -\sqrt{26})$

Foci: $(0, 2\sqrt{3})$ $(0, -2\sqrt{3})$

Graph the figure.



12) Find *linear* and *exponential* equations that pass through (1, 50) and (2, 25).

Algebra II Review 004

13) What is the remainder of $2x^5 + 3x^4 - x^3 + 9$ divided by $(x - 1)$

14) Find a 3rd degree polynomial with zeros 2, 4, and -1 that goes through the point (1, 18)

15) Solve $\frac{6x}{x^2 + 2} - 2 = 0$

16) a) Solve $x^4 + 5x^2 - 36 = 0$

b) Factor $a^2 - 14a + 49 - 9b^2$

17) Find the center and radius of the sphere $x^2 + y^2 + z^2 - 4x + 10y + 8z - 27 = 0$

Algebra II Review 004

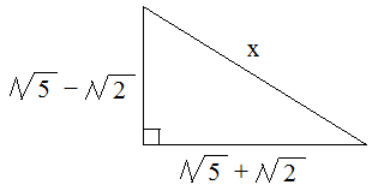
18) Find x:

a) $\log_4 (x^2 - 4) = 1$

b) $x = 3\log_2 4$

c) $\ln(e^3) + \log 100 + \log_5 \sqrt{5} = x$

19) What is the length of X?



20) A ball is dropped from 44 cm above the ground.
Each time it bounces, the ball retraces 60% of its previous height.

a) What is the height of the ball *after* the 8th bounce?

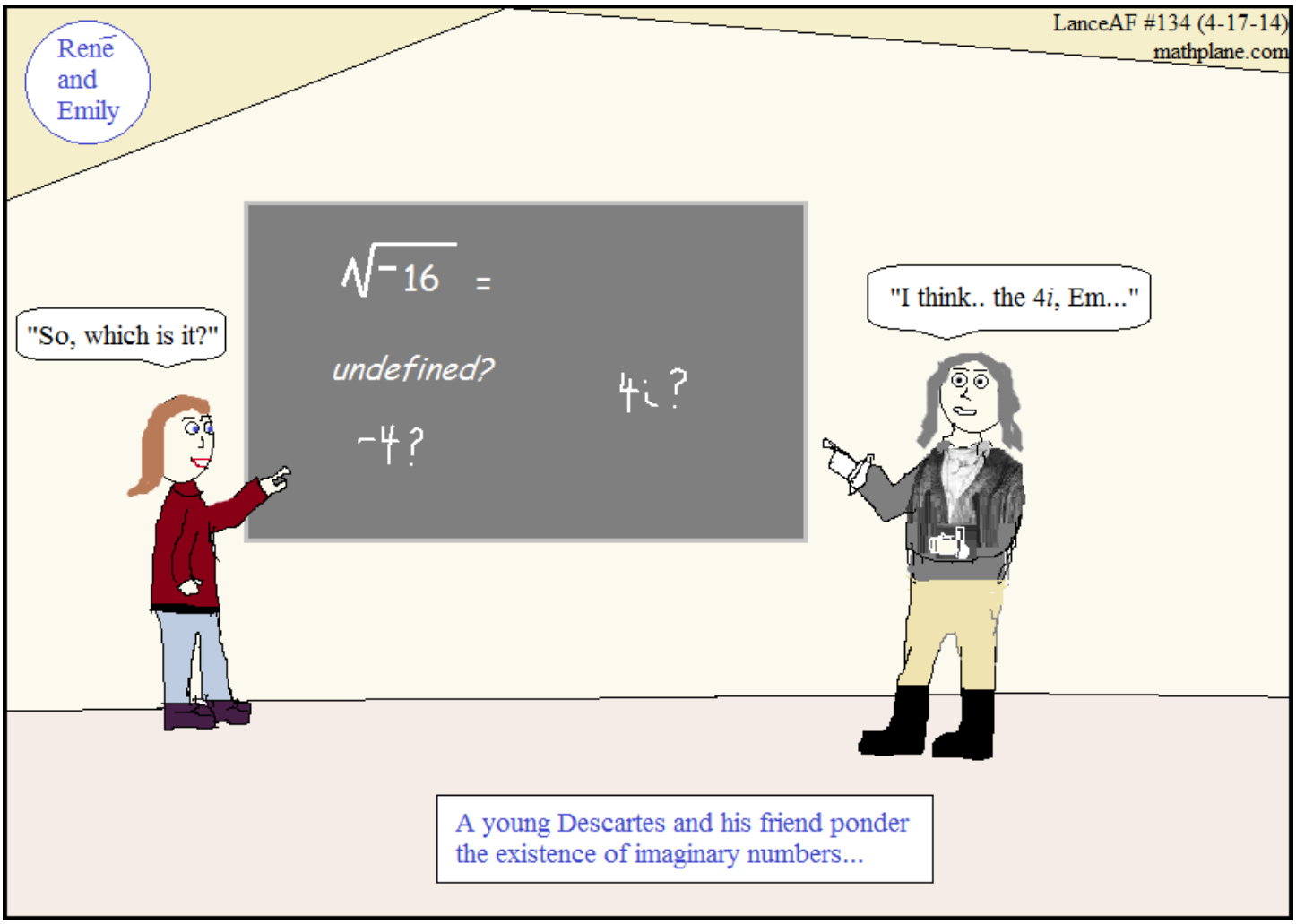
b) How far will the ball travel before it 'comes to rest'?

