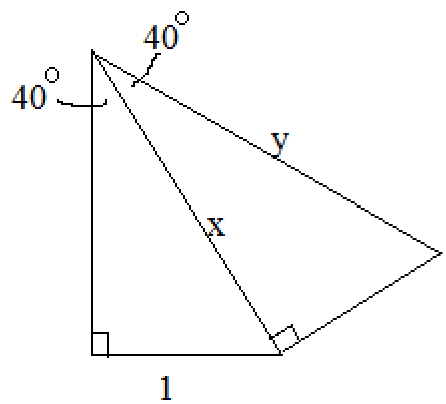
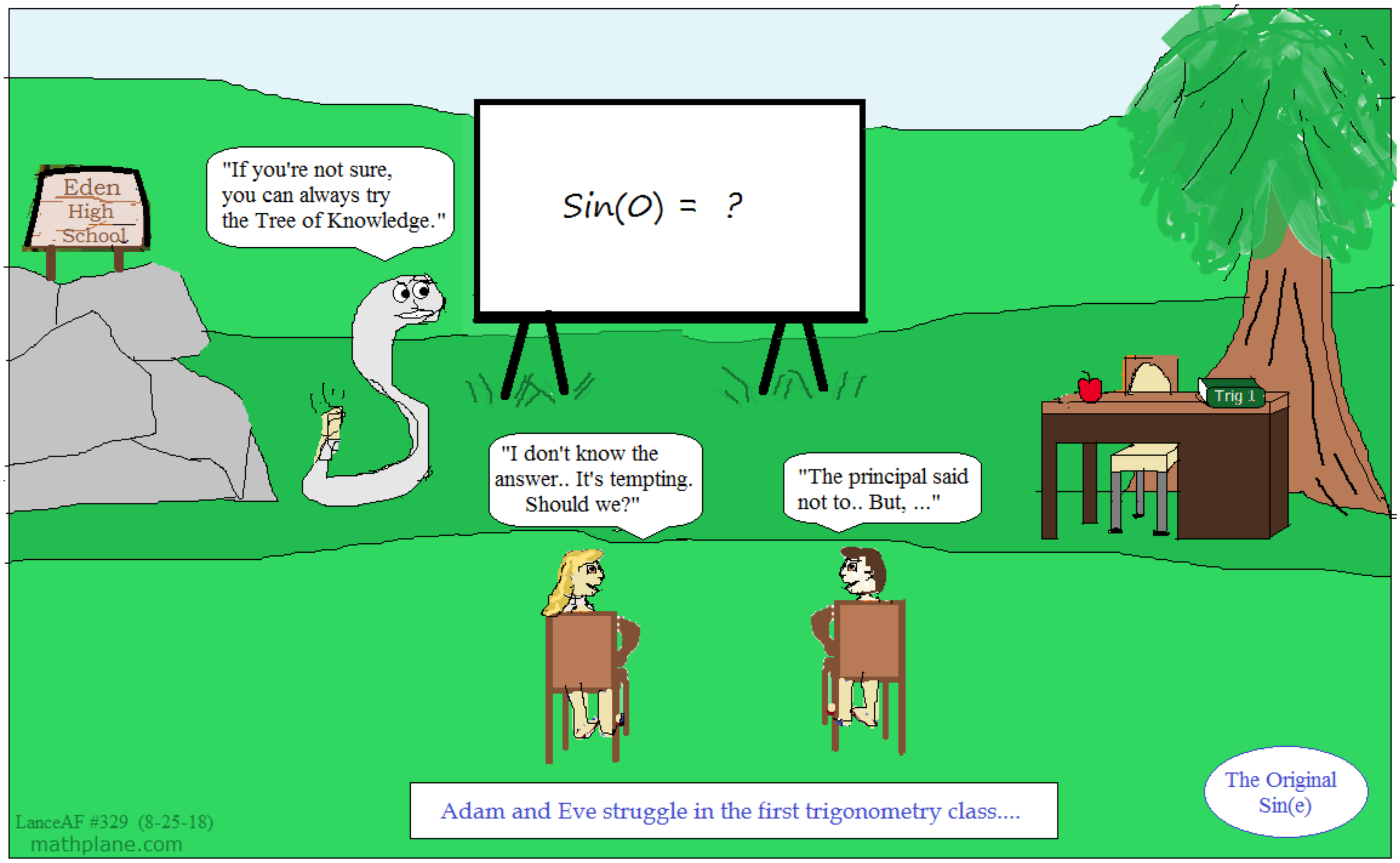
Trigonometry (Honors) Review 3

Practice Questions (and Answers)



Topics include trig values, half-angle identities, angular distance, quadrants and intervals, inverses, and more.



Practice Questions -→

Can you answer these questions?

1) Evaluate

a) $\sec\theta = \dfrac{2\sqrt{3}}{3}$

Find θ

where $0^\circ < \theta \leq 360^\circ$

b) $\tan x = \dfrac{\sqrt{3}}{3}$

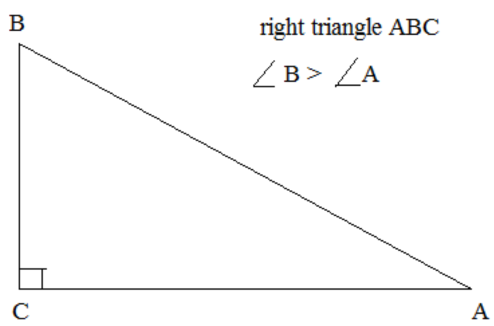
Find x

where $0 < x \leq 2\pi$

2) Given: $\tan\theta = -4/3$ $\cos\theta > 0$

Find θ for $0^\circ < \theta < 360^\circ$

3)



right triangle ABC

$\angle B > \angle A$

True or false:

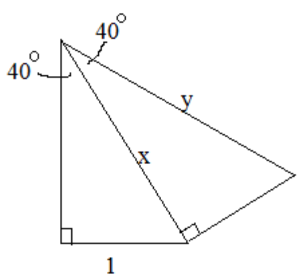
a) SinA > CosA ______

b) TanA < CosB ______

c) CosC > CosA ______

d) TanB > SinB ______

4)



Find x and y:

5) fill in with <, >, or =

cos(54) _____ cos(-54)

sin(260) _____ sin(150)

cos(10) _____ cos(12)

6) What is sin(-270) ?

