Topics include Pythagorean Triples, Word Problems, distance/rate, geometry applications, and more.
Pythagorean Theorem:

\[ a^2 + b^2 = c^2 \]

where \( a \) and \( b \) are lengths of the legs of a right triangle

and \( c \) is the length of the hypotenuse

"sum of the squares of the legs is equal to the square of the hypotenuse"

---

**Example:**

\[ (4)^2 + (7)^2 = c^2 \]

\[ 16 + 49 = 65 \]

\[ c = \sqrt{65} \]

---

Identifying triangles by their sides:

- \( a^2 + b^2 = c^2 \) right triangle
- \( a^2 + b^2 > c^2 \) acute triangle
- \( a^2 + b^2 < c^2 \) obtuse triangle

---

Distance Formula illustrates Pythagorean Theorem!

**point A:** \((x_1, y_1)\)

**point B:** \((x_2, y_2)\)

**distance**

\[ AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \]
"I have anxiety... Can't eat... Can't sleep... I feel tense and stiff..."

"I see yellow discoloration, and a bit of a pink rash -- which is common. But, let's get to the point, on top, where it matters, you look sharp!"

"I'm feeling drained. Hopefully, the doc can give me a boost."

"Practice Questions--→"
1) The following are sides of a triangle. Determine whether the triangle is right, obtuse, acute, not possible.
   a) 2, 7, 10
   b) 4, 5, 8
   c) 10, 6, 8
   d) 7, 8, 9
   e) 11, 11, 11

2) A 1-foot thick wooden platform is set 10 feet from a loading dock. If the dock is 4 feet high, how long must the ramp be to connect the platform and dock?

3) Multiple Choice: Jack traveled through D to get from F to B. How much shorter is the direct route versus the route he took?
   a) 5
   b) 10
   c) 15
   d) 20
   e) 25

4) Pythagorean Theorem rate question:
   A boy stands on the shore of a one-mile wide lake. He wants to reach camp down shore 3 miles on the opposite side. He can swim 2mph and walk 4mph. Is it quicker to swim across and then walk OR swim directly to the camp?
5) Find the length of the diagonal of the rectangular prism.

6) A biker riding at 10 miles per hour must take a road around the hills to reach a lake.
(15 miles due East, then, 25 miles due North)...
Meanwhile, a bird flying at 7 miles per hour can go directly over the hills.
Who would reach the water first?

7) Given: Circle Q
\[ \overline{PS} \perp \overline{SR} \]
\[ \overline{PS} = 36 \]
\[ \overline{SR} = 15 \]

Find: The area of circle Q

8) A 9 x 12 rectangle is inscribed in a circle.

What is the circumference of the circle?
Identify a related Pythagorean Triple. Then, find x.

**a)** \[ 15 - 20 - x \] \[ 3 - 4 - 5 \] \[ x = 25 \]

**b)** \[ 9 - x - 15 \]

**c)** \[ x - 30 - 34 \]

**d)** \[ 24 - 32 - x \]

**e)** \[ 10 - x - 26 \]

**f)** \[ x - 60 - 65 \]

**g)** \[ 40 - x - 85 \]

**h)** \[ 18 - 80 - x \]

**i)** \[ 14 - x - 50 \]

**j)** \[ 100 - 105 - x \]

**k)** \[ x - 70 - 74 \]

**l)** \[ 35 - x - 125 \]

**m)** \[ 2.5 - x - 6.5 \]

Pythagorean Triple (or, Triplet)
consists of 3 positive integers a, b, c
that satisfy the Pythagorean Theorem
\[ a^2 + b^2 = c^2 \]
Math Mixer in Ancient Greece

Dionysus Sports Bar
EST. 675 BC

Watch the Olympic Games here!

Specials:
\[ \text{Ale}^2 \text{ for } 1 \]
\[ + \text{ Beer}^2 \text{ for } 1 \]
\[ \frac{\text{Chips}^2}{\text{silver pieces}} \]

"...hey, under the bar, look at those legs..."
"I know! I know!"

Pythagoras gets an idea...

ANSWERS—→
1) The following are sides of a triangle.
Determine whether the triangle is right, obtuse, acute, not possible.

   a) 2, 7, 10  not possible  \( 2 + 7 < 10 \)  \( \sqrt{7} \)  \( \sqrt{10} \)  \( a^2 + b^2 < c^2 \)  \( \text{right} \)
   b) 4, 5, 8  obtuse  \( 16 + 25 < 64 \)
   c) 10, 6, 8  right  \( (3-4-5\text{'}\triangle) \)  \( 36 + 64 = 100 \)
   d) 7, 8, 9  acute  \( 49 + 64 > 81 \)
   e) 11, 11, 11  acute  \( \text{equilateral triangle} \)

\[ 121 + 121 > 121 \]

2) A 1-foot thick wooden platform is set 10 feet from a loading dock. If the dock is 4 feet high, how long must the ramp be to connect the platform and dock?

\[
R^2 = (\text{height})^2 + (\text{distance})^2
\]

\[
R = \sqrt{109}
\]

3) Multiple Choice: Jack traveled through D to get from F to B.
How much shorter is the direct route versus the route he took?

   a) 5
   b) 10
   c) 15
   d) 20
   e) 25

\[
\text{long route} = 15\text{ miles} + 20\text{ miles} = 35\text{ miles}
\]

\[
\text{short route}: 15^2 + 20^2 = \overline{FB}^2
\]

\[
\overline{FB} = 25\text{ miles}
\]

\[
\text{FB is 10 miles shorter than FDB}
\]

4) Pythagorean Theorem rate question:
A boy stands on the shore of a one-mile wide lake.
He wants to reach camp down shore 3 miles on the opposite side.
He can swim 2mph and walk 4mph.
Is it quicker to swim across and then walk OR swim directly to the camp?