21) Find the inverse of each function:

a) $f(x) = e^{x+4} - 5$

b) $g(x) = 2x^3 + 1$

c) $h(x) = \frac{2x + 5}{x - 7}$


LanceAF #134 (4-17-14)
mathplane.com
René and Emily
$\sqrt{-16}$ =
undefined?
4i?
-4?
"So, which is it?"
"I think.. the 4i, Em..."
A young Descartes and his friend ponder the existence of imaginary numbers...

SOLUTIONS -→

Algebra II Review 004

1) What is the domain of $\sqrt{6x - x^2}$

For real numbers, there cannot be negative numbers under the radical sign:

$(6x - x^2) \geq 0$

To solve, factor and find critical values

$x(6 - x) = 0$

$x = 0, 6$

Then, test regions and check..

-5 doesn't work | 3 works | 10 doesn't work

domain: $0 \leq x \leq 6$

or $[0, 6]$

2) Simplify $\frac{5 - 3i}{3 + 4i}$

To simplify, a complex number must be in the form $a + bi$

multiply the numerator and denominator by the conjugate:

$$\frac{5 - 3i}{3 + 4i} \cdot \frac{3 - 4i}{3 - 4i} = \frac{15 - 9i - 20i + 12i^2}{9 - 12i + 12i - 16i^2}$$

$$= \frac{15 - 29i + 12(-1)}{9 - 16(-1)} = \boxed{\frac{3}{25} - \frac{29}{25}i}$$

3) Find the average rate of change of $f(t) = t^3 + 2t + 4$

between $t = 1$ and $t = 3$

The *average* rate of change is the slope between 2 points!

At $t = 1$, $f(1) = 1 + 2(1) + 4 = 7$ $(1, 7)$

At $t = 3$, $f(3) = 27 + 2(3) + 4 = 37$ $(3, 37)$

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{37 - 7}{3 - 1} = \boxed{15}$$

4) For $y = 3(x + 2)^2 + 6$, determine any x and y-intercepts.

The y-intercept is the point where an equation crosses the y-axis. Since the coordinate will be (0, ?), substitute 0 into the equation:

$y = 3(0 + 2)^2 + 6$

$y = 18$

(0, 18) is the y-intercept

And, since the x-intercept is a point where an equation crosses the x-axis, the coordinate(s) will be (? , 0). Substitute 0 into the equation:

$0 = 3(x + 2)^2 + 6$

$0 = 3x^2 + 12x + 12 + 6$

$0 = 3(x^2 + 4x + 6)$

$0 = (x^2 + 4x + 6)$

Note: the discriminant is $b^2 - 4ac = (4)^2 - (4)(1)(6) = -16$

Since the discriminant is *less than zero*, there is no x-intercept!

