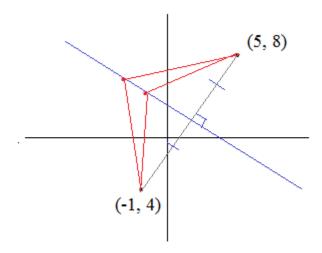
Algebra I / Geometry Partial Review

Examples and practice questions (with solutions)



Topics include coordinate plane, linear equations, order of operations, quadratics, graphing, and more.

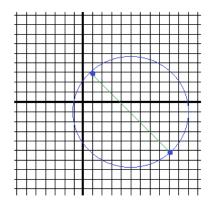
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Distance, Midpoint, and Circles

Example: The diameter of a circle has endpoints (1, 3) and (9, -5). What is the center of the circle?

What is the area of the circle?

Step 1: Draw a picture



Step 3: Solve and answer the questions

To find the center, we'll take the midpoint of (1, 3) and (9, -5)

center =
$$\left(\frac{1+9}{2}, \frac{3+-5}{2}\right) = (5, -1)$$

note: 5 is halfway between 1 and 9
-1 is four units from 3 and
four units from -5

Step 2: Outline the problem; identify formulas

All diameters go through the center of the circle. So, the midpoint of these endpoints will be the center.

Then, to find the area of a circle, we need the length of the diameter or radius (use distance formula).

Midpoint
$$(x, y) = \left\langle \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right\rangle$$
Area of circle = $\prod (radius)^2$

Distance = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

To determine the length of the radius, let's find the distance from (5, -1) to (1, 3):

radius =
$$\sqrt{(5-1)^2 + (-1-3)^2}$$

= $\sqrt{16+16}$ = $4\sqrt{2}$

Then, the area of the circle is

$$17(4\sqrt{2})^2 = 3217$$

Step 4: Check your answer.

Looking at the graph, we can see the answers are 'reasonable'

The radius length is a bit over 4 and, the square units are approx. 100...

Example: Describe the line parallel to x + 3y = 6 that passes through the origin?

What line is perpendicular to the above line and goes through (2, 1)?

Step 1: Set up the problem

The origin is (0, 0).. A *parallel* line has the *same slope*.

Then, *perpendicular* lines have slopes that are *opposite reciprocals*.

To describe a line, you need a point and the slope.

Step 2: Solve and answer

$$x + 3y = 6$$

$$3y = -x + 6$$
 slope is -1/3
$$y = \frac{-x}{3} + 2$$

parallel line through the origin:

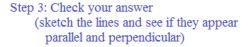
$$y - 0 = -1/3(x - 0)$$

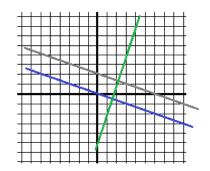
 $y = \frac{-1}{3}x$

perpendicular line through (2, 1):

$$3x - y = 5$$
 $y - 1 = 3(x - 2)$ pt. slope form
standard form $y = 3x - 5$ slope intercept form

Linear Equations, slope, perpendicular/parallel





Algebra/Geometry Review

Example: Graph the equation y = 3(x - 2)(x - 8);

Identify the x-intercept(s), y-intercept, and vertex.

(optional) The above equation is in factored form. (or, intercept form)

Express the equation in standard form and vertex form.

What is the x-intercept(s)?

It's the point(s) where a line or curve crosses the x-axis. In other words, any coordinate (?, 0)...

To find the x-intercept, set y = 0:

$$0 = 3(x - 2)(x - 8)$$

$$x = 2, 8$$

x-intercepts: (2, 0) (8, 0)

What is the y-intercept?

It's the point where a line or curve crosses the y-axis. In other words, any coordinate (0, ?)

To identify the y-intercept, set x = 0

$$y = 3(0-2)(0-8)$$

$$y = 48$$

How do you find the vertex?

When given a quadratic in factored form, you can find the axis of symmetry by recognizing the midpoint of 2 (horizontal/mirror') points!

