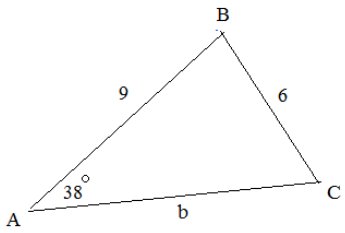
Trigonometry: Law of Sines and Cosines III

Example: Solve the triangle (finding all angles and sides) using law of cosines (only)



$6^2 = 9^2 + b^2 - 2(9)(b)(\cos 38)$

$36 = 81 + b^2 - 14.1842b$

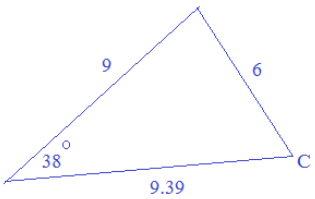
$b^2 - 14.1842b + 45 = 0$

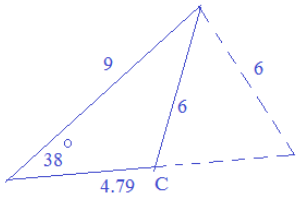
quadratic formula; 2 solutions

$b = 4.79$ or 9.39

Note: This is "angle-side-side", so it's the 'ambiguous case'...

If b = 9.39,
it's an acute triangle





If b = 4.79,
it's an obtuse triangle

$9^2 = 9.39^2 + 6^2 - 2(9.39)(6)(\cos C)$

$81 = 124.172 - 112.68(\cos C)$

$\cos C = .383138$

Angle C = 67.47 degrees....

and, Angle B = 180 - 67.47 - 38 = 74.53°

supplements!

$9^2 = 4.79^2 + 6^2 - 2(4.79)(6)(\cos C)$

$81 = 58.944 - 57.48(\cos C)$

$\cos C = -.3837$

Angle C = 112.53 degrees

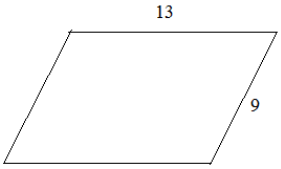
and, Angle B = 180 - 112.53 - 38 = 29.47°

Law of Cosines and Sines (advanced)

mathplane.com

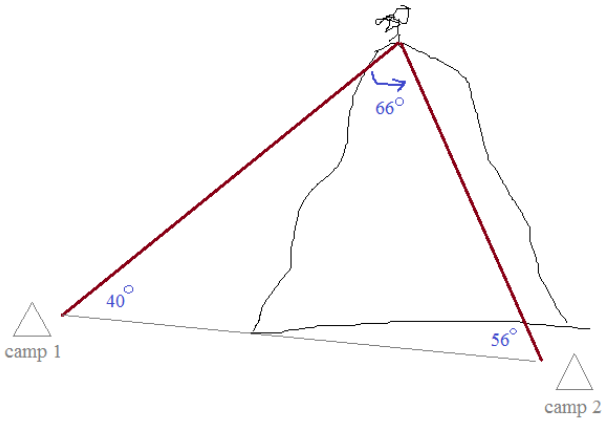
1) The parallelogram has side lengths 9 and 13.

If one diagonal is 16, what is the other diagonal?



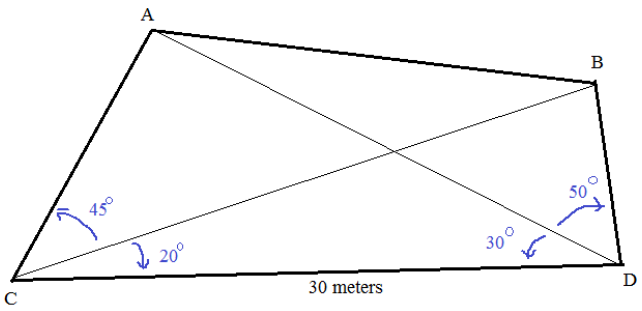
2) A surveyor stands atop a 2400 foot mountain...
He can look down at an angle of depression of 40 degrees and see Base Camp 1.
If the surveyor swivels 66 degrees to his left,
he can look down at an angle of depression of 56 degrees and see Base Camp 2.

What is the approximate distance between Base Camps 1 and 2?



Law of Cosines and Sines (advanced)

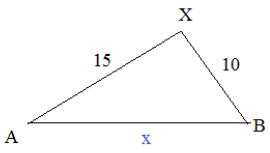
3) Can you find the length of $\overline{AB}$?