5) $\sqrt{7x + 4} = x + 2$

square both sides $7x + 4 = (x + 2)^2$

collect "like" terms $0 = x^2 + 4x + 4 - 7x - 4$

$x^2 - 3x = 0$

factor and solve $x(x + 3) = 0$

$x = 0$ or 3

check answers $\sqrt{7(0) + 4} = (0) + 2$

$2 = 2$ ✓

$\sqrt{7(3) + 4} = (3) + 2$

$5 = 5$ ✓

Both 0 and 3 are solutions

Algebra II Review 004

6) For the function $f(x) = \dfrac{6x^2 - 96}{2x^2 + 5x - 12}$

Identify the intercepts, asymptotes, holes (if any), and graph.

factor the numerator and denominator

$$f(x) = \frac{6(x + 4)(x - 4)}{(2x - 3)(x + 4)}$$

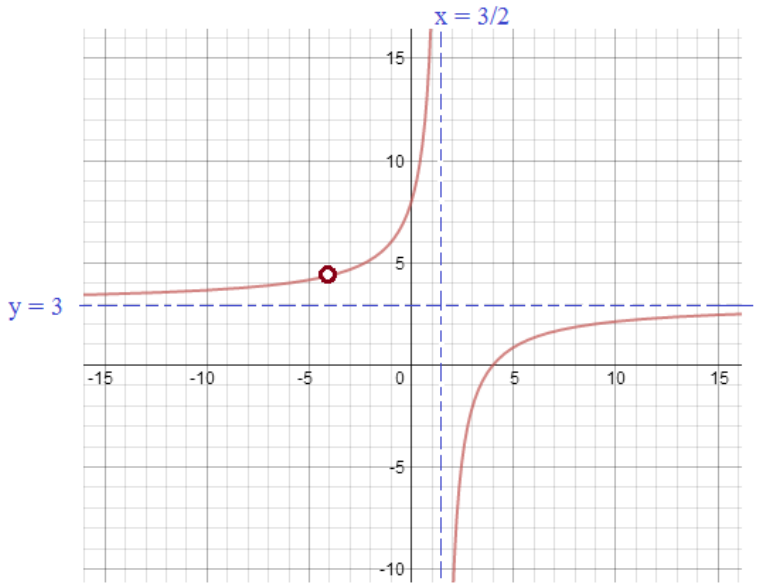
horizontal asymptote: y = 3 (lead coefficients: 6/2)

vertical asymptote: x = 3/2

y-intercept: (0, 8)

x-intercept: (4, 0)

"hole": (-4, 48/11)



7) $3|x - 2| + 6 < 12$

Solve and graph:

find critical points ---

isolate absolute value: $3|x - 2| + 6 = 12$

$3|x - 2| = 6$

$|x - 2| = 2$

separate:

$x - 2 = 2$ $\qquad$ $x - 2 = -(2)$

$x = 4$ $\qquad$ $x = 0$

test points:

$0 < x < 4$

since it is < , use open circles...

8) $A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & -4 & 6 \end{bmatrix}$ $\qquad$ $B = \begin{bmatrix} 1 & 1 \\ 6 & 3 \\ 2 & -5 \end{bmatrix}$

Find AB $\begin{bmatrix} 20 & 11 \\ -11 & -41 \end{bmatrix}$

Find BA $\begin{bmatrix} 1 & 1 \\ 6 & 3 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} 2 & 3 & 0 \\ 1 & -4 & 6 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 6 \\ 15 & 6 & 18 \\ -1 & 26 & -30 \end{bmatrix}$

9) $\dfrac{x^2 - 2x - 15}{x^2 - 6x + 5} \div \dfrac{x^2 - x - 12}{x^2 - 1}$

Factor.. then, invert and multiply

$$\frac{(x - 5)(x + 3)}{(x - 5)(x - 1)} \cdot \frac{(x + 1)(x - 1)}{(x - 4)(x + 3)}$$

Cancel and simplify

$$\frac{\cancel{(x - 5)}(x + 3)}{\cancel{(x - 5)}\cancel{(x - 1)}} \cdot \frac{(x + 1)\cancel{(x - 1)}}{(x - 4)\cancel{(x + 3)}} = \frac{(x + 1)}{(x - 4)}$$

Algebra II Review 004

10) How long will it take for a $1000 deposit to increase to $1800 if the bank offers 7% interest compounded annually?
How long if the bank compounds the deposit *continuously*?