7) $2\sin\frac{\pi}{12}\cos\frac{\pi}{12} =$?

8) $\sin^2(67^\circ) + \cos^2(67^\circ) =$?

9) $\text{Sec}x = \frac{5}{2}$ $\quad$ $\text{Csc}y = 3$ $\quad$ in Quadrant I:

Without a calculator, find sin(x + y)

10) SecU = 3/2 in quadrant IV

What is $\text{Cos}\,\frac{U}{2}$?

11) $2\cos x - 3\tan x = 0$

Find x on the interval $[0, 2\pi]$

12) Solve for the interval $0 \leq x < 2\pi$

$\tan x + \sec x = 1$

13) $\sin A = \frac{4}{5}$ $\quad$ $\cos B = \frac{-5}{13}$

where A $90^\circ \leq A < 180^\circ$ and $180^\circ \leq B < 270^\circ$

a) Sin2A =

b) Cos2B =

14) Transform the equation by solving for θ:

then, find the first *three positive* values for which y = 5

$$y = 3 + 9\cos 4(\theta - 50^{\circ})$$

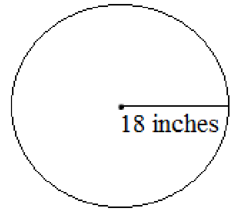
15) Find θ in the interval $[0, 2\pi)$

$$\cos\theta \cos 3\theta - \sin\theta \sin 3\theta = 0$$

16) The following wheel turns at 35 revolutions per minute.

What is the linear distance of the wheel in 1 minute?

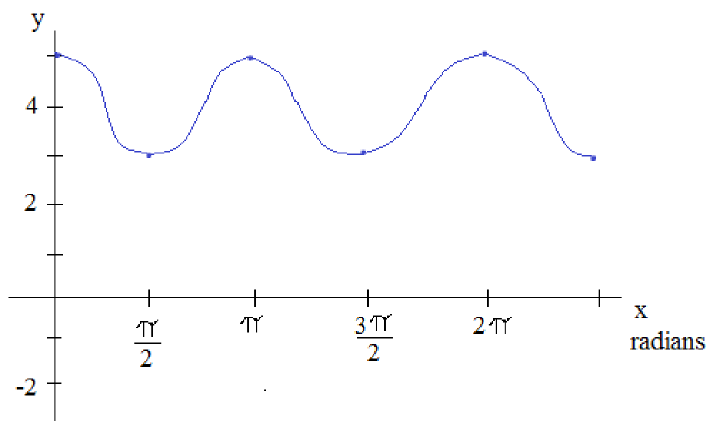
What is the angular distance of the wheel in 1 minute?



17) $f(x) = a\sin b(x - c) + d$

Write an equation where a < 0.

Write an equation where a > 0.



18) Find the acute angle formed by the line $y - \sqrt{3}x + 1 = 0$ and the x-axis.

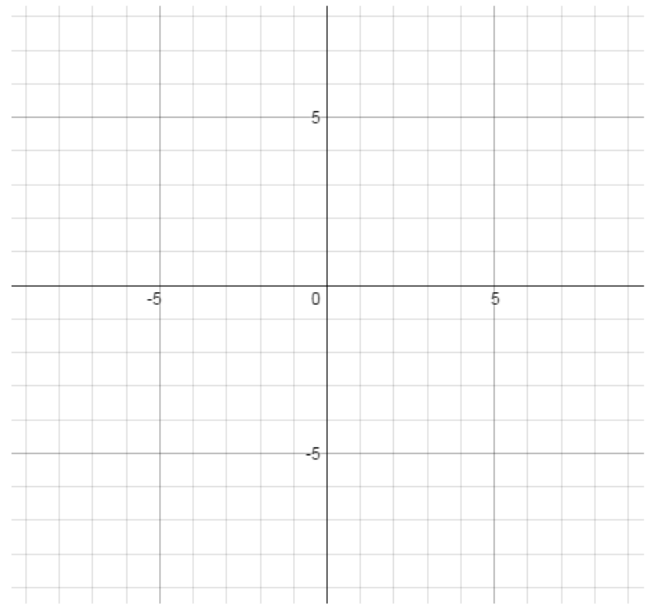
19) Find $\sin(195^\circ)$ (WITHOUT a calculator)

Trigonometry Review Concepts: Area

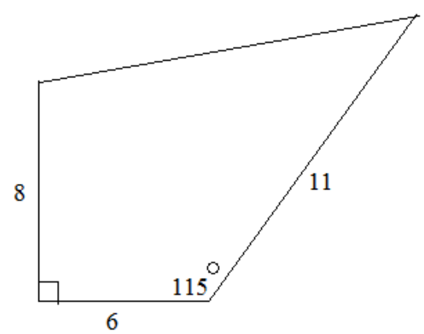
Graph and find the area of the region

20) $x^2 + y^2 \le 36$

$y \ge 3$



21) Find the area of the figure:



22) $\sin(-x)\tan(-x) + \cos(-x) = ?$

23) For $0 < x < 360^{\circ}$,

what is $2\sin(x + 42^{\circ}) = 1$?

$\sec(x - 35^{\circ}) = 2$?

24) $8 - 2\tan x - 5\sec^{2} x = 0$

where $x \in (0^{\circ}, 360^{\circ})$

25) A parallelogram has side lengths 12 and 15.
The longer diagonal has length 20...
What is the length of the shorter diagonal?

26) As $\theta \to 0^-$,

what is $\csc\theta$?

As $\theta \to 0^+$,

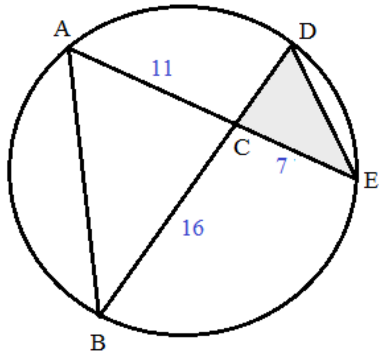
what is $\csc\theta$?