4) Given: Angle A = 40 degrees

side a = 10

side b = 15



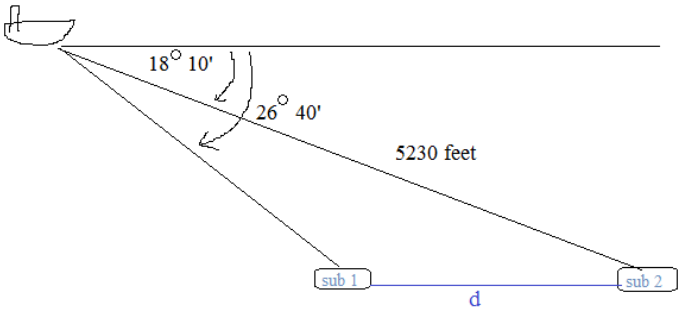
In the following 'ambiguous case', find the length of x

a) using law of sines

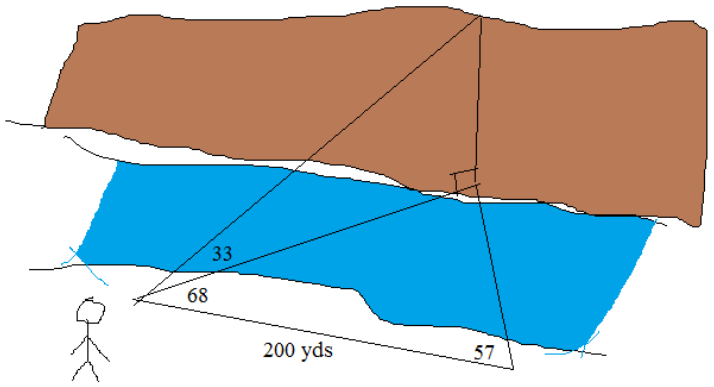
b) using law of cosines

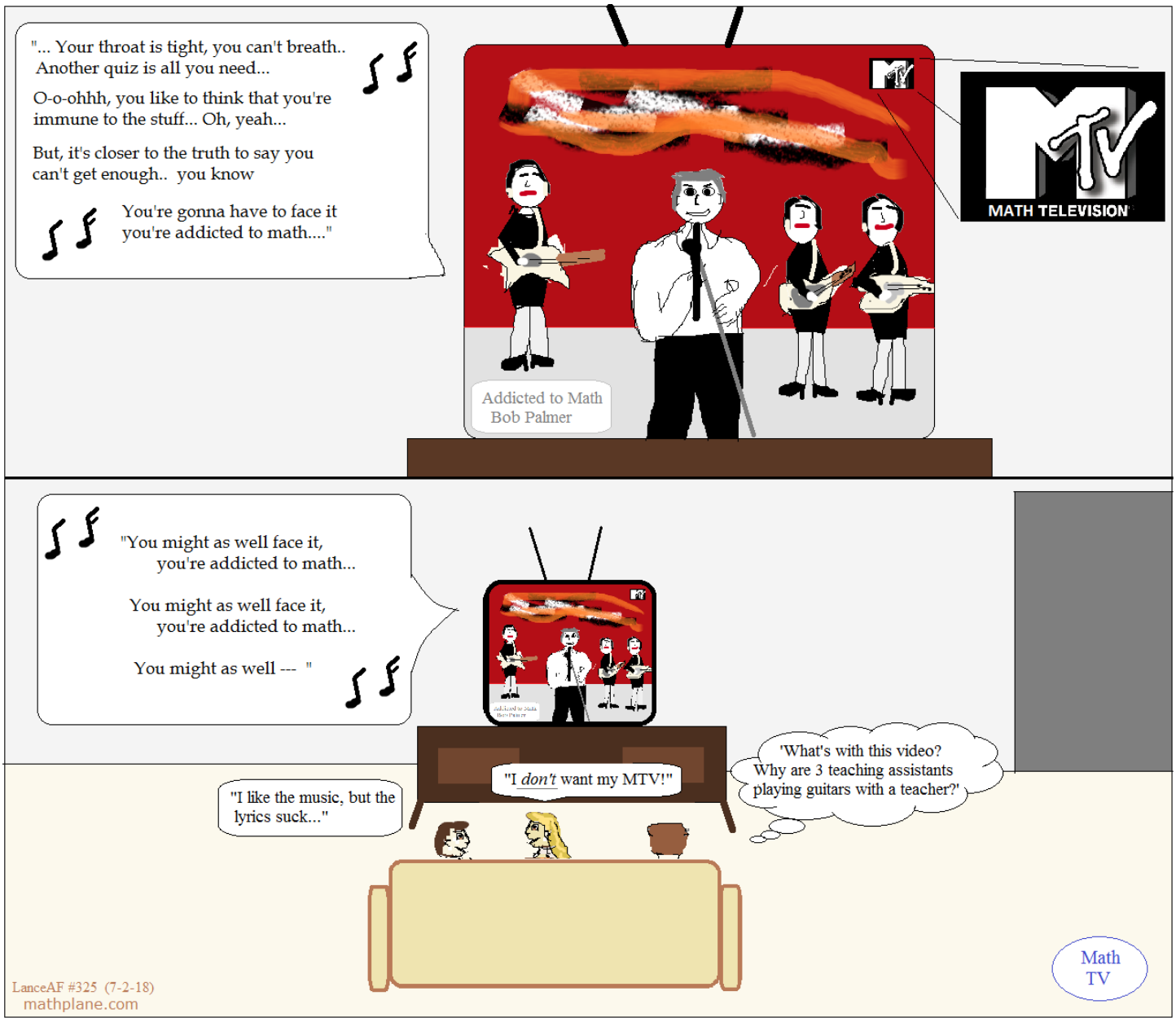
Law of Cosines and Sines (advanced)