7% compounded annually: $A = P(1 + r)^t$

$1800 = 1000(1.07)^t$

$1.8 = (1.07)^t$

$\log(1.8) = t(\log(1.07))$

$t = 8.69$ years

7% compounded continuously: $A = Pe^{rt}$

$\ln(1.8) = .07t(\ln e)$

$\ln(1.8) = .07t$

$t = 8.39$ years

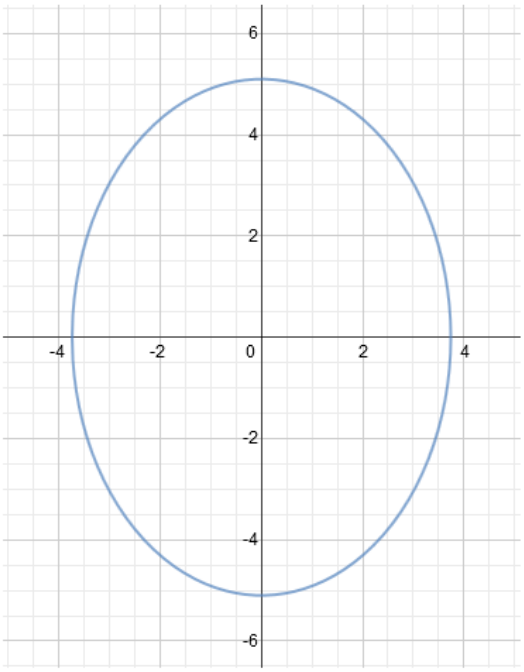
quick check: "Rule of 72" 72/7 = 10.2 years for an amount to double..
(1000 to increase to 2000)..
so, 1000 to 1800 in 8.7 years seems reasonable.

11) What is the equation of the ellipse?

Vertices: $(0, \sqrt{26})$ $(0, -\sqrt{26})$

Foci: $(0, 2\sqrt{3})$ $(0, -2\sqrt{3})$

Graph the figure.



major semi-axis

$a = \sqrt{26}$

center is midpoint of vertices: (0, 0)

$c = 2\sqrt{3}$

$c^2 = a^2 - b^2$

$12 = 26 - b^2$

$b = \pm\sqrt{14}$

minor semi-axis

$$\frac{y^2}{26} + \frac{x^2}{14} = 1$$

since foci are between vertices above and below, this is a vertical ellipse.

12) Find *linear* and *exponential* equations that pass through (1, 50) and (2, 25).

Line: we need the slope

$m = \frac{50 - 25}{1 - 2} = -25$

point slope form

$y - 50 = -25(x - 1)$

or

$y = -25x + 75$

slope intercept form

Exponential equation: $y = ab^x$

Using substitution

$50 = ab^1$

$a = \frac{50}{b}$

$25 = ab^2$

$25 = \frac{50}{b}b^2$

$25 = 50b$

$b = 1/2$

if b = 1/2, then a = 100

$y = 100(1/2)^x$

Algebra II Review 004

13) What is the remainder of $2x^5 + 3x^4 - x^3 + 9$ divided by $(x - 1)$

Using "Remainder Theorem"

$f(1) = 2(1)^5 + 3(1)^4 - 1(1)^3 + 9 =$ **13**

Using Synthetic Division

1	2	3	-1	0	0	9
		2	5	4	4	4
	2	5	4	4	4	**13**

Using Long Division

$$x - 1 \;\overline{)\; 2x^5 + 3x^4 - x^3 + 9}$$

$$2x^4 + 5x^3 + 4x^2 + 4x + 4$$

$$-(2x^5 - 2x^4)$$

$$5x^4 - x^3$$

$$-(5x^4 - 5x^3)$$

$$4x^3 + 0x^2$$

$$-(4x^3 - 4x^2)$$

$$4x^2 + 0x$$

$$-(4x^2 - 4x)$$

$$4x + 9$$

$$-(4x - 4)$$

13

14) Find a 3rd degree polynomial with zeros 2, 4, and -1 that goes through the point (1, 18)