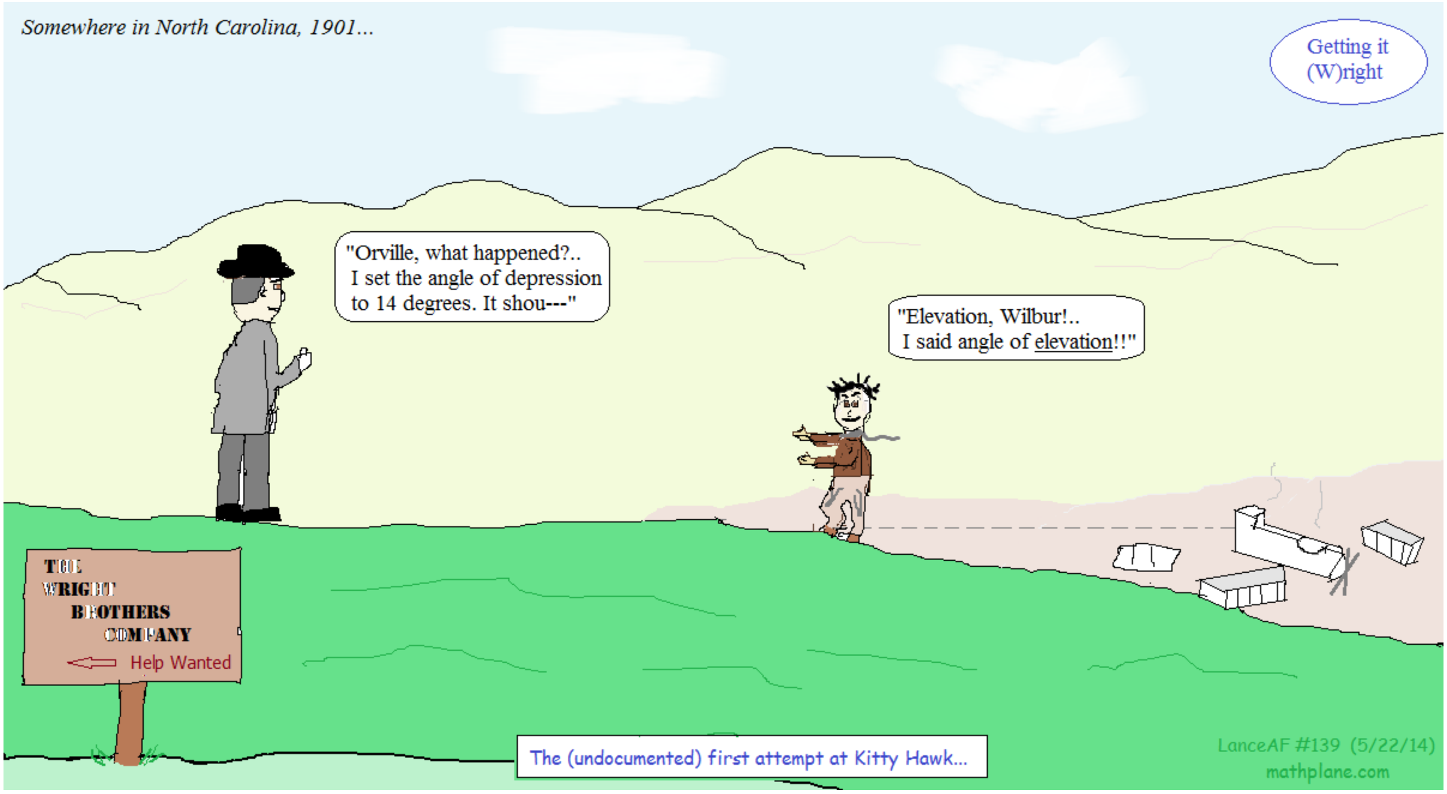
27) If the area of triangle ABC is 86,
what is the area of triangle CDE?

$\overline{AC} = 11$

$\overline{BC} = 16$

$\overline{CE} = 7$




Somewhere in North Carolina, 1901...
Getting it (W)right
"Orville, what happened?.. I set the angle of depression to 14 degrees. It shou---"
"Elevation, Wilbur!.. I said angle of elevation!!"
THE WRIGHT BROTHERS COMPANY
Help Wanted
The (undocumented) first attempt at Kitty Hawk...
LanceAF #139 (5/22/14)
mathplane.com

Solutions -→

Can you answer these questions?

SOLUTIONS

Trigonometry Concepts Review

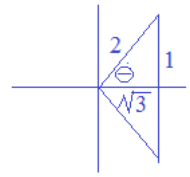
1) Evaluate

a) $\sec\theta = \dfrac{2\sqrt{3}}{3}$

Find θ where $0^\circ < \theta \le 360^\circ$

$\sec\theta = \dfrac{2}{\sqrt{3}}$

(secant is positive in I and IV)



(reference angle is 30°)

30° and 330°

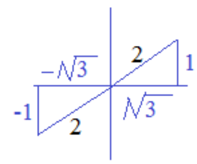
b) $\tan x = \dfrac{\sqrt{3}}{3}$

Find x where $0 < x \le 2\pi$

It's helpful to 'unrationalize the denominator' to recognize the ratio

$\tan x = \dfrac{1}{\sqrt{3}}$

(tangent is positive in I and III)

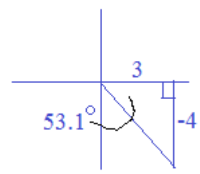


$\dfrac{\pi}{6}$ and $\dfrac{7\pi}{6}$

2) Given: $\tan\theta = -4/3$ $\cos\theta > 0$

Find θ for $0^\circ < \theta < 360^\circ$

Since tan is negative and cos is positive, the angle must be in quad IV..



