# Algebra I / Geometry 

Partial Review
Examples and practice questions (with solutions)


Topics include coordinate plane, linear equations, order of operations, quadratics, graphing, and more.

## Algebra/Geometry Review

Example: The diameter of a circle has endpoints $(1,3)$ and $(9,-5)$.
What is the center of the circle?
What is the area of the circle?

Step 1: Draw a picture


Step 3: Solve and answer the questions
To find the center, we'll take the midpoint of $(1,3)$ and $(9,-5)$

$$
\text { center }=\left\langle\frac{1+9}{2}, \frac{3+-5}{2}\right\rangle=(5,-1)
$$

note: 5 is halfway between 1 and 9
-1 is four units from 3 and four units from -5

Step 2: Outline the problem; identify formulas
All diameters go through the center of the circle.
So, the midpoint of these endpoints will be the center.

Then, to find the area of a circle, we need the length of the diameter or radius (use distance formula).
$\operatorname{Midpoint}(x, y)=\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right) \quad$ Area of circle $=\Pi$ (radius) ${ }^{2}$
Distance $=\sqrt{\left(x_{1}-x_{2}\right)^{2}+\left(y_{1}-y_{2}\right)^{2}}$

$$
\begin{aligned}
& \text { To determine the length of } \\
& \text { the radius, let's find the } \\
& \text { distance from }(5,-1) \text { to }(1,3) \text { : } \\
& \begin{aligned}
\text { radius } & =\sqrt{(5-1)^{2}+(-1-3)^{2}} \\
& =\sqrt{16+16}=4 \sqrt{2}
\end{aligned}
\end{aligned}
$$

Step 4: Check your answer.
Looking at the graph, we can see the answers are 'reasonable'

The radius length is a bit over 4 and, the square units are approx. 100...

Then, the area of the circle is

$$
T(4 \sqrt{2})^{2}=32 T
$$

Example: Describe the line parallel to $\mathrm{x}+3 \mathrm{y}=6$
that passes through the origin?
What line is perpendicular to the above line
and goes through $(2,1)$ ?

Step 1: Set up the problem
The origin is $(0,0)$..
A parallel line has the same slope.
Then, perpendicular lines have slopes that are opposite reciprocals.

To describe a line, you need a point and the slope.

Step 2: Solve and answer

$$
\begin{aligned}
x+3 y & =6 \\
3 y & =-x+6 \quad \text { slope is }-1 / 3 \\
y & =\frac{-x}{3}+2
\end{aligned}
$$

parallel line through the origin:

$$
\begin{gathered}
y-0=-1 / 3(x-0) \\
y=\frac{-1}{3} x
\end{gathered}
$$

perpendicular line through $(2,1)$ :

| $3 x-y=5$ | $y-1=3(x-2) \quad$ pt. slope form |  |
| ---: | :---: | :---: |
| standard form | $y=3 x-5$ | slope intercept form |

Linear Equations, slope, perpendicular/parallel

Step 3: Check your answer (sketch the lines and see if they appear parallel and perpendicular)


## Algebra/Geometry Review

Example: Graph the equation $\mathrm{y}=3(\mathrm{x}-2)(\mathrm{x}-8)$;
Quadratics (parabolas), graphing, completing the square
Identify the x -intercept(s), y -intercept, and vertex.
(optional) The above equation is in factored form. (or, intercept form)
Express the equation in standard form and vertex form.
What is the x -intercept(s)?
It's the point(s) where a line or curve crosses the x -axis.
In other words, any coordinate (?, 0)...
To find the x -intercept, set $\mathrm{y}=0$ :

$$
\begin{gather*}
0=3(x-2)(x-8)  \tag{2,0}\\
x=2,8
\end{gather*}
$$

What is the y-intercept?
It's the point where a line or curve crosses the $y$-axis.
In other words, any coordinate ( 0, ?)
To identify the y -intercept, set $\mathrm{x}=0$

$$
\mathrm{y} \text {-intercept: }(0,48)
$$

$$
\begin{gathered}
y=3(0-2)(0-8) \\
y=48
\end{gathered}
$$

How do you find the vertex?
When given a quadratic in factored form, you can find the axis of symmetry by recognizing the midpoint of 2 (horizontal/'mirror') points!