5) The diagram depicts a boat on the surface of the sea, and 2 submarines below the water level.
What is the distance between the submarines?



6) A surveyor is standing across a (blue) river, facing (brown) cliffs..
Taking 2 measurements from spots 200 yards apart, he finds the measures are 68 degrees and 57 degrees to a specific point across the river.
If the angle of elevation from the surveyor to the top of the cliffs is 33 degrees, then what is the approximate height of the cliffs?




"... Your throat is tight, you can't breath.. Another quiz is all you need...
O-o-ohhh, you like to think that you're immune to the stuff... Oh, yeah...
But, it's closer to the truth to say you can't get enough.. you know
You're gonna have to face it you're addicted to math...."
MTV
MATH TELEVISION
Addicted to Math
Bob Palmer
"You might as well face it, you're addicted to math...
You might as well face it, you're addicted to math...
You might as well --- "
"I like the music, but the lyrics suck..."
"I don't want my MTV!"
'What's with this video? Why are 3 teaching assistants playing guitars with a teacher?'
Math TV
LanceAF #325 (7-2-18)
mathplane.com

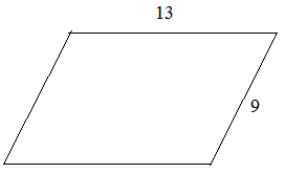
SOLUTIONS-→

SOLUTIONS

Law of Cosines and Sines (advanced)

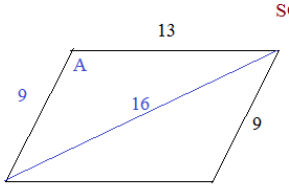
1) The parallelogram has side lengths 9 and 13.

If one diagonal is 16, what is the other diagonal?



law of cosines

$c^2 = a^2 + b^2 - 2(a)(b)(\cos C)$



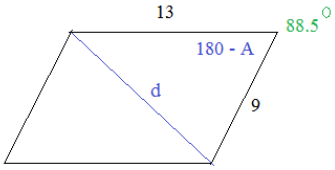
use law of sines/cosines to find angles...

$16^2 = 9^2 + 13^2 - 2(9)(13)\cos(A)$

$256 = 81 + 169 - 234\cos(A)$

$A = 91.5^\circ$

Note: we know that consecutive angles in a parallelogram are supplementary...



then, use law of cosines to find diagonal...

$d^2 = 9^2 + 13^2 - 2(9)(13)\cos(88.5)$

$d = 15.6$ (approx.)

2) A surveyor stands atop a 2400 foot mountain...
He can look down at an angle of depression of 40 degrees and see Base Camp 1.
If the surveyor swivels 66 degrees to his left,
he can look down at an angle of depression of 56 degrees and see Base Camp 2.

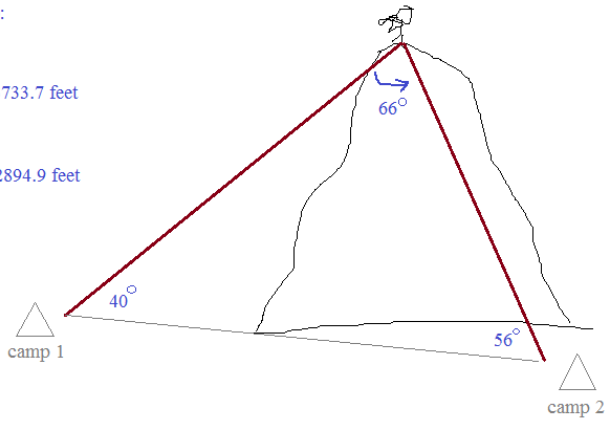
What is the approximate distance between Base Camps 1 and 2?