In other words, what is the midpoint of (2, 0) and (8, 0)?

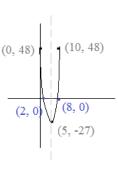
The middle value is (5, 0) ----> The axis of symmetry is x = 5... (going through that middle value)

Since the vertex lies on the axis of symmetry, the vertex is (5, ?)

$$y = 3(5 - 2)(5 - 8)$$

$$y = -27$$

Vertex: (5, -27)



Quadratics (parabolas), graphing, completing the square

Let's check the answers:

$$y = 3(x - 2)(x - 8)$$

convert to standard form:
$$y = 3(x^2 - 2x - 8x + 16)$$

$$y = 3x^2 - 30x + 48$$
 standard form

y-intercept is shown: 48

axis of symmetry:
$$\frac{-b}{2a} = \frac{-(-30)}{2(3)} = 5$$

convert to vertex form:
$$y = 3x^2 - 30x + 48$$
 (complete the square)

$$y = 3(x^2 - 10x) + 48$$

$$y = 3(x^2 - 10x + 25) + 48 - 3(25)$$

$$y = 3(x-5)^2 - 27$$
 vertex form

the vertex is (5, -27)

Given:
$$\overline{MT} \cong \overline{MH}$$
 \overline{AS} bisects \overline{MH}
 \overline{AS} bisects \overline{MT}

What is the perimeter of \triangle MTH?

Solution:

$$y + 5 = x + 4$$
 (because A is the midpoint of MT)

$$2x = y + 5$$
 (because S is midpoint of MH and MT = MH)

Since we have 2 equations and 2 unknowns, we can solve:

$$x - y = 1$$

 $2x - y = 5$

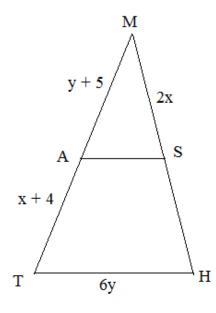
(elimination method)

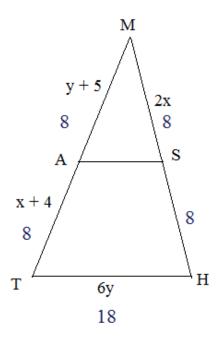
$$-x = -4$$

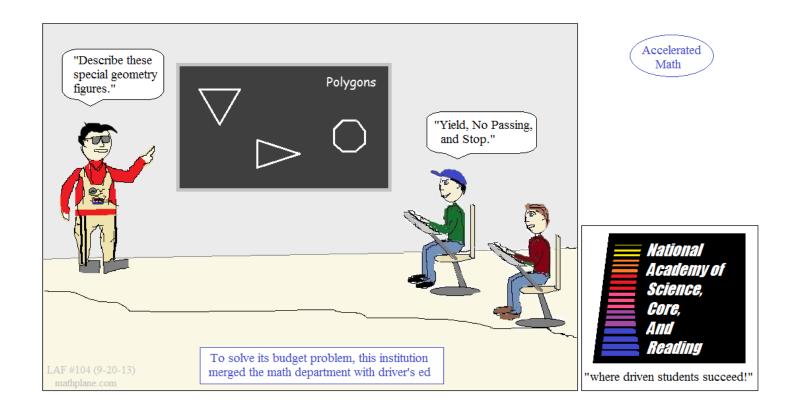
$$x = 4$$

$$y = 3$$

Substitute into the triangle, add up the segments: 8 + 8 + 18 + 8 + 8 = 50







Practice Questions →

Solve the following:

1)
$$\frac{1}{3} - 2 \div -3 \frac{1}{4} =$$

2)
$$\left(4\frac{1}{3}\right)\left(-3\frac{1}{3}\right) - \left(-3\frac{4}{7}\right) =$$