In other words, what is the midpoint of $(2,0)$ and $(8,0)$ ?
The middle value is $(5,0) \cdots$ The axis of symmetry is $x=5 \ldots$ (going through that middle value)
Since the vertex lies on the axis of symmetry, the vertex is $(5, ?)$

$$
\begin{gathered}
y=3(5-2)(5-8) \\
y=-27
\end{gathered}
$$

Vertex: $(5,-27)$


Let's check the answers:
$y=3(x-2)(x-8)$
convert to standard form: $y=3\left(x^{2}-2 x-8 x+16\right)$

$$
y=3 x^{2}-30 x+48 \quad \text { standard form }
$$

y-intercept is shown: 48
axis of symmetry: $\frac{-\mathrm{b}}{2 \mathrm{a}}=\frac{-(-30)}{2(3)}=5$
convert to vertex form: $\quad y=3 x^{2}-30 x+48$
(complete the square)

$$
\begin{aligned}
& y=3\left(x^{2}-10 x\right)+48 \\
& y=3\left(x^{2}-10 x+25\right) \quad+48-3(25) \\
& y=3(x-5)^{2}-27 \\
& y
\end{aligned}
$$

the vertex is $(5,-27)$

Given: $\overline{\mathrm{MT}} \xlongequal{\cong} \overline{\mathrm{MH}}$
$\overline{\mathrm{AS}}$ bisects $\overline{\mathrm{MH}}$

What is the perimeter of $\triangle \mathrm{MTH}$ ?

Solution:
$y+5=x+4$
(because A is the midpoint of MT)
$2 x=y+5$
(because S is midpoint of MH and $\mathrm{MT}=\mathrm{MH}$ )

Since we have 2 equations and 2 unknowns, we can solve:

$$
\begin{aligned}
& \text { (rewrite equations) } \\
& \begin{array}{l}
x-y=1 \\
2 x-y=5 \\
\text { (elimination method) } \\
-x=-4 \\
x=4 \\
y=3
\end{array}
\end{aligned}
$$

Substitute into the triangle, add up the segments:
$8+8+18+8+8=50$



## Practice Questions $\rightarrow$

Solve the following:

1) $\frac{1}{3}-2 \div-3 \frac{1}{4}=$
2) $\left(4 \frac{1}{3}\right)\left(-3 \frac{1}{3}\right)-\left(-3 \frac{4}{7}\right)=$
3) $(-1.5) \times 3.5+2.9=$
4) $2.4 \times 3.6 \times(-1.7)=$
5) $10=2|x-4|+4$ $\mathrm{x}=$
6) $3|x+4|+8=2$
$\mathrm{x}=$
7) Solve using the quadratic formula
a) $2 x^{2}+7 x-3=0$
$\mathrm{x}=$
b) $-x^{2}+11 x=8$ $\mathrm{x}=$

Solving Quadratic Equations Exercise

1) $x^{2}+11 x+10=0$
2) $x^{2}-16 x+28=0$
3) $x^{2}-6 x-7=0$
4) $3 x^{2}+9 x+6=0$
5) $2 x^{2}-7 x+5=0$
6) $x^{2}+4 x-7=0$

Find the equations of the lines.
Linear Equations Review Exercise
Compare the slopes, and determine if they are parallel, perpendicular, or neither.


Given: $\triangle \mathrm{ABC} \quad \mathrm{A}=(1,2)$
$B=(5,6)$

$$
C=(6,5)
$$

Find: The equation for the median from C

Given: $\triangle$ LMN

$$
\begin{aligned}
& \mathrm{L}=(1,1) \\
& \mathrm{M}=(5,-3) \\
& \mathrm{N}=(2,8)
\end{aligned}
$$

Find: The equation of the perpendicular bisector of $\overline{\mathrm{LM}}$

Systems of (Linear) Inequalities
Solve and Graph the following:

1) $y<2 x+4$
$2 x-y \leq 4$

2) A tortilla recipe consists of any combination of flour and corn. Flour costs $\$ 1.50$ per pound; and, corn costs $\$ 2.50$ per pound. If you need over 4 pounds of flour and corn, but you must spend no more than $\$ 9.50$, find and graph the production possibilities.
a) Suppose you prefer corn. How much corn would you use?
b) Suppose you have no preference. Identify the maximum weight that could be used within the budget.
3) Mathflix offers 2 rental plans: Plan A: $\$ 20$ per month and $\$ 1$ per movie

Plan B: $\$ 10$ per month and $\$ 1.50$ per movie
Your household movie budget is $\$ 50$ per month.
You watch at least 10 movies per month.
a) Assume Mathflix allows you to select a Plan at the end of the month. Using the graph, show how much you'll pay Mathflix after one month.
b) What is the 'break-even' number of movies?
(At what point is the cost of each plan the same?)