First, find the distances from the surveyor to each base camp:

$\sin(40^\circ) = \frac{2400 \text{ feet}}{\text{distance to camp 1}}$ distance is approx. 3733.7 feet

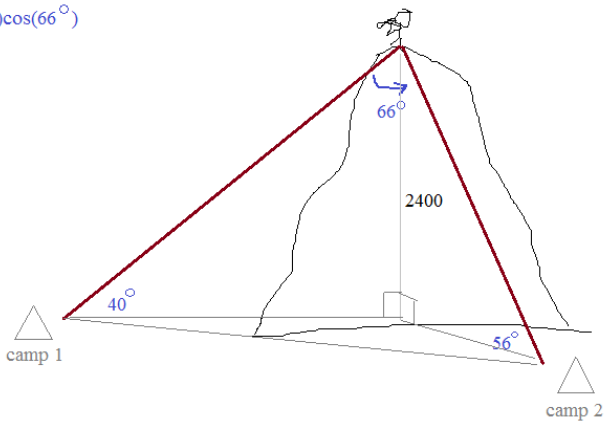
$\sin(56^\circ) = \frac{2400 \text{ feet}}{\text{distance to camp 1}}$ distance is approx. 2894.9 feet

Then, using law of cosines, we can find the distance between the camps!



$\left\langle \begin{matrix} \text{distance} \\ \text{between camps} \end{matrix} \right\rangle^2 = (3733.7)^2 + (2894.9)^2 - 2(3733.7)(2894.9)\cos(66^\circ)$

distance between camps is approximately 3678 feet



3) Can you find the length of $\overline{AB}$?

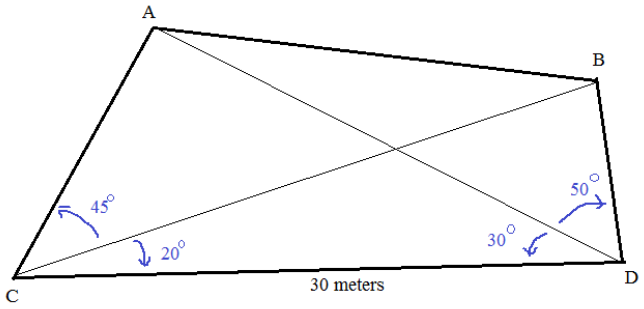
SOLUTIONS

Step 1: Using Law of Sines and $\triangle ACD$

find $\overline{AD}$

$$\frac{\sin(85)}{30} = \frac{\sin(65)}{AD}$$

$\overline{AD} = 27.3$ approx.

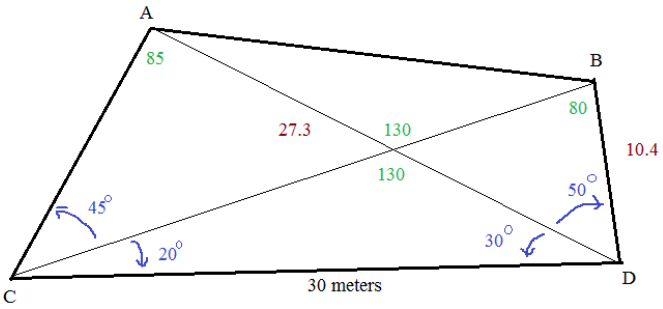


Step 2: Using Law of Sines and $\triangle BDC$

find $\overline{BD}$

$$\frac{\sin(80)}{30} = \frac{\sin(20)}{BD}$$

$\overline{BD} = 10.4$ approx.



Step 3: Using Law of Cosines and $\triangle ABD$

find $\overline{AB}$

$$\overline{AB}^2 = (10.4)^2 + (27.3)^2 - 2(10.4)(27.3)\cos(50^\circ)$$

$\overline{AB} = 22.1$ approximately

4) Given: Angle A = 40 degrees

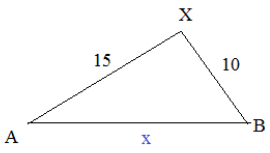
side a = 10

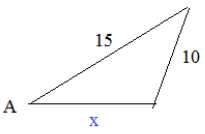
side b = 15

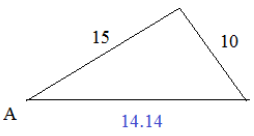
In the following 'ambiguous case', find the length of x

a) using law of sines

b) using law of cosines







Method 1: Using law of sines...