Decimals, Fractions, and Order of Operations

3)
$$(-1.5) \times 3.5 + 2.9 =$$

4)
$$2.4 \times 3.6 \times (-1.7) =$$

5)
$$10 = 2|x - 4| + 4$$

 $x =$

6)
$$3|x + 4| + 8 = 2$$

 $x =$

7) Solve using the quadratic formula

a)
$$2x^2 + 7x - 3 = 0$$

b)
$$-x^2 + 11x = 8$$

 $\mathbf{x} =$

$$\mathbf{x} =$$

Solving Quadratic Equations Exercise

1)
$$x^2 + 11x + 10 = 0$$

2)
$$x^2 - 16x + 28 = 0$$

3)
$$x^2 - 6x - 7 = 0$$

4)
$$3x^2 + 9x + 6 = 0$$

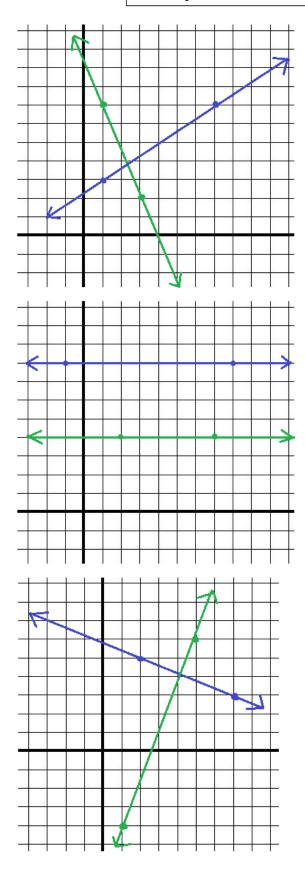
5)
$$2x^2 - 7x + 5 = 0$$

6)
$$x^2 + 4x - 7 = 0$$

Find the equations of the lines.

Linear Equations Review Exercise

Compare the slopes, and determine if they are parallel, perpendicular, or neither.



Linear Equations/Geometry Problems

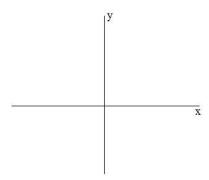
Given:
$$\triangle$$
 ABC $A = (1, 2)$
 $B = (5, 6)$
 $C = (6, 5)$

Find: The equation for the median from C

Find: The equation of the perpendicular $\underline{ \text{bisector} \text{ of } \overline{LM} }$

Solve and Graph the following:

1) y < 2x + 4 $2x - y \le 4$



- 2) A tortilla recipe consists of any combination of flour and corn. Flour costs \$1.50 per pound; and, corn costs \$2.50 per pound. If you need over 4 pounds of flour and corn, but you must spend no more than \$9.50, find and graph the production possibilities.
 - a) Suppose you prefer corn. How much corn would you use?
 - b) Suppose you have no preference. Identify the maximum weight that could be used within the budget.
- 3) Mathflix offers 2 rental plans: Plan A: \$20 per month and \$1 per movie Plan B: \$10 per month and \$1.50 per movie

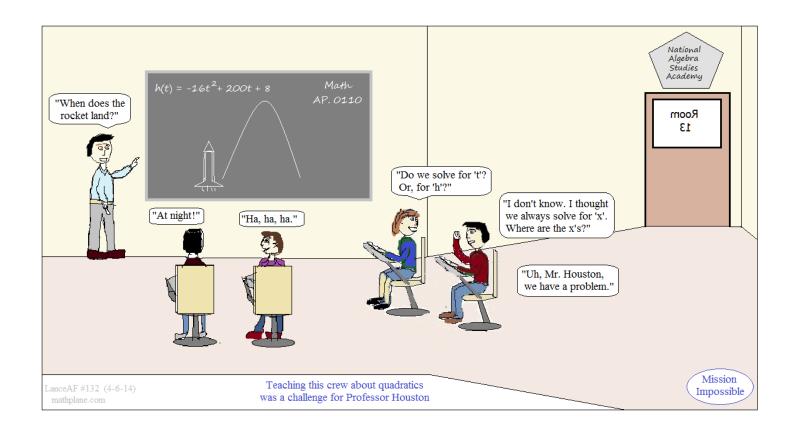
Your household movie budget is \$50 per month. You watch at least 10 movies per month.