$y = a(x - x_1)(x - x_2)(x - x_3)$

substitute the 3 zeros

$y = a(x - 2)(x - 4)(x + 1)$

Then, to find a, insert the point into the equation

$18 = a(1 - 2)(1 - 4)(1 + 1)$

$18 = a(6)$

$a = 3$

$3(x - 2)(x - 4)(x + 1)$

$3x^3 - 15x^2 + 6x + 24$

15) Solve $\dfrac{6x}{x^2 + 2} - 2 = 0$

$\dfrac{6x}{x^2 + 2} = \dfrac{2}{1}$

$2(x^2 + 2) = 6x(1)$

$2x^2 + 4 = 6x$

$2x^2 - 6x + 4 = 0$

$2(x^2 - 3x + 2) = 0$

$2(x - 2)(x - 1) = 0$

$x = 1, 2$

16) a) Solve $x^4 + 5x^2 - 36 = 0$

$(x^2 + 9)(x^2 - 4) = 0$

$(x^2 + 9)(x + 2)(x - 2) = 0$

real solutions: -2, 2

imaginary solutions: $3i$, $-3i$

b) Factor $a^2 - 14a + 49 - 9b^2$

group: $a^2 - 14a + 49 - 9b^2$

$(a - 7)(a - 7) - 9b^2$

$(a - 7)^2 - 9b^2$ (difference of squares)

$(a - 7 + 3b)(a - 7 - 3b)$

17) Find the center and radius of the sphere $x^2 + y^2 + z^2 - 4x + 10y + 8z - 27 = 0$

(similar to a circle in a plane), the best way to find the center and radius is to express the equation in standard form...

complete the square:

$x^2 - 4x + 4 + y^2 + 10y + 25 + z^2 + 8z + 16 = 27 + 4 + 25 + 16$

$(x - 2)^2 + (y + 5)^2 + (z + 4)^2 = 72$

Center: $(2, -5, -4)$

Radius: $\sqrt{72} = 6\sqrt{2}$

Algebra II Review 004

18) Find x:

a) $\log_4 (x^2 - 4) = 1$

$4^1 = (x^2 - 4)$

$x^2 = 8$

$\sqrt{8}$ and $-\sqrt{8}$

b) $x = 3\log_2 4$

power rule:

$x = \log_2 4^3$

$x = \log_2 64$

$2^x = 64$

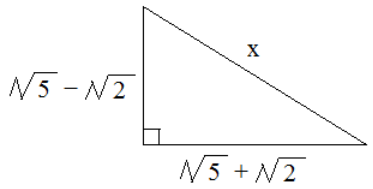
$x = 6$

c) $\ln(e^3) + \log 100 + \log_5 \sqrt{5} = x$

$3\ln e + \log_{10} 10^2 + \log_5 (5)^{1/2}$

$3 + 2 + 1/2 = 5\frac{1}{2}$

19) What is the length of X?



Use Pythagorean Theorem to find x

$a^2 + b^2 = c^2$

$(\sqrt{5} + \sqrt{2})^2 + (\sqrt{5} - \sqrt{2})^2 = x^2$

$(\sqrt{5} + \sqrt{2})(\sqrt{5} + \sqrt{2}) + (\sqrt{5} - \sqrt{2})(\sqrt{5} - \sqrt{2}) = x^2$

$5 + \sqrt{10} + \sqrt{10} + 2 \quad 5 - \sqrt{10} - \sqrt{10} + 2 = x^2$

$5 + \sqrt{10} + \sqrt{10} + 2 \quad 5 - \sqrt{10} - \sqrt{10} + 2 = x^2$

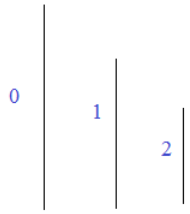
$14 = x^2$

$\sqrt{14} = x$

20) A ball is dropped from 44 cm above the ground.
Each time it bounces, the ball retraces 60% of its previous height.

a) What is the height of the ball *after* the 8th bounce?

b) How far will the ball travel before it 'comes to rest'?



$S_\infty = \frac{a_1}{(1 - r)}$

The height after a particular bounce can be expressed as

$a_0 = 44$

$a_1 = 44\ (.60) = 26.2$ cm

$a_n = 44(.60)^n$

$a_2 = 44\ (.60)(.60)$

$a_8 = 44(.60)^8 =$ .739 cm above ground

The distance travel will be the sum of all the bounces (up and down).
And, the number of bounces will be infinite...

distance traveled (going down): initial move: 44 cm..