1) Given a line segment with endpoints $A=(-4,-3)$ and $B=(8,6)$.

Find the equation of the line perpendicular to segment $\overline{\mathrm{AB}}$, going through the point that is three-fourths of the distance from $B$ to $A$.
Write your answer in standard form (i.e. $\mathrm{Ax}+\mathrm{By}=\mathrm{C}$, where $\mathrm{A}, \mathrm{B}$, and C are integers)
2) Given $\mathrm{z}=\mathrm{a}+\mathrm{b} i$ and $\overline{\mathrm{z}}=\mathrm{a}-\mathrm{b} i$. (i.e. z and $\overline{\mathrm{z}}$ are complex conjugates)

Solve for z , where

$$
5 z-3 \bar{z}=3+2 i
$$



SOLUTIONS - -

Solve the following:
SOLUTIONS

1) $\frac{1}{3}-2 \div-3 \frac{1}{4}=$

Change to (similar) improper fractions

$$
\frac{1}{3}-\frac{2}{1} \div \frac{13}{4}=
$$

(Order of operations) -- divide first!

$$
\frac{1}{3}-\frac{-8}{13}=\frac{13}{39}-\frac{24}{39}=\frac{37}{39}
$$

3) $(-1.5) \times 3.5+2.9=$

| -1.5 <br> $\times \quad 3.5$ <br> 75 <br> +450 <br> -5.25 | -5.25 <br> -2.35 <br> (2 decimal <br> places) |
| :--- | :--- |

(sign is negative because neg x pos)

$$
\text { 5) } \begin{aligned}
10 & =2|x-4|+4 \\
x & =1,7
\end{aligned}
$$

("isolate the absolute value")

$$
\begin{gathered}
6=2|x-4| \\
3=|x-4|
\end{gathered}
$$

(then, "split and solve")

$$
\begin{array}{cc}
3=x-4 \\
-3=x-4 & x=7 \\
x=1
\end{array}
$$

7) Solve using the quadratic formula
a) $2 x^{2}+7 x-3=0$
$\mathrm{x}=$
$\begin{aligned} & \mathrm{a}=2 \\ & \mathrm{~b}=7 \\ & \mathrm{c}=-3\end{aligned} \quad \mathrm{x}=\frac{-\mathrm{b} \pm \sqrt{\mathrm{b}^{2}-4 \mathrm{ac}}}{2 \mathrm{a}}$

$$
\begin{aligned}
& x=\frac{-7 \pm \sqrt{49-4(2)(-3)}}{2(2)} \\
& x=\frac{-7 \pm \sqrt{73}}{4}
\end{aligned}
$$

b) $-x^{2}+11 x=8$
$\mathrm{x}=$
$-x^{2}+11 x-8=0$
$\mathrm{a}=-1$
$\mathrm{b}=11$
$c=-8$

$$
x=\frac{-11 \pm \sqrt{89}}{-2}
$$

2) $\left(4 \frac{1}{3}\right)\left(-3 \frac{1}{3}\right)-\left(-3 \frac{4}{7}\right)=$
$\frac{13}{3} \cdot \frac{-10}{3}-\frac{-25}{7}=$

$$
\frac{-130}{9}+\frac{25}{7}=\frac{-910}{63}+\frac{225}{63}=\frac{-685}{63} \text { or }-10 \frac{55}{63}
$$

4) $2.4 \times 3.6 \times(-1.7)=$

|  | 2.4 | 8.64 |
| :---: | :---: | :---: |
| x | 3.6 | x -1.7 |
|  | 144 | 6048 |
| $+$ |  | 8640 |
| 8.64 |  | -14.688 |
|  |  | (3 decim |

6) $3|x+4|+8=2$

$$
\mathrm{x}=\text { no solution }
$$

$$
\begin{array}{r}
3|x+4|=-6 \\
|x+4|=-2 \\
\hline
\end{array}
$$

## NO REAL SOLUTION

absolute value must be positive!

Notice, $x+4=-2 \quad x=-6$
$x+4=2 \quad x=-2$
But, when you check your answers, neither solution will work...