- a) Assume Mathflix allows you to select a Plan at the end of the month. Using the graph, show how much you'll pay Mathflix after one month.
- b) What is the 'break-even' number of movies? (At what point is the cost of each plan the same?)

Given a line segment with endpoints A = (-4, -3) and B = (8, 6).
 Find the equation of the line perpendicular to segment AB, going through the point that is three-fourths of the distance from B to A.
 Write your answer in standard form (i.e. Ax + By = C, where A, B, and C are integers)

2) Given z = a + bi and $\overline{z} = a - bi$. (i.e. z and \overline{z} are complex conjugates) Solve for z, where

 $5z - 3\overline{z} = 3 + 2i$



SOLUTIONS -→

1)
$$\frac{1}{3} - 2 \div -3\frac{1}{4} =$$

Change to (similar) improper fractions

$$\frac{1}{3} - \frac{2}{1} \cdot \frac{\cdot}{\cdot} \frac{13}{4} =$$

(Order of operations) -- divide first!

$$\frac{1}{3} - \frac{-8}{13} = \frac{13}{39} - \frac{24}{39} = \frac{37}{39}$$

3)
$$(-1.5) \times 3.5 + 2.9 =$$

(2 decimal places)

(sign is negative because neg x pos)

5)
$$10 = 2|x - 4| + 4$$

$$x = 1, 7$$

("isolate the absolute value")

$$6 = 2|x - 4|$$

$$3 = |x - 4|$$

(then, "split and solve")

$$3 = x - 4$$
 $x = 7$
 $-3 = x - 4$ $x = 1$

7) Solve using the quadratic formula

a)
$$2x^2 + 7x - 3 = 0$$

$$x = a = 2
b = 7
c = -3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{49 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{-7 \pm \sqrt{73}}{4}$$

2)
$$\left(4\frac{1}{3}\right)\left(-3\frac{1}{3}\right) - \left(-3\frac{4}{7}\right) = \frac{13}{3} \cdot \frac{-10}{3} - \frac{-25}{7} =$$

$$\frac{-130}{9} + \frac{25}{7} = \frac{-910}{63} + \frac{225}{63} = \frac{-685}{63}$$
 or $-10\frac{55}{63}$

4)
$$2.4 \times 3.6 \times (-1.7) =$$

6)
$$3|x+4|+8=2$$

x = no solution

$$3|x + 4| = -6$$
$$|x + 4| = -2$$
NO REAL SOLUTION

Notice, x + 4 = -2 x = -6x + 4 = 2 x = -2

But, when you check your answers, neither solution will work...

absolute value must be positive!

b)
$$-x^2 + 11x = 8$$

$$-x^2 + 11x - 8 = 0$$

$$\begin{array}{c} a = -1 \\ b = 11 \\ c = -8 \end{array} \qquad x = \frac{-11 \pm \sqrt{121 - 4(-1)(-8)}}{2(-1)}$$

$$x = \frac{-11 \pm \sqrt{89}}{-2}$$

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Solving Quadratic Equations Exercise

SOLUTIONS

1)
$$x^2 + 11x + 10 = 0$$

plus plus

"what multiplies to 10 and adds to 11?"

$$(x + 10)(x + 1) = 0$$

$$x = -10, -1$$

(use 'zero product property' to solve factored equation!)