using a calculator,

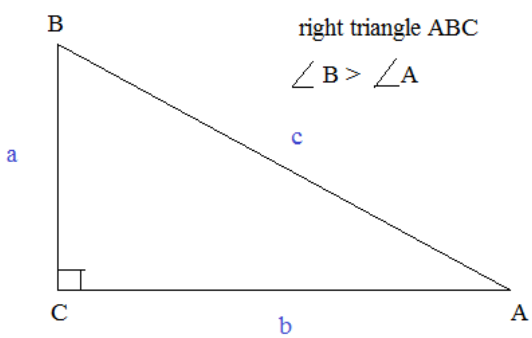
$\tan^{-1}\left(\dfrac{-4}{3}\right) = -53.1^\circ$

but, the angle is between 0 and 360!

the coterminal angle is

$360 + (-53.1) =$ **306.9°**

3)



right triangle ABC

$\angle B > \angle A$

True or false:

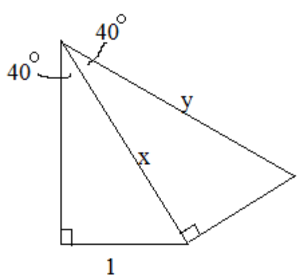
a) SinA > CosA — false — $\dfrac{\text{opposite}}{\text{hypotenuse}}$ is NOT > $\dfrac{\text{adjacent}}{\text{hypotenuse}}$

b) TanA < CosB — false — $\dfrac{a}{b} > \dfrac{a}{c}$ (because c > b)

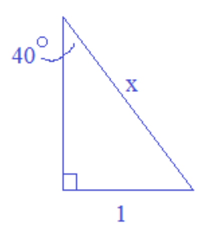
c) CosC > CosA — false — $\cos(90) = 0$

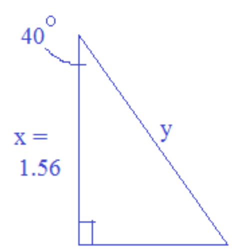
d) TanB > SinB — true — $\dfrac{b}{a} > \dfrac{b}{c}$

4)



Find x and y:





$\sin(40) = \dfrac{1}{x}$

$x = \dfrac{1}{.643} =$ **1.56**

$\cos(40) = \dfrac{x}{y} = \dfrac{1.56}{y}$

$y = \dfrac{1.56}{.766} =$ **2.04**

SOLUTIONS

5) fill in with <, >, or =

$\cos(54)$ __=__ $\cos(-54)$ quad I and quad IV are the same (for cosine)

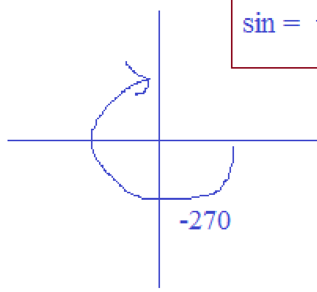
$\sin(260)$ __<__ $\sin(150)$ sine in quad II is positive.. sine in quad III is negative!

$\cos(10)$ __>__ $\cos(12)$ as angle gets larger, the adjacent side gets smaller!

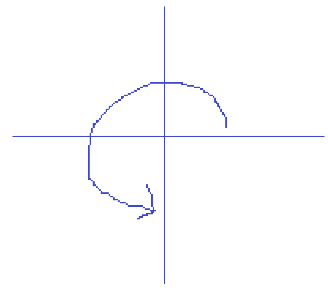
6) What is $\sin(-270)$?

-270 begins in standard position and moves *clockwise*

$\sin = \frac{y}{r} = 1$



notice: $\sin(270) = -1$



7) $2\sin\frac{\pi}{12}\cos\frac{\pi}{12} = ?$

$\sin 2\left(\frac{\pi}{12}\right) = \sin\frac{\pi}{6} = \boxed{\frac{1}{2}}$

$\sin 2x = 2\sin x\cos x$

8) $\sin^2(67^\circ) + \cos^2(67^\circ) = ?$

$\sin^2(x) + \cos^2(x) = \boxed{1}$

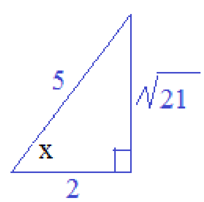
9) $\sec x = \frac{5}{2}$ $\csc y = 3$ in Quadrant I:

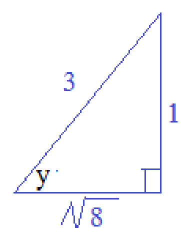
Without a calculator, find $\sin(x + y)$

$\sin(x + y) = \sin x\cos y + \cos x\sin y$

$= \frac{\sqrt{21}}{5} \cdot \frac{\sqrt{8}}{3} + \frac{2}{5} \cdot \frac{1}{3}$

$= \frac{14.96}{15} = \boxed{.997 \text{ (approx)}}$





(to check, use a calculator to find x and y)
then, calculate $\sin(x + y)$

SOLUTIONS

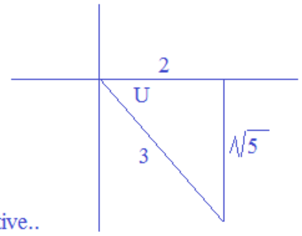
Trigonometry Concepts Review

10) SecU = 3/2 in quadrant IV

What is $\cos\frac{U}{2}$?

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

since U is in quad IV, U/2 will be in quad II... therefore, the result is negative..



$-\sqrt{\frac{1+\frac{2}{3}}{2}} = -\sqrt{\frac{5}{6}} = \boxed{-\frac{\sqrt{30}}{6}}$

to check, (using a calculator), find U --- 312 degrees. then, find cos(312/2) = cos(156) = -.91 ✓

11) $2\cos x - 3\tan x = 0$

Find x on the interval [0 , 2π]

$2\cos x = 3\tan x$

$2\cos x = \frac{3\sin x}{\cos x}$

$2\cos^2 x = 3\sin x$

$2(1 - \sin^2 x) = 3\sin x$

$2\sin^2 x + 3\sin x - 2 = 0$

$(2\sin x - 1)(\sin x + 2) = 0$

$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{6},\ \frac{5\pi}{6}}$

$\sin x + 2 = 0$

$\sin x = -2$

no solution
(must be between -1 and 1)

12) Solve for the interval $0 \le x < 2\pi$

$\tan x + \sec x = 1$

$\sec x = 1 - \tan x$

goal: try to get one trig function to solve

$\sec^2 x = (1 - \tan x)(1 - \tan x)$

Note: when you square a function, you often "create another solution".. It may be extraneous...