1) $x^{2}+11 x+10=0 \quad$ plus plus
"what multiplies to 10 and adds to 11 ?"

$$
\begin{array}{r}
(x+10)(x+1)=0 \\
x=-10,-1
\end{array}
$$

(use 'zero product property'
to solve factored equation!)
2) $x^{2}-16 x+28=0$
3) $x^{2}-6 x-7=0$
plus minus
minus minus
"what multiplies to 28
and adds to -16?"

$$
\begin{gathered}
(x-2)(x-14)=0 \\
x=2,14
\end{gathered}
$$


multiplies to -7

$$
\begin{array}{r}
(x-7)(x+1)=0 \\
x=-1,7
\end{array}
$$

4) $3 x^{2}+9 x+6=0$

## greatest common factor

$$
\begin{gathered}
3\left(x^{2}+3 x+2\right)=0 \\
3(x+2)(x+1)=0 \\
x=-2,-1
\end{gathered}
$$

5) $2 x^{2}-7 x+5=0$

> split and divide or logic method

> "split and regroup"
> $\mathrm{AC}=10: \quad$ "what multiplies to $10 .$.
> $\mathrm{B}=-7: \quad \ldots$ and, adds to -7 "
split the middle term: $\quad 2 x^{2}-2 x-5 x+5=0$ regroup (factor): $\quad 2 x(x-1)+-5(x-1)=0$
6) $x^{2}+4 x-7=0 \quad$ quadratic formula

$$
\mathrm{x}=\frac{-\mathrm{b} \pm \sqrt{\mathrm{b}^{2}-4 \mathrm{ac}}}{2 \mathrm{a}}
$$

$$
\begin{aligned}
& \mathrm{a}=1 \\
& \mathrm{~b}=4 \\
& \mathrm{c}=-7
\end{aligned} \quad \mathrm{x}=\frac{-4 \pm \sqrt{(4)^{2}-4(1)(-7)}}{2(1)}
$$

$x=\frac{-4 \sum_{-} \sqrt{44}}{2}$

$$
\mathrm{x}=\frac{-4 \pm 2 \sqrt{11}}{2}=-2 \pm \sqrt{11}
$$

"logic method"
First terms must multiply to $2 x^{2}$

$$
(2 \mathrm{x} \quad)(\mathrm{x} \quad)
$$

Signs must be 'minus minus'

$$
(2 x-\quad)(x-\quad)
$$

Last terms must multiply to 5

$$
(2 x-5)(x-1)
$$

$$
\begin{gathered}
2\left[x^{2}-3.5 x+2.5\right]=0 \\
2(x-1)(x-2.5)=0 \\
x=1,2.5
\end{gathered}
$$

$$
\begin{gathered}
(x-1)(2 x-5)=0 \\
x=1,5 / 2
\end{gathered}
$$

"GCF method"

$$
\begin{aligned}
& \text { OR } \\
& 2 x-5=0 \\
& x-1=0
\end{aligned} \quad 5 / 2
$$

Find the equations of the lines.
SOLUTIONS
Compare the slopes, and determine if they are parallel, perpendicular, or neither.
(To find the equation of a line, you need the slope and a point)
$(1,3)$ and $(7,7)$

$$
\text { slope }=\frac{\text { "rise" }}{\text { "run" }}=\frac{y_{1}-y_{2}}{x_{1}-x_{2}}=\frac{(7-3)}{(7-1)}=\frac{4}{6}=\frac{2}{3}
$$

equation of blue line: $y-3=\frac{2}{3}(x-1)$ (point slope form)
$(3,2)$ and $(1,7)$

$$
\frac{(2-7)}{(3-1)}=\frac{-5}{2}(\text { slope of green line })
$$ equation of green line: $y-2=\frac{-5}{2}(x-3)$ also,

$$
y=\frac{-5}{2} x+\frac{19}{2} \text { or } 5 x+2 y=19
$$

(slope intercept form) (standard form)
horizontal line through $(-1,8)$ and $(8,8)$

$$
\text { slope is } 0 \quad y=8 \quad y=0 x+8
$$

horizontal line through $(2,4)$ and $(7,4)$
slope is $0 \quad y=4 \quad$ Lines are parallel
(note: vertical lines have "no slope" or "undefined slope" while horizontal lines have slope 0 )
$(2,5)$ and $(7,3)$
slope is $\frac{(5-3)}{(2-7)}=\frac{2}{-5}$

$$
y-5=\frac{2}{-5}(x-2)
$$

$(1,-4)$ and $(5,6)$
slope is $\frac{(-4-6)}{(1-5)}=\frac{-10}{-4}=\frac{5}{2}$

$$
\begin{aligned}
& y-(-4)=\frac{5}{2}(x-1) \\
& y+4=\frac{5}{2}(x-1)
\end{aligned}
$$