$S_{down} = \frac{44}{(1 - .60)} = 110$ cm

distance traveled (going up): inital move: 26.2 cm..

$S_{up} = \frac{26.2}{(1 - .60)} = 65.5$ cm

Total distance traveled will approach 175.5 cm

21) Find the inverse of each function:

a) $f(x) = e^{x+4} - 5$

let $y = e^{x+4} - 5$

then, switch x and y...

$x = e^{y+4} - 5$

solve for y....

$x + 5 = e^{y+4}$

$\ln(x + 5) = \ln e^{y+4}$

$\ln(x + 5) = (y + 4)\ln e$

$\ln(x + 5) = y + 4$

$y = \ln(x + 5) - 4$

b) $g(x) = 2x^3 + 1$

$y = 2x^3 + 1$

$x = 2y^3 + 1$

$\frac{x - 1}{2} = y^3$

$g^{-1}(x) = \sqrt[3]{\frac{x - 1}{2}}$

c) $h(x) = \frac{2x + 5}{x - 7}$

let $h(x) = y$
and
"flip" the x and y..

Then, solve for y

$x = \frac{2y + 5}{y - 7}$

$x(y - 7) = 2y + 5$

$xy - 7x = 2y + 5$

$xy - 2y = 5 + 7x$

$y(x - 2) = 7x + 5$

$y = \frac{7x + 5}{x - 2}$

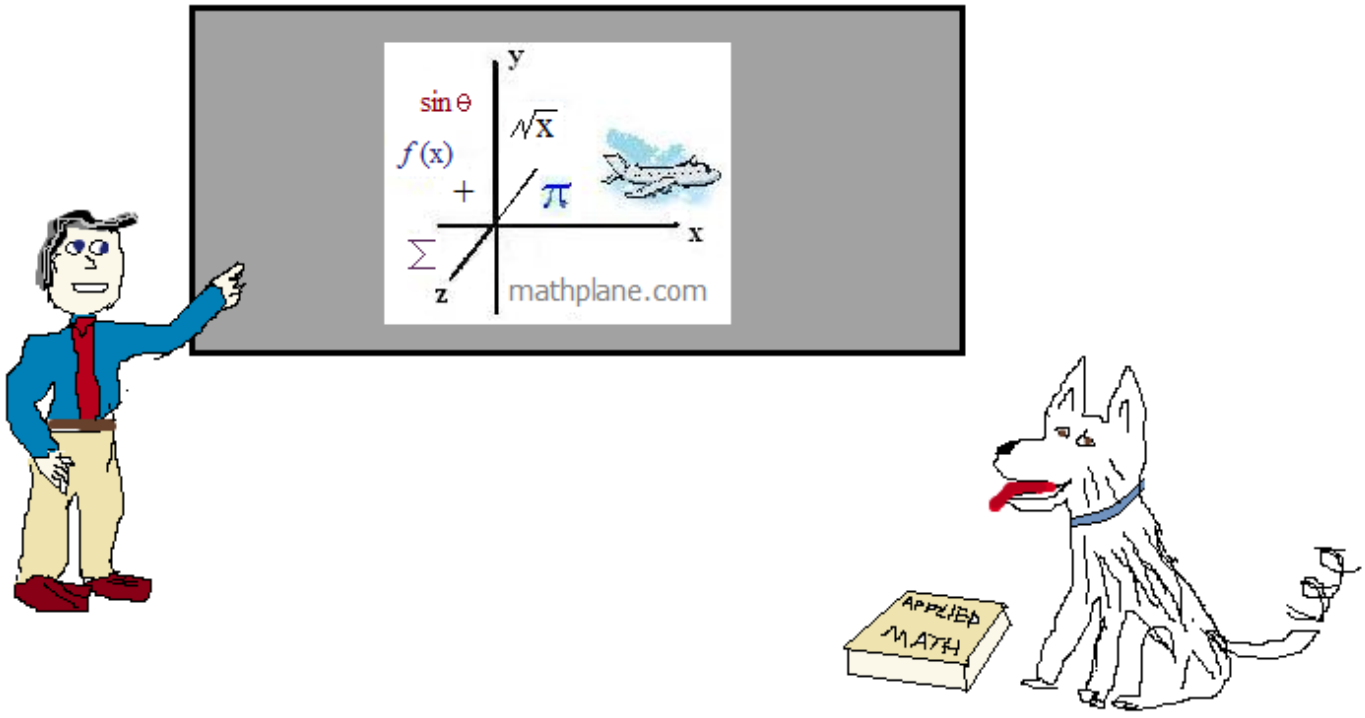
change back to function notation

$h^{-1}(x) = \frac{7x + 5}{x - 2}$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Enjoy.