$\sec^2 x = 1 - 2\tan x + \tan^2 x$

$1 + \tan^2 x = 1 - 2\tan x + \tan^2 x$

$0 = -2\tan x$

$\tan x = 0$

For the interval, $0 \le x < 2\pi$

$\boxed{x = 0}$ or ~~π~~

Check your solutions!

x = 0: tan(0) + sec(0) =
0 + 1 = 1 ✓

x = π: tan(π) + sec(π) =
0 + (-1) = -1 ✗

13) $\sin A = \frac{4}{5}$ $\cos B = \frac{-5}{13}$

where A $90^\circ \le A < 180^\circ$ and $180^\circ \le B < 270^\circ$

a) Sin2A = $\boxed{\frac{-24}{25}}$

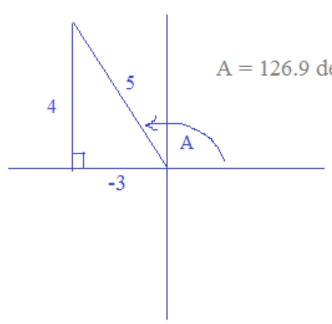
$\sin 2A = 2\sin A\cos A$

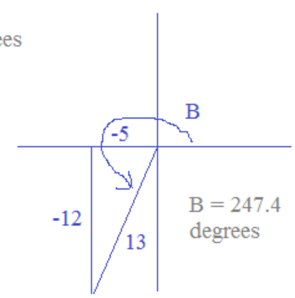
$2 \cdot \frac{4}{5} \cdot \frac{-3}{5} = \frac{-24}{25}$

b) Cos2B = $\boxed{\frac{-119}{169}}$

$\cos 2B = \cos^2 B - \sin^2 B$

$\left(\frac{-5}{13}\right)^2 - \left(\frac{-12}{13}\right)^2 = \frac{-119}{169}$





SOLUTIONS

14) Transform the equation by solving for θ:

then, find the first *three positive* values for which y = 5

$$y = 3 + 9\cos 4(\theta - 50^\circ)$$

$$y - 3 = 9\cos 4(\theta - 50^\circ)$$

$$\frac{y-3}{9} = \cos 4(\theta - 50^\circ)$$

$$\cos^{-1}\left(\frac{y-3}{9}\right) = 4(\theta - 50^\circ)$$

$$\frac{1}{4}\cos^{-1}\left(\frac{y-3}{9}\right) = (\theta - 50^\circ)$$

$$\theta = \frac{1}{4}\cos^{-1}\left(\frac{y-3}{9}\right) + 50^\circ$$

If y = 5:

$$\theta = \frac{1}{4}\cos^{-1}\left(\frac{5-3}{9}\right) + 50^\circ$$

$$= \frac{1}{4}(77.16^\circ) + 50^\circ$$

$$= 19.29^\circ + 50^\circ = 69.29^\circ$$

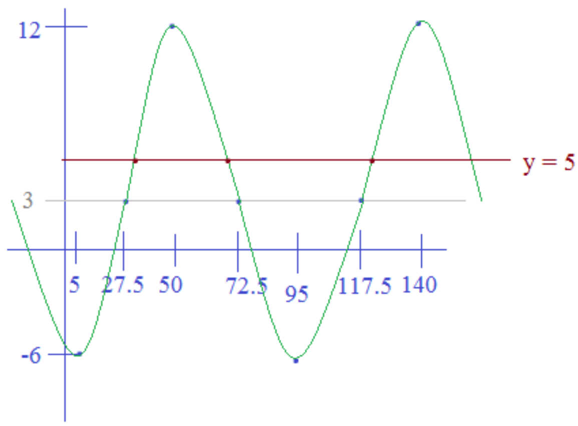
That's the *2nd positive* value

$$27.5 + 3.21 = 30.71^\circ$$

That's the *1st positive* value

30.71 + (1 period) = 120.71 degrees

that's the *3rd positive* value



15) Find θ in the interval $[0, 2\pi)$

$$\cos\theta\cos 3\theta - \sin\theta\sin 3\theta = 0$$

Recognizing the sum/addition trig identity, we can combine the terms..

$$\cos(\theta + 3\theta) = 0$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\cos(4\theta) = 0$$

Suggestion: use substitution to evaluate cosine.. let $A = 4\theta$ $\cos A = 0$

$$A = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, \frac{13\pi}{2}, \frac{15\pi}{2}$$

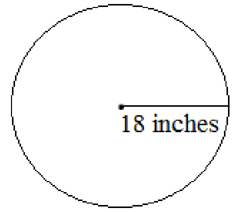
$$\text{Therefore,}\quad 4\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, \frac{13\pi}{2}, \frac{15\pi}{2}$$

$$\text{and,}\quad \theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$$

16) The following wheel turns at 35 revolutions per minute. SOLUTIONS

What is the linear distance of the wheel in 1 minute?

What is the angular distance of the wheel in 1 minute?



Linear is the distance travelled, in terms of a point on the circle.. Every revolution, a point will travel the circumference, 36π inches..