The slopes are opposite reciprocals, therefore the lines are perpendicular

Not parallel nor perpendicular




Given: $\triangle \mathrm{ABC} \quad \mathrm{A}=(1,2)$
$B=(5,6)$
$\mathrm{C}=(6,5)$


Find: The equation for the median from C
Step 1: Sketch the triangle
Step 2: Draw median from C
(median is drawn from vertex to
midpoint of opposite side)
Step 3: Find midpoint M
Using midpoint formula:
$\left(\frac{\mathrm{X}_{1}+\mathrm{X}_{2}}{2}, \frac{\mathrm{Y}_{1}+\mathrm{Y}_{2}}{2}\right)=\left(\frac{5+1}{2}, \frac{6+2}{2}\right)=(3,4)$

Step 4: Find equation of line passing through $(3,4)$ and $(6,5)$
Find Slope:

$$
\mathrm{m}=\frac{\mathrm{y}_{1}-\mathrm{y}_{2}}{\mathrm{x}_{1}-\mathrm{x}_{2}}=\frac{4-5}{3-6}=\frac{1}{3}
$$

Use slope and one of the points to make equation of the line

$$
(y-4)=\frac{1}{3}(x-3)
$$

Given: $\triangle \mathrm{LMN} \quad \mathrm{L}=(1,1)$

$$
\begin{aligned}
& \mathrm{M}=(5,-3) \\
& \mathrm{N}=(2,8)
\end{aligned}
$$

Find: The equation of the perpendicular bisector of LM

Step 1: Sketch the triangle
Step 2: Construct the $\perp$ bisector
(perpendicular bisector is a perpendicular line drawn through the midpoint of LM)


Step 3: Find midpoint of LM

$$
\left(\frac{(1+5)}{2}, \frac{(1+-3)}{2}\right)=(3,-1)
$$

Step 4: Find slope of line perpendicular to $\overline{\mathrm{LM}}$
Slope of LM: $\frac{-3-1}{5-1}=-1$
Slope of line $\perp$ to LM :
(opposite inverse) $\quad=1$
Step 5: Determine equation of line

$$
\mathrm{m}=1 \text { through }(3,-1)
$$

$$
y+1=1(x-3)--->y=x-4
$$

(note: the lines are parallel, because they have the same slope. There is no solution)


Solve and Graph the following:

1) $y<2 x+4 \quad$ step 1: rewrite equations (using equal $2 x-y \leq 4 \quad$ signs)

$$
\begin{aligned}
y=2 x+4 \quad-y & =-2 x+4 \\
y & =2 x-4
\end{aligned}
$$

step 2: graph (and identify solution and intercepts)
step 3: replace the inequalities and test points

$$
\begin{aligned}
& y<2 x+4 \quad(\text { dashed line }) \\
& 2 x-y \leq 4 \text { (solid line) }
\end{aligned}
$$

test $(0,0):(0)<2(0)+4 \quad 0<4$ L
test $(0,0): 2(0)-0 \leq 4 \quad 0 \leq 4$

2) A tortilla recipe consists of any combination of flour and corn. Flour costs $\$ 1.50$ per pound; and, corn costs $\$ 2.50$ per pound. If you need over 4 pounds of flour and corn, but you must spend no more than $\$ 9.50$, find and graph the production possibilities.
Let $\mathrm{C}=$ pounds of corn

$$
\mathrm{C}+\mathrm{F}>4
$$

$\mathrm{F}=$ pounds of flour
$\$ 2.50 \mathrm{C}=$ cost of corn
$\$ 1.50 \mathrm{~F}+\$ 2.50 \mathrm{C} \leq \$ 9.50$

$$
1.5 \mathrm{~F}+2.5 \mathrm{C} \leq 9.5
$$

a) Suppose you prefer corn. How much corn would you use?