2)
$$x^2 - 16x + 28 = 0$$

minus minus

"what multiplies to 28 and adds to -16?"

$$(x-2)(x-14)=0$$

$$x = 2, 14$$

3)
$$x^2 - 6x - 7 = 0$$

plus minus

multiplies to -7 and adds to -6

$$(x-7)(x+1) = 0$$

$$x = -1, 7$$

4)
$$3x^2 + 9x + 6 = 0$$

greatest common factor

$$3(x^2 + 3x + 2) = 0$$

$$3(x+2)(x+1) = 0$$

$$x = -2, -1$$

5)
$$2x^2 - 7x + 5 = 0$$

split and divide or logic method

"split and regroup"

AC = 10: "what multiplies to 10..

B = -7: ...and, adds to -7"

split the middle term:

$$2x^2 - 2x - 5x + 5 = 0$$

$$2x(x-1) + -5(x-1) = 0$$

6)
$$x^2 + 4x - 7 = 0$$

quadratic formula

$$(x-1)(2x-5) = 0$$

$$x = 1, 5/2$$

 $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

$$2[x^2 - 3.5x + 2.5] = 0$$

$$2(x-1)(x-2.5) = 0$$

$$x = 1, 2.5$$

"logic method"

First terms must multiply to 2x²

Signs must be 'minus minus'

$$(2x -)(x -)$$

Last terms must multiply to 5

$$(2x - 5)(x - 1)$$

$$OR$$

$$(2x-1)(x-5)$$

$$2x - 5 = 0$$
 5/2

$$x - 1 = 0$$
 1

$$x = \frac{-4 + \sqrt{(4)^2 - 4(1)(-7)}}{2(1)}$$

$$x = \frac{-4 + \sqrt{44}}{2}$$

$$x = \frac{-4 + 2 \sqrt{11}}{2} = -2 + \sqrt{11}$$

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Compare the slopes, and determine if they are parallel, perpendicular, or neither.

(To find the equation of a line, you need the slope and a point)

(1, 3) and (7, 7)

slope =
$$\frac{\text{"rise"}}{\text{"run"}} = \frac{y_1 + y_2}{x_1 + x_2} = \frac{(7-3)}{(7-1)} = \frac{4}{6} = \frac{2}{3}$$

equation of blue line: $y - 3 = \frac{2}{3}(x - 1)$ (point slope form)

(3, 2) and (1, 7)
$$\frac{(2-7)}{(3-1)} = \frac{-5}{2}$$
 (slope of green line)

equation of green line: $y - 2 = \frac{-5}{2}(x - 3)$

nor perpendicular

also,
$$y = \frac{-5}{2}x + \frac{19}{2}$$
 or $5x + 2y = 19$

(slope intercept form) (standard form)

horizontal line through (-1, 8) and (8, 8)

$$y = 0x + 8$$

horizontal line through (2, 4) and (7, 4)

Lines are parallel

(note: vertical lines have "no slope" or "undefined slope" while horizontal lines have slope 0)

(2, 5) and (7, 3)

slope is
$$\frac{(5-3)}{(2-7)} = \frac{2}{-5}$$

$$y - 5 = \frac{2}{-5}(x - 2)$$

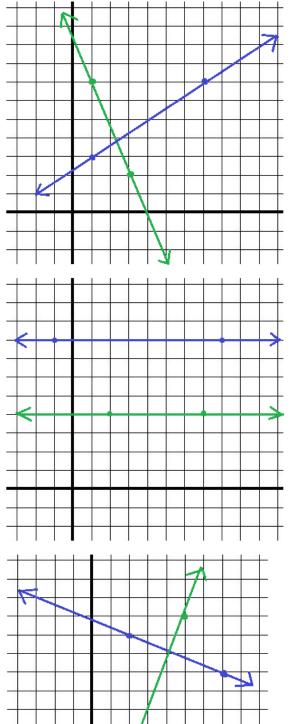
(1, -4) and (5, 6)

slope is
$$\frac{(-4-6)}{(1-5)} = \frac{-10}{-4} = \frac{5}{2}$$
 $y - (-4) = \frac{5}{2} (x-1)$ $y + 4 = \frac{5}{2} (x-1)$

$$y - (-4) = \frac{5}{2} (x - 1)$$

$$y+4=\frac{5}{2}(x-1)$$

The slopes are opposite reciprocals, therefore the lines are perpendicular



Given:
$$\triangle$$
 ABC $A = (1, 2)$
 $B = (5, 6)$

C = (6, 5)