So, in 1 minute, a point will travel $35 \times 36\pi$ or 1260π inches approx. 3958.4 inches

Angular is the distance traveled, in terms of angle rotation inside the circle... Every revolution is 360 degrees... So, in 1 minute, the angular distance is 35 x 360 degrees or 12,600 degrees per minute

17) $f(x) = a\sin b(x - c) + d$

Write an equation where a < 0.

the period is π, b = 2

the vertical shift is 4, so d = 4

If a < 0, and amplitude is 1, then a "starting point" is $\frac{\pi}{4}$ $-\sin 2(x - \frac{\pi}{4}) + 4$

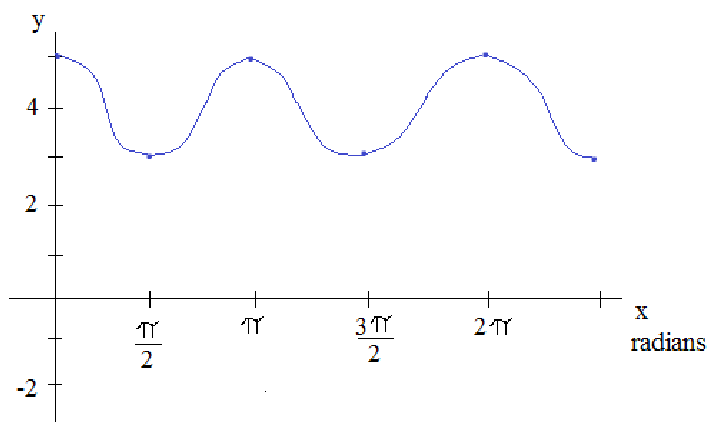
Write an equation where a > 0.

amplitude is 1

same period: b = 2

same vertical shift: d = 4

If a > 0, then a "starting point" is $\frac{3\pi}{4}$ $\sin 2(x - \frac{3\pi}{4}) + 4$



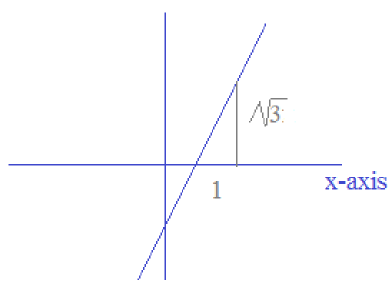
18) Find the acute angle formed by the line $y - \sqrt{3}x + 1 = 0$ and the x-axis.

$y = \sqrt{3}x - 1$

y-intercept: (0, -1)

**slope is $\sqrt{3}$

60 degree angle



19) Find sin(195°) (WITHOUT a calculator)

First, find sin(15°)

$\sin(45 - 30) = \sin(45)\cos(30) - \cos(45)\sin(30)$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

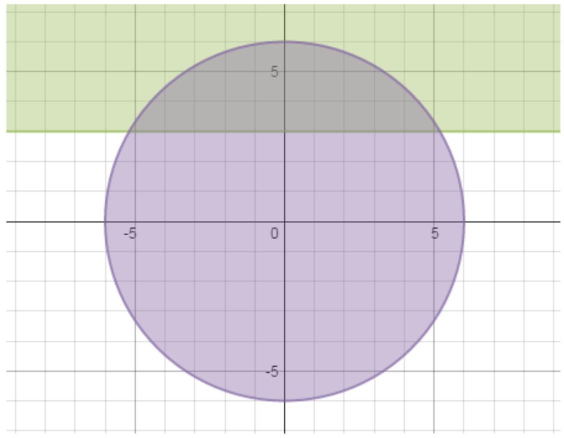
since 195 degrees is in quadrant III, sine is negative...

so, $\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4}$

SOLUTIONS

20) Graph and find the area of the region

$$x^2 + y^2 \le 36$$
$$y \ge 3$$



Step 1: Find the sector area

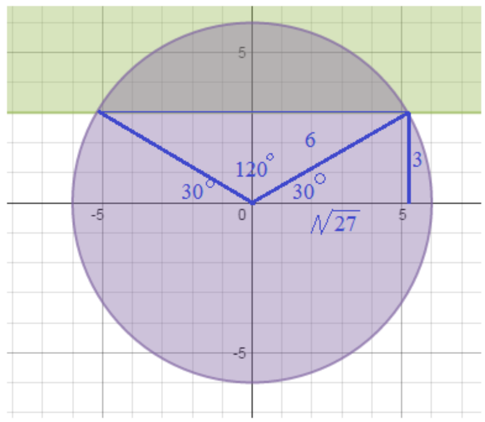
area of the circle: $\pi(\text{radius})^2$

If $y = 3$, then $x^2 + (3)^2 = 36$

$x = \sqrt{27}$ or $-\sqrt{27}$

30-60-90 triangle... and, sector is 120°

$$\text{sector area} = \frac{120}{360}\pi(\text{radius})^2$$
$$= \frac{1}{3}\pi(6)^2 = 12\pi$$



Step 2: Find the area of the (upside down) triangle

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$$
$$\frac{1}{2}(2 \times \sqrt{27})(3) = 9\sqrt{3}$$

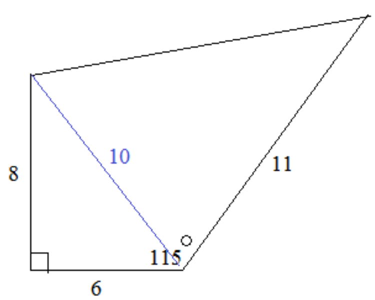
Step 3: Combine to find area of the segment

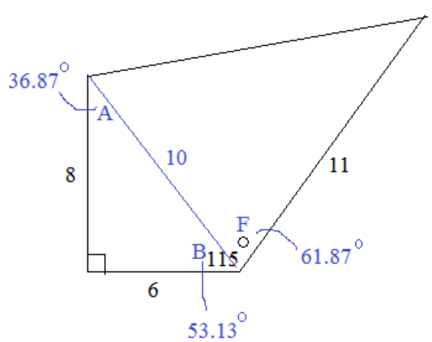
Area = sector area − triangle

$$\text{Area of the 'segment'} = 12\pi - 9\sqrt{3}$$

approx. 22.1

21) Find the area of the figure:





Step 1: Divide the figure into triangles!

Step 2: Find area of the right triangle

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$$
$$= \frac{1}{2}(6)(8) = 24 \text{ square units}$$

Step 3: Identify angle and use area formula

$\tan A = \frac{6}{8}$ $\quad A = \tan^{-1}(.75) = 36.87^\circ$

$A + B + 90 = 180$ (triangle)

$36.87 + B = 90$ $\quad B = 53.13^\circ$

$B + F = 115^\circ$

$53.13^\circ + F = 115^\circ$ $\quad F = 61.87^\circ$

Area = ghSinF ("Law of Sines Area formula")

$= (10)(11)\text{Sin}(61.87) = 97$ square units

Step 4: Combine areas of 2 triangles

24 + 97 = 121 square units (approx.)