If we find the solution (the intersection), we'll know the breakdown of corn and flour.

$$
\begin{aligned}
& \mathrm{C}=4-\mathrm{F} \\
& 1.5 \mathrm{~F}+2.5 \mathrm{C}=9.5
\end{aligned}
$$

(substitution) $1.5 \mathrm{~F}+2.5(4-\mathrm{F})=9.5$

$$
\begin{aligned}
1.5 \mathrm{~F}+10-2.5 \mathrm{~F} & =9.5 \\
\mathrm{~F} & =.5
\end{aligned}
$$

If $\mathrm{F}=.5$, then $\mathrm{C}=3.5$
**3.5 pounds of corn is the most you could use. Any more than 3.5 would violate the constraints!

Flour (F)
$\left(0,6 \frac{1}{3}\right)$
b) Suppose you have no preference. Identify the maximum weight that could be used within the budget.
The maximum weight would be all flour:

## $61 / 3$ pounds

3) Mathflix offers 2 rental plans: Plan A: $\$ 20$ per month and $\$ 1$ per movie Plan B: $\$ 10$ per month and $\$ 1.50$ per movie

Your household movie budget is $\$ 50$ per month. You watch at least 10 movies per month.
$\begin{aligned} \text { let } \mathrm{M} & =\# \text { of movies } \\ \mathrm{C} & =\text { Cost in } \$\end{aligned}$
C (ost of plan A) $=\$ 20+\$ 1 \mathrm{M}$
C (ost of plan B) $=\$ 10+\$ 1.50 \mathrm{M}$
Budget: $\mathrm{C} \leq \$ 50$
Movies $\geq 10$
a) Assume Mathflix allows you to select a Plan at the end of the month. Using the graph, show how much you'll pay Mathflix after one month.
Since you'll choose the cheaper plan, watch at least 10 movies, and stay within your $\$ 50$ budget, the solid purple line will represent possible payments.
b) What is the 'break-even' number of movies?

(At what point is the cost of each plan the same?)

Set Cost of plan $A=$ Cost of plan $B$

$$
\begin{aligned}
20+\mathrm{M} & =10+1.5 \mathrm{M} \\
10 & =.5 \mathrm{M} \\
\mathrm{M} & =20
\end{aligned}
$$

20 movies: cost of either plan is $\$ 40$ $<20$ movies: cost of $\mathrm{B}<$ cost of A $>20$ movies: cost of $\mathrm{B}>\cos$ of A

1) Given a line segment with endpoints $A=(-4,-3)$ and $B=(8,6)$.

Find the equation of the line perpendicular to segment $\overline{\mathrm{AB}}$, going through the point

## SOLUTIONS

 that is three-fourths of the distance from $B$ to $A$.Write your answer in standard form (i.e. $\mathrm{Ax}+\mathrm{By}=\mathrm{C}$, where $\mathrm{A}, \mathrm{B}$, and C are integers)

Step 1: Draw a picture


Step 2: Determine strategy and formulas
To find equation of a line, we need slope and a point.
Since $L$ is perpendicular to $\overline{A B}$, slope of $L$
is opposite reciprocal of slope of $\overline{A B}$.
Then, use ratios to find the point.

Step 3: Solve

$$
\text { slope }_{A B}=\frac{y_{1}-y_{2}}{x_{1}-x_{2}}=\frac{-3-6}{-4-8}=\frac{3}{4}
$$

Slope of $L$ is $-4 / 3$ (opposite reciprocal)
To find the point, we must use proportions to determine where the lines cross..
$x$-coordinate: from $B$ to $A$ is $8 \leftrightharpoons-4$ distance is 12 Then $3 / 4$ of 12 is $9 \ldots$
Therefore, the x -coordinate is 9 units from 8: -1
$y$-coordinate: from $B$ to $A$ is $6 \approx-3$ distance is 9
Then $3 / 4$ of 9 is $27 / 4$
Therefore, the $y$-coordinate is $27 / 4$ units from 6: $-3 / 4$
L has slope $-4 / 3$ and contains point $(-1,-3 / 4)$

$$
\begin{array}{lc}
y-(-3 / 4)=-4 / 3(x-(-1)) & \\
y+3 / 4=-4 / 3(x+1) & \begin{array}{cc}
\text { (multiply by 12 to get } \\
y=-\frac{4}{3} x-4 / 3-3 / 4 & \text { into standard form) }
\end{array} \\
y=\frac{-4}{3} x-\frac{25}{12} & 12 y=-16 x-25
\end{array}
$$

2) Given $\mathrm{z}=\mathrm{a}+\mathrm{b} i$ and $\overline{\mathrm{z}}=\mathrm{a}-\mathrm{b} i$. (i.e. z and $\overline{\mathrm{z}}$ are complex conjugates)