Find: The equation for the median from C

Step 1: Sketch the triangle Step 2: Draw median from C

> (median is drawn from vertex to midpoint of opposite side)

Step 3: Find midpoint M

Using midpoint formula:

Given: △LMN

$$\left(\frac{X_1 + X_2}{2}, \frac{Y_1 + Y_2}{2}\right) = \left(\frac{5+1}{2}, \frac{6+2}{2}\right) = (3, 4)$$

$$L = (1, 1)$$

 $M = (5, -3)$

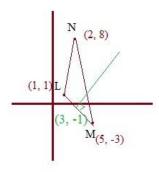
$$N = (2, 8)$$

Find: The equation \underline{of} the perpendicular $\underline{bisector}$ of \underline{LM}

Step 1: Sketch the triangle

Step 2: Construct the ⊥bisector

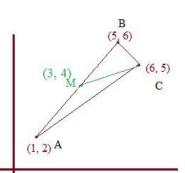
(perpendicular bisector is a perpendicular line drawn through the midpoint of LM)





Linear Equation/Geometry Problems

Step 4: Find equation of line passing through (3, 4) and (6, 5)



Find Slope:

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{4 - 5}{3 - 6} = \frac{1}{3}$$

Use slope and one of the points to make equation of the line

$$(y-4) = \frac{1}{3}(x-3)$$

Step 3: Find midpoint of LM

$$\left(\frac{(1+5)}{2}, \frac{(1+-3)}{2}\right) = (3, -1)$$

Step 4: Find slope of line perpendicular to LM

Slope of LM:
$$\frac{-3 - 1}{5 - 1} = -1$$

Slope of line ⊥ to LM:

(opposite inverse)

Step 5: Determine equation of line

$$m = 1$$
 through $(3, -1)$

$$y + 1 = 1(x - 3)$$
 ----> $y = x - 4$

(note: the lines are parallel, because they have the same slope. There is no solution)

Solve and Graph the following:

1) y < 2x + 4 $2x - y \le 4$

step 1: rewrite equations (using equal signs)

$$y = 2x + 4$$
 $-y = -2x + 4$
 $y = 2x - 4$

step 2: graph (and identify solution and intercepts)

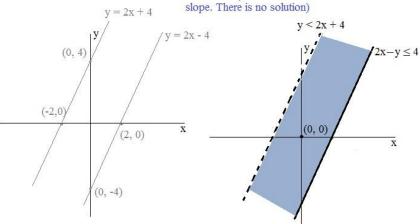
step 3: replace the inequalities and test points

$$y < 2x + 4$$
 (dashed line)

$$2x - y \le 4$$
 (solid line)

test
$$(0, 0)$$
: $(0) < 2(0) + 4 0 < 4$

test
$$(0, 0)$$
: $2(0) - 0 \le 4$ $0 \le 4$ tortilla recipe consists of any combinati



2) A tortilla recipe consists of any combination of flour and corn. Flour costs \$1.50 per pound; and, corn costs \$2.50 per pound. If you need over 4 pounds of flour and corn, but you must spend no more than \$9.50, find and graph the production possibilities.

Let C = pounds of com

$$C+F>4$$

$$F = pounds of flour$$

$$1.50F + 2.50C \le 9.50$$

\$2.50C = cost of corn\$1.50F = cost of flour

$$1.5F + 2.5C \le 9.5$$

a) Suppose you prefer corn. How much corn would you use?