22) $\sin(-x)\tan(-x) + \cos(-x) = \ ?$ SOLUTIONS

$(-\sin x)(-\tan x) + \cos x$ odd/even identities

$\sin x \tan x + \cos x$

$\sin x \left(\frac{\sin x}{\cos x}\right) + \cos x$

$\frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$

23) For $0 < x < 360^\circ$,

what is $2\sin(x + 42^\circ) = 1$?

Let $A = x + 42$, $2\sin(A) = 1$

$\sin(A) = \frac{1}{2}$

$A = 30^\circ + 360K$ or $150^\circ + 360K$

Therefore, $(x + 42^\circ) = -330, 30, 390, 750, \ldots$

or $(x + 42^0) = -210, 150, 510, 870, \ldots$

$x = -372, -12, \boxed{348}, 708, \ldots$

or $x = -252, \boxed{108}, 468, 828, \ldots$

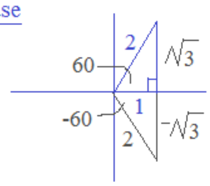
$\boxed{x = 108^\circ, 348^\circ}$

$\sec(x - 35^\circ) = 2$?

Let $B = x - 35$, $\sec(B) = 2$

$B = \sec^{-1}(2) = 60^\circ + 360K$ or $-60^\circ + 360K$

$\sec = \frac{\text{hypotenuse}}{\text{adjacent}}$



Therefore, $(x - 35^\circ) = -300, 60, 420, 780, \ldots$

$(x - 35^0) = -420, -60, 300, 660, \ldots$

$x = -265, \boxed{95}, 455, 815, \ldots$

or $x = -385, -25, \boxed{335}, 695, \ldots$

$\boxed{x = 95^0, 335^\circ}$

24) $8 - 2\tan x - 5\sec^2 x = 0$

where $x \in (0^\circ, 360^\circ)$

$5\sec^2 x + 2\tan x - 8 = 0$ use trig identity / substitution

$5(1 + \tan^2 x) + 2\tan x - 8 = 0$

distribute and collect 'like' terms

$5\tan^2 x + 2\tan x - 3 = 0$

$5U^2 + 2U - 3 = 0$ factor

$(5U - 3)(U + 1) = 0$

$(5\tan x - 3)(\tan x + 1) = 0$

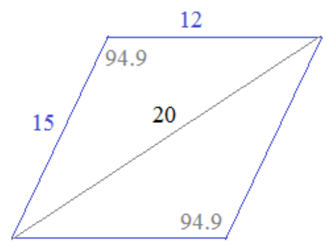
$\tan x = 3/5$ or $.6$ $x = \tan^{-1}(.6)$

$\tan x = -1$ $x = \tan^{-1}(-1)$

$\boxed{x = 30.96^\circ, 210.96^\circ \quad x = 135^\circ, 315^\circ}$

SOLUTIONS

25) A parallelogram has side lengths 12 and 15.
The longer diagonal has length 20...
What is the length of the shorter diagonal?



We need to find the angles of the parallelogram...

Using law of cosines:

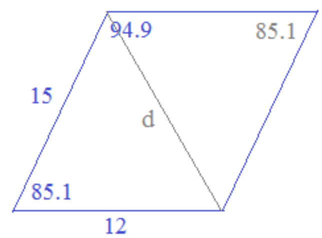
$$c^2 = a^2 + b^2 - 2(a)(b)\cos C$$

$$20^2 = 12^2 + 15^2 - 2(12)(15)\cos C$$

$$400 = 144 + 225 - 360\cos C$$

$$\frac{31}{-360} = \cos C \qquad C = 94.9^{\circ}$$

If C = 94.9 degrees, then the other angles are 180 - 94.9 = 85.1 degrees
(consecutive angles in parallelogram are supplementary)



Then, use law of cosines again to find the other diagonal...

$$d^2 = 12^2 + 15^2 - 2(12)(15)\cos(85.1)$$

$$= 144 + 225 - 360\cos(85.1)$$

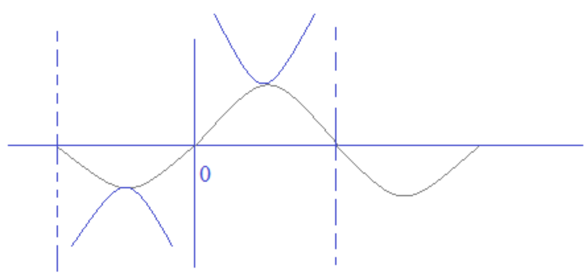
$$\boxed{d = 18.39}$$

26) As $\theta \to 0^-$,

what is $\csc\theta$? $\boxed{-\infty}$

As $\theta \to 0^+$,

what is $\csc\theta$? $\boxed{\infty}$

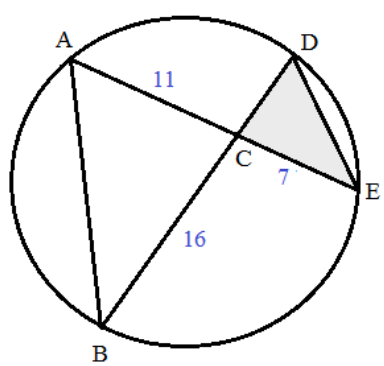


27) If the area of triangle ABC is 86,
what is the area of triangle CDE?

$\overline{AC} = 11$

$\overline{BC} = 16$

$\overline{CE} = 7$



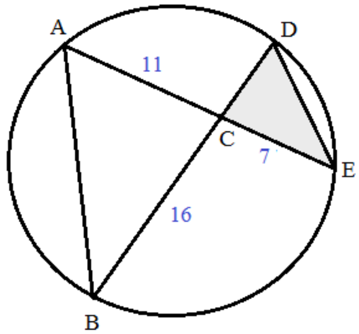
There are two approaches:
a 'trigonometry approach'
and
a 'geometry approach'.....

Solution...