Solve for $z$, where

$$
5 z-3 \bar{z}=3+2 i
$$

Since $\mathrm{z}=\mathrm{a}+\mathrm{b} i$

$$
\begin{gathered}
2 \mathrm{a}+8 \mathrm{bi}=3+2 \mathrm{i} \\
2 \mathrm{a}=3 \\
\mathrm{a}=3 / 2
\end{gathered}
$$

Since $\overline{\mathrm{Z}}=\mathrm{a}-\mathrm{b} i$

$$
3 \bar{z}=3 \mathrm{a}-3 \mathrm{bi}
$$

Given $5 z-3 \bar{z}=3+2 i$

$$
\begin{array}{r}
2 \mathrm{a}+8 \mathrm{bi}=3+2 \mathrm{i} \\
\begin{array}{l}
8 \mathrm{bi}=2 \mathrm{i} \\
\mathrm{~b}=1 / 4
\end{array}
\end{array}
$$

$$
\begin{aligned}
5 a+5 b i-(3 a-3 b i) & =3+2 i \\
2 a+8 b i & =3+2 i
\end{aligned}
$$

To check:

$$
\begin{gathered}
5 z-3 \bar{z}= \\
15 / 2+5 / 4 \mathrm{i}-(9 / 2-3 / 4 \mathrm{i}) \\
6 / 2+8 / 4 \mathrm{i} \\
3+2 \mathrm{i}
\end{gathered}
$$

So, $z=\frac{3}{2}+\frac{1}{4}$ i
$\bar{z}=\frac{3}{2}-\frac{1}{4} \mathrm{i}$

Thanks for visiting (hope it helps!)
If you have questions, suggestions, or requests, let us know. Enjoy


One more question:
Write an equation that describes the set of points equidistant from both $(-1,4)$ and $(5,8)$

Answer on next page $-\rightarrow$

Write an equation that describes the set of points equidistant from both $(-1,4)$ and $(5,8)$.

## Solution



Step 1: Graph and apply Geometric theorem

$$
\begin{aligned}
& \text { Perpendicular Bisector Theorem: } \begin{array}{l}
\text { The perpendicular bisector of a line segment } \\
\text { is the locus of all points that equidistant from } \\
\text { the endpoints }
\end{array}
\end{aligned}
$$

Step 2: Establish strategy and lists formulas or variables

To find the equation of a line, we need the slope and a point.
The bisector is the midpoint of $(-1,4)$ and $(5,8)$
The slope of a perpendicular segment is the opposite reciprocal.

any point on the perpendicular bisector is equidistant from both points!

Step 3: Solve

The midpoint of $(5,8)$ and $(-1,4)$

$$
\begin{aligned}
\text { midpoint formula: } & \left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right) \\
& \left(\frac{-1+5}{2}, \frac{4+8}{2}\right)=(2,6)
\end{aligned}
$$

The slope of segment joining $(5,8)$ and $(-1,4)$

$$
\begin{aligned}
\text { slope }= & \frac{y_{1}-y_{2}}{x_{1}-x_{2}} \\
& \frac{8-4}{5-(-1)}=\frac{4}{6}=\frac{2}{3}
\end{aligned}
$$

Therefore, the slope of the perpendicular line is $\frac{-3}{2}$

> Equation of a line: $y-y_{1}=m\left(x-x_{1}\right)$ (pt. slope form)

$$
\begin{aligned}
& \text { slope } \mathrm{m}=\frac{-3}{2} \text { through point }(2,6) \\
& \qquad y-6=\frac{-3}{2}(x-2)
\end{aligned}
$$

Step 4: Quick check

Pick a random point on the line. then, see if it is equidistant from $(-1,4)$ and $(5,8)$

$$
\text { If } x=8, \quad y-6=\frac{-3}{2}(8-2)
$$

then $y=-3$
Let's test (8, -3)
distance formula: $d=\sqrt{\left(x_{1}-x_{2}\right)^{2}+\left(y_{1}-y_{2}\right)^{2}}$
distance between $(8,-3)$ and $(-1,4)$

$$
d=\sqrt{(8-(-1))^{2}+(-3-4)^{2}}=\sqrt{130}
$$

distance between $(8,-3)$ and $(5,8)$

$$
d=\sqrt{(8-5)^{2}+(-3-8)^{2}}=\sqrt{130}
$$