If we find the solution (the intersection), we'll know the breakdown of corn and flour.

$$C = 4 - F$$

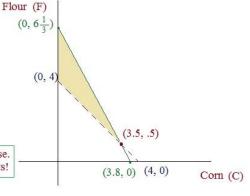
$$1.5F + 2.5C = 9.5$$

(substitution)
$$1.5F + 2.5(4 - F) = 9.5$$

 $1.5F + 10 - 2.5F = 9.5$

If
$$F = .5$$
, then $C = 3.5$

**3.5 pounds of corn is the most you could use. Any more than 3.5 would violate the constraints!



 Suppose you have no preference. Identify the maximum weight that could be used within the budget.

The maximum weight would be all flour:

3) Mathflix offers 2 rental plans:

Plan A: \$20 per month and \$1 per movie

Plan B: \$10 per month and \$1.50 per movie

Your household movie budget is \$50 per month. You watch at least 10 movies per month.

$$C(\text{ost of plan A}) = $20 + $1M$$

$$C(\text{ost of plan B}) = \$10 + \$1.50M$$

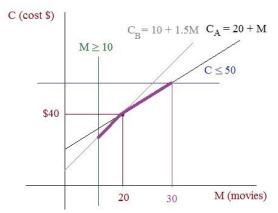
Budget:
$$C \le $50$$

a) Assume Mathflix allows you to select a Plan at the end of the month. Using the graph, show how much you'll pay Mathflix after one month.

Since you'll choose the cheaper plan, watch at least 10 movies, and stay within your \$50 budget, the solid purple line will represent possible payments.

b) What is the 'break-even' number of movies?

(At what point is the cost of each plan the same?)



Set Cost of plan A = Cost of plan B

$$20 + M = 10 + 1.5M$$

$$10 = .5M$$

$$M = 20$$

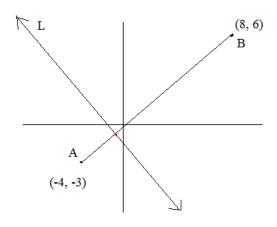
20 movies: cost of either plan is \$40 < 20 movies: cost of B < cost of A

> 20 movies: cost of B > cost of A

Given a line segment with endpoints A = (-4, -3) and B = (8, 6).
 Find the equation of the line perpendicular to segment AB, going through the point that is three-fourths of the distance from B to A.
 Write your answer in standard form (i.e. Ax + By = C, where A, B, and C are integers)

SOLUTIONS

Step 1: Draw a picture



Step 2: Determine strategy and formulas

To find equation of a line, we need slope and a point.

Since L is perpendicular to \overline{AB} , slope of L is opposite reciprocal of slope of \overline{AB} .

Then, use ratios to find the point.

Step 3: Solve

slope
AB =
$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{-3 - 6}{-4 - 8} = \frac{3}{4}$$

Slope of L is -4/3 (opposite reciprocal)

To find the point, we must use proportions to determine where the lines cross..

x-coordinate: from B to A is 8 ⇒ -4 distance is 12 Then 3/4 of 12 is 9...

Therefore, the x-coordinate is 9 units from 8: -1

y-coordinate: from B to A is 6 ⇒ -3 distance is 9 Then 3/4 of 9 is 27/4

Therefore, the y-coordinate is 27/4 units from 6: -3/4

$$y - (-3/4) = -4/3(x - (-1))$$

$$y + 3/4 = -4/3 (x + 1)$$

into standard form)
$$12y = -16x - 25$$

$$y = \frac{-4}{3} x - 4/3 - 3/4$$
$$y = \frac{-4}{3} x - \frac{25}{12}$$

$$16x + 12y = -25$$

(multiply by 12 to get

2) Given z = a + bi and $\overline{z} = a - bi$. (i.e. z and \overline{z} are complex conjugates) Solve for z, where