Trigonometry Concepts Review

SOLUTION

27) If the area of triangle ABC is 86, what is the area of triangle CDE?



AE and BD are intersecting chords... therefore, $(AC)(CE) = (BC)(CD)$

$$(11)(7) = (16)(CD)$$

$$CD = 4.8125$$

$$\text{Area of triangle} = \frac{1}{2}ab(\sin C)$$

$$86 = \frac{1}{2}(11)(16)\sin C$$

$$.977 = \sin C$$

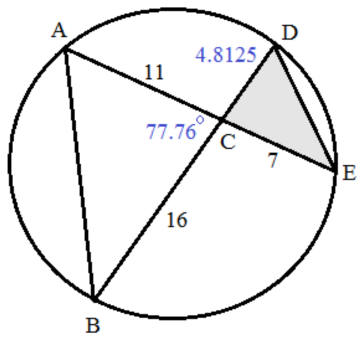
$$C = 77.76^{\circ}$$

$\angle ACB = \angle DCE$ (vertical angles)

77.76 degrees

$$\text{Area } \triangle CDE = \frac{1}{2}(4.8125)(7)\sin(77.76)$$

$$= 16.46$$

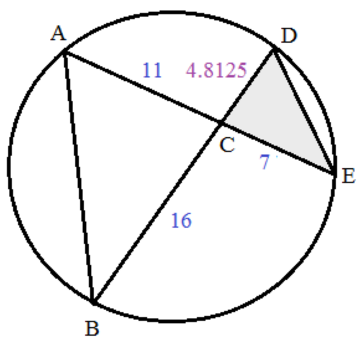


'Trigonometry Approach'

AE and BD are intersecting chords... therefore, $(AC)(CE) = (BC)(CD)$

$$(11)(7) = (16)(CD)$$

$$CD = 4.8125$$



$\angle BAE \cong \angle EDB$

because the inscribed angles share the same arc $\overset{\frown}{BE}$

$\angle ACB \cong \angle DCE$

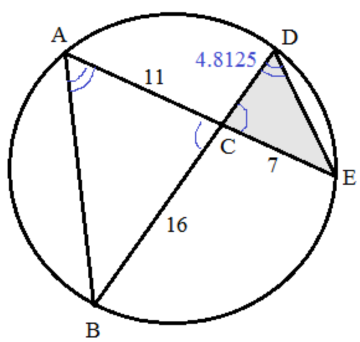
because vertical angles are congruent

Therefore, the triangles are similar (angle-angle)

Since the triangles are similar, the ratio of the areas is "ratio of the squared sides"

$$\frac{(16)^2}{(7)^2} = \frac{(11)^2}{(4.8125)^2} = \frac{86}{\text{Area of CDE}}$$

$$= 16.46$$

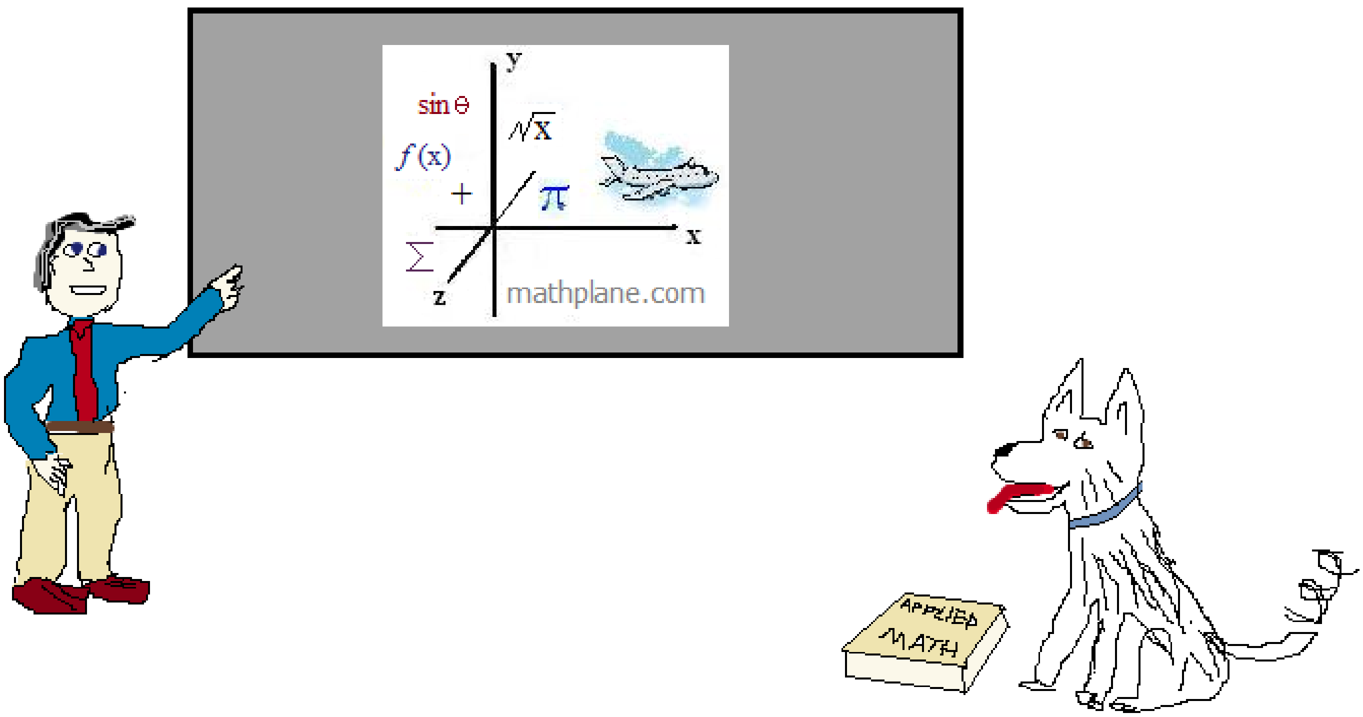


'Geometry Approach'

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, at Facebook, Google+, Pinterest, TES and TeachersPayTeachers

And, Mathplane *Express* for mobile at Mathplane.ORG