$$\frac{\sin(40)}{10} = \frac{\sin(B)}{15}$$

$\sin(B) = .9641$

$B = 74.61^\circ$ or 105.39°

If $B = 74.61^\circ$, $X = 65.39^\circ$

$$\frac{\sin(40)}{10} = \frac{\sin(65.39)}{x}$$

$x = 14.14$

If $B = 105.39^\circ$, $X = 34.61^\circ$

$$\frac{\sin(40)}{10} = \frac{\sin(34.61)}{x}$$

$x = 8.84$

Method 2: Using law of cosines...

$$100 = 15^2 + x^2 - 2(15)(x)\cos(40)$$

$$-125 = x^2 - 30(x)(.766)$$

$$x^2 - 22.98x + 125 = 0$$

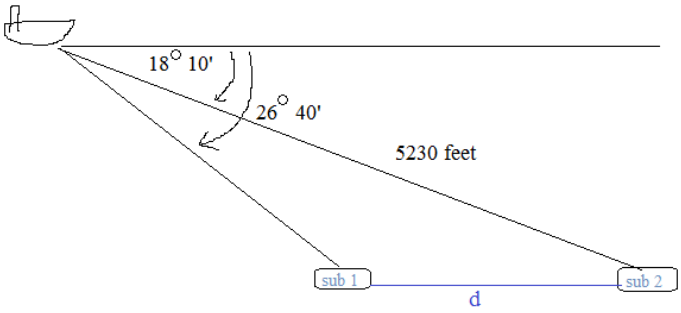
quadratic formula (calculator)

$x = 8.84$ or 14.14

Law of Cosines and Sines (advanced)

5) The diagram depicts a boat on the surface of the sea, and 2 submarines below the water level.
What is the distance between the submarines?

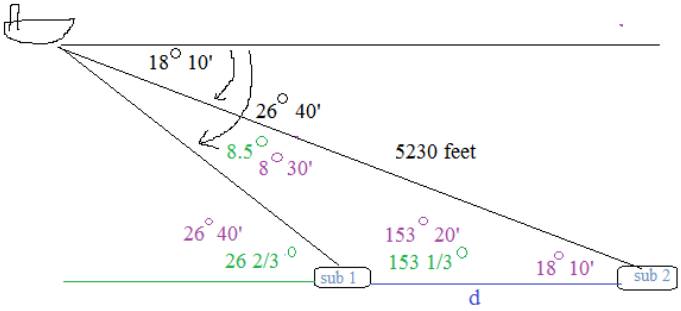
SOLUTIONS

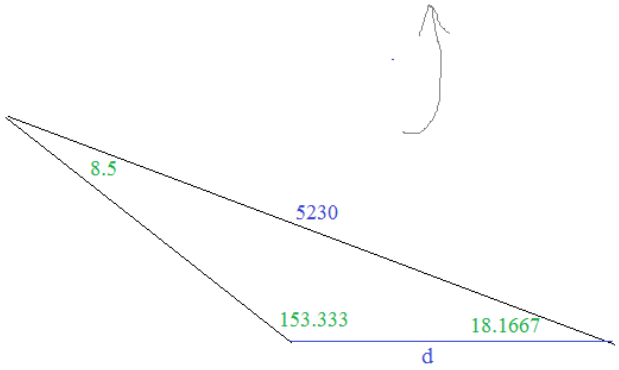


Using Law of Sines

$$\frac{\sin(8.5)}{d} = \frac{\sin(153.333)}{5230}$$

d = 1722 feet (approx.)





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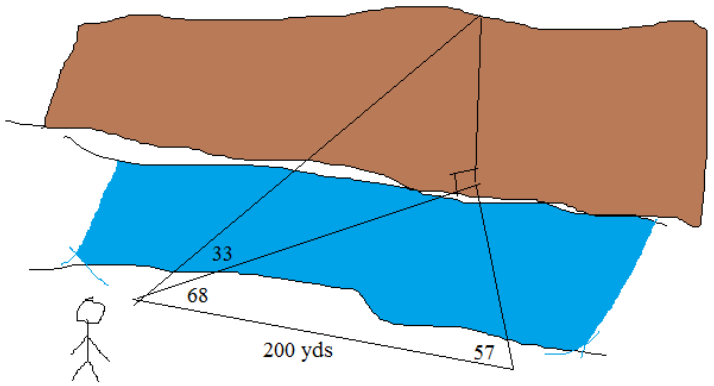

Step 1: use law of sines to get the base of the triangle...

$$\frac{\sin(55)}{200} = \frac{\sin(57)}{d}$$

d = 204.8

Step 2: Using trig ratios, find the height of the cliffs..

$$\tan(33) = \frac{\text{height}}{204.8}$$

height = 133 yards (approx.)