$$5z - 3\overline{z} = 3 + 2i$$

Since
$$z = a + bi$$

$$5z = 5a + 5bi$$

Since
$$\overline{z} = a - bi$$

$$3\overline{z} = 3a - 3bi$$

Given
$$5z - 3\overline{z} = 3 + 2i$$

$$5a + 5bi - (3a - 3bi) = 3 + 2i$$

$$2a + 8bi = 3 + 2i$$

$$2a + 8bi = 3 + 2i$$
 $2a = 3$
 $a = 3/2$

$$2a + 8bi = 3 + 2i$$

 $8bi = 2i$

$$b = 1/4$$

So,
$$z = \frac{3}{2} + \frac{1}{4}i$$

 $\overline{z} = \frac{3}{2} + \frac{1}{4}i$

$$5z - 3\overline{z} =$$

$$15/2 + 5/4i - (9/2 - 3/4i)$$

$$6/2 + 8/4i$$

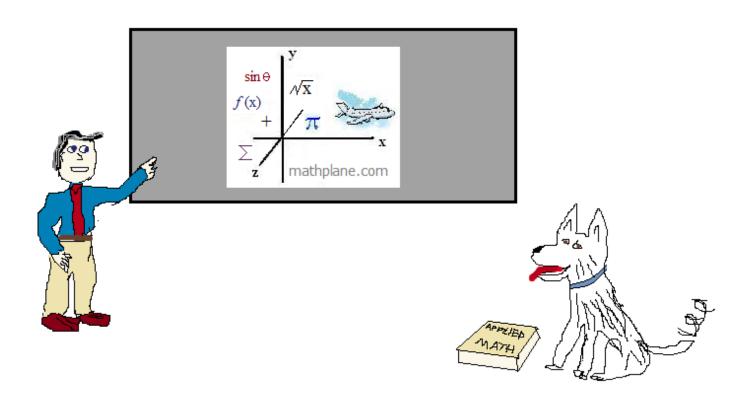
$$3 + 2i$$

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Thanks for visiting (hope it helps!)

If you have questions, suggestions, or requests, let us know.

Enjoy



One more question:

Write an equation that describes the set of points equidistant from both (-1, 4) and (5, 8)

Answer on next page -→

Write an equation that describes the set of points equidistant from both (-1, 4) and (5, 8).

f

(5, 8)



Solution

Step 1: Graph and apply Geometric theorem

Perpendicular Bisector Theorem: The perpendicular bisector of a line segment is the locus of all points that equidistant from the endpoints

Step 2: Establish strategy and lists formulas or variables

To find the equation of a line, we need the slope and a point.

The bisector is the midpoint of (-1, 4) and (5, 8)

The slope of a perpendicular segment is the opposite reciprocal.

(-1, 4) (perpendicular bisector)

any point on the perpendicular bisector is equidistant from both points!

Step 3: Solve

The midpoint of (5, 8) and (-1, 4)

midpoint formula:
$$\left\langle \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right\rangle$$

$$\left\langle \frac{-1+5}{2}, \frac{4+8}{2} \right\rangle = (2, 6)$$

The slope of segment joining (5, 8) and (-1, 4)

slope =
$$\frac{y_1 - y_2}{x_1 - x_2}$$

 $\frac{8 - 4}{5 - (-1)} = \frac{4}{6} = \frac{2}{3}$

Therefore, the slope of the perpendicular line is $\frac{-3}{2}$

Equation of a line:
$$y - y_1 = m(x - x_1)$$
 (pt. slope form)

slope
$$m = \frac{-3}{2}$$
 through point (2, 6)

$$y-6 = \frac{-3}{2}(x-2)$$

Step 4: Quick check

Pick a random point on the line, then, see if it is equidistant from (-1, 4) and (5, 8)

If
$$x = 8$$
, $y - 6 = \frac{4 - 3}{2} (8 - 2)$

then
$$y = -3$$

distance formula:
$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

distance between (8, -3) and (-1, 4)

$$d = \sqrt{(8 - (-1))^2 + (-3 - 4)^2} = \sqrt{130}$$

distance between (8, -3) and (5, 8)

$$d = \sqrt{(8-5)^2 + (-3-8)^2} = \sqrt{130}$$