Also, at Facebook, Google+, TeachersPayTeachers, and Pinterest

Hidden message Puzzle-→

Hidden Message

Hint: "counting math treats?"

Letter/Number Key									
1	2	3	4	5	6	7	8	9	0
A	E	G	I	M	N	R	S	T	U

Solve the 12 problems below....
Then, convert the numbers into letters to reveal the answer!

1) $\log(100{,}000)$ → ☐ ____

2) $6! =$ 7 ☐ 0 → ____

3) Evaluate the determinant: $D = \begin{vmatrix} 3 & 5 \\ 1 & 2 \end{vmatrix}$ → ☐ ____

4) What is the *zero* (i.e. x-intercept) in the quadratic equation $x^2 - 16x + 64$? → ☐ ____

5) $5^2 + (5i)^2 =$ → ☐ ____

6) If $f(x) = 3(x + 7) - 20$, then what is $f(2)$? → ☐ ____

7) $(4\sqrt{2})^2$ → 3 ☐ ____

8) 250% of 2 → ☐ ____

9) $64^{\frac{1}{3}} =$ → ☐ ____

10) The geometric mean between 2 and 18. → ☐ ____

11) Find the discriminant in the following quadratic: $2x^2 + 7x + 5$ → ☐ ____

12) The y-intercept of $y = 2|x - 3| + 2$ → ☐ ____

Hidden Message

SOLUTIONS

Hint: "counting math treats?"

Letter/Number Key									
1	2	3	4	5	6	7	8	9	0
A	E	G	I	M	N	R	S	T	U

Solve the 12 problems below....
Then, convert the numbers into letters to reveal the answer!

---

1) log(100,000) $\log_{10}(100{,}000) = X$ (change to exponent form) $10^X = 100{,}000$, $X = 5$ → 5 → M

2) $6! =$ $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ → 7 2 0 → E

3) Evaluate the determinant: $D = \begin{vmatrix} 3 & 5 \\ 1 & 2 \end{vmatrix}$ $(3 \times 2) - (5 \times 1) = 1$ → 1 → A

4) What is the *zero* (i.e. x-intercept) in the quadratic equation $x^2 - 16x + 64$? (x, 0) $x^2 - 16x + 64 = 0$, $(x - 8)(x - 8) = 0$, $x = 8$ → 8 → S

5) $5^2 + (5i)^2 =$ $(5 \cdot 5) + (5i \cdot 5i) = 25 + (-25) = 0$ → 0 → U

6) If $f(x) = 3(x + 7) - 20$, then what is $f(2)$? $3((2) + 7) - 20 = 3(9) - 20 = 7$ → 7 → R

7) $(4\sqrt{2})^2$ $(4\sqrt{2})(4\sqrt{2}) = 16 \cdot 2 = 32$ → 3 2 → E

8) 250% of 2 $2.50 \times 2 = 5$ 100% of 2 is 2, 200% of 2 is 4, 250% of 2 is 5 → 5 → M

Measure-mints

9) $64^{\frac{1}{3}} =$ 4 → 4 → I

10) The geometric mean between 2 and 18. the *arithmetic* mean is 10, but the *geometric* mean is 6 $\frac{2}{X} = \frac{X}{18}$ → 6 → N

11) Find the discriminant in the following quadratic: $2x^2 + 7x + 5$ discriminant is $b^2 - 4ac$, $(7)^2 - 4(2)(5) = 9$ → 9 → T

12) The y-intercept of $y = 2|x - 3| + 2$ → 8 → S

y-intercept: (0, y) $2|0 - 3| + 2$, $2(3) + 2 = 8$