\[
distance = rate \times time
\]

\[
time = \frac{distance}{rate}
\]

swim directly:
\[
time = \frac{\overline{10}}{2\text{ mph}} = 1.58\text{ hours}
\]

swim and walk:
\[
time (\text{swim}) = \frac{1\text{ mile}}{2\text{ mph}} = .5\text{ hours}
\]

\[
time (\text{walk}) = \frac{3\text{ miles}}{4\text{ mph}} = .75\text{ hours}
\]

It's faster to swim across and then walk....
5) Find the length of the diagonal of the rectangular prism.

\[
\begin{align*}
\triangle & = \sqrt{161} \\
\triangle & = \sqrt{125}
\end{align*}
\]

**SOLUTIONS**

Pythagorean Theorem and Distance Practice

-one method: first, find diagonal of bottom:

\[
d^2 = a^2 + b^2
\]
\[
d^2 = 10^2 + 5^2 = 125
\]
\[
d = \sqrt{125}
\]

then, find the prism's diagonal:

\[
D^2 = d^2 + c^2
\]
\[
D^2 = 125 + 6^2
\]
\[
D = \sqrt{161}
\]

shortcut:

\[
\sqrt{10^2 + 5^2 + 6^2} = \sqrt{161}
\]

6) A biker riding at 10 miles per hour must take a road around the hills to reach a lake.
(15 miles due East. Then, 25 miles due North)... Meanwhile, a bird flying at 7 miles per hour can go directly over the hills. Who would reach the water first?

6) A biker riding at 10 miles per hour must take a road around the hills to reach a lake.
(15 miles due East. Then, 25 miles due North)... Meanwhile, a bird flying at 7 miles per hour can go directly over the hills. Who would reach the water first?

**Pythagorean Theorem:**

\[
a^2 + b^2 = c^2
\]
\[
15^2 + 25^2 = c^2
\]
\[
c = \frac{5 \sqrt{34}}{2} \text{ or approx. } 29.15 \text{ miles}
\]

**distance = rate x time**

-biker: 40 miles = (10m/hr)(time)

-time = 4 hours

-bird: 29.15 miles = (7m/hr)(time)

-time = 4.16 hours (approx.)

The biker will reach the lake first!

7) Given: Circle Q  
\[
P \perp S R
\]
\[
PS = 36 \\
SR = 15
\]

Find: The area of circle Q

\[
\text{Area} = \pi (\text{radius})^2
\]
\[
\text{Area} = \pi (19.5)^2
\]
\[
= 380.25 \pi \text{ square units}
\]

8) A 9 x 12 rectangle is inscribed in a circle.

What is the circumference of the circle?


diameter of circle is 15

\[
\text{circumference} = \pi \cdot \text{(diameter)}
\]

\[
15 \pi
\]
Identify a related Pythagorean Triple. Then, find x.

a) \(15 - 20 - x\)  \(3 - 4 - 5\)  \(x = 25\)

b) \(9 - x - 15\)  \(3 - 4 - 5\)  \(x = 12\)

c) \(x - 30 - 34\)  \(8 - 15 - 17\)  \(x = 16\)

d) \(24 - 32 - x\)  \(3 - 4 - 5\)  \(x = 40\)

e) \(10 - x - 26\)  \(5 - 12 - 13\)  \(x = 24\)

f) \(x - 60 - 65\)  \(5 - 12 - 13\)  \(x = 25\)

g) \(40 - x - 85\)  \(8 - 15 - 17\)  \(x = 75\)

h) \(18 - 80 - x\)  \(9 - 40 - 41\)  \(x = 82\)

i) \(14 - x - 50\)  \(7 - 24 - 25\)  \(x = 48\)

j) \(100 - 105 - x\)  \(20 - 21 - 29\)  \(x = 145\)

k) \(x - 70 - 74\)  \(12 - 35 - 37\)  \(x = 24\)

l) \(35 - x - 125\)  \(7 - 24 - 25\)  \(x = 120\)

m) \(2.5 - x - 6.5\)  \(5 - 12 - 13\)  \(x = 6\)

SOLUTIONS

A few Pythagorean Triples:

- \(3, 4, 5\)
- \(5, 12, 13\)
- \(8, 15, 17\)
- \(7, 24, 25\)
- \(9, 40, 41\)
- \(12, 35, 37\)
- \(20, 21, 29\)
Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Cheers

Also, at Facebook, Google+, Pinterest, and TeachersPayTeachers

One more question:

The perimeter of an isosceles triangle is 50, and the length of the altitude to the base is 10. What is the measure of each leg and base?

ANSWER→
One More Question:

The perimeter of an isosceles triangle is 50, and the length of the altitude to the base is 10.

What is the measure of each leg and base?

Step 1: Draw a picture and label parts

Step 2: Solve (applying Pythagorean Theorem)

base + leg + leg = 50
Therefore, base = 50 - 2L
and half the base is \(\frac{1}{2}(50 - 2L) = 25 - L\)

\[a^2 + b^2 = c^2\]
\[10^2 + (25 - L)^2 = L^2\]
\[100 + 625 - 50L + L^2 = L^2\]
\[725 = 50L\]
\[L = 14.5\]

Each leg is 14.5

\[B = 50 - 2L\]
\[= 50 - 29 = 21\]

The base is 21

The perimeter of triangle = 14.5 + 14.5 + 21 = 50

Since legs are congruent, it's an isosceles triangle.

\[(10.5)^2 + (10)^2 = (14.5)^2\]
\[110.25 + 100 = 210.25\]

Pythagorean Theorem confirms right and left triangles