Geometry: Proofs and Postulates Worksheet

Practice Exercises (w/ Solutions)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{AD}$ and $\overline{BC}$ bisect each other</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\overline{AM} \cong \overline{DM}$; $\overline{CM} \cong \overline{BM}$</td>
<td>2. Definition of bisector</td>
</tr>
<tr>
<td>3. $\angle AMC \cong \angle BMD$</td>
<td>3. Vertical angles are congruent</td>
</tr>
<tr>
<td>4. $\triangle AMC \cong \triangle DMB$</td>
<td>4. Side-Angle-Side (SAS) (2, 3, 2)</td>
</tr>
<tr>
<td>5. $\overline{AC} \cong \overline{BD}$</td>
<td>5. CPCTC (Corresponding Parts of Congruent Triangles are Congruent)</td>
</tr>
</tbody>
</table>

Topics include triangle characteristics, quadrilaterals, circles, midpoints, SAS, and more.
... Stranded somewhere in the (Bermuda) Triangle...

SAS is not a distress signal

... the Math Guy -- losing his mind -- mistakenly builds a geometry message

When you’re in the Bermuda Triangle, SOS is more useful than SAS!!
Introduction to proofs: Identifying theorems and postulates

1) Why is $\angle AOB \simeq \angle COD$?

2) Why are $\overline{LQ}$ and $\overline{PN}$ congruent segments?

3) Angles 1 and 2 are congruent. Why are $\angle KPM$ and $\angle LQJ$ congruent?

4) $DV \perp VF$ and $EV \perp VG$. Why is angle 1 congruent to angle 2?

5) If $TR = RI$, and $AB$ and $CD$ are trisectors, why are $CR$ and $BD$ congruent segments?

6) $KM$ and $LN$ bisect each other. Is $KM$ congruent to $NL$?
Introduction to proofs: Identifying geometry theorems and postulates

Explain using geometry concepts and theorems:

1) Why is the triangle isosceles?

2) Why is $AB$ an altitude?

3) Why are the triangles congruent?

4) Why is $NM$ a median?

5) If $ABCD$ is a parallelogram, why are $\angle A$ and $\angle C$ congruent?

6) Why are the triangles congruent?
A) Given: \( AB \cong EG \)
\( CD \cong EG \)
Prove: \( AC \cong BD \)

B) Given: \( \angle M \) is the complement to \( \angle MQN \)
\( \angle P \) is the complement to \( \angle PQN \)
\( NQ \) bisects \( \angle MQP \)
Prove: \( \angle M \cong \angle P \)

C) Given: \( BE \) bisects \( \angle FMD \)
Prove: \( \angle 1 \cong \angle 3 \)
1) Given: \( \odot O; \overline{DK} \perp \overline{OK} \)
Prove: \( \overline{DK} \cong \overline{KR} \)

![Diagram](image1.png)

---

2) Given: \( \overline{AD} \) and \( \overline{BC} \) bisect each other
Prove: \( \overline{AC} \cong \overline{BD} \)

![Diagram](image2.png)

---

3) Given: \( \overline{RS} \cong \overline{RT} \)
\( AT \) and \( CS \) are medians
Prove: \( \overline{AT} \) and \( \overline{CS} \) are congruent

![Diagram](image3.png)
4) Given: \( AC \cong AB \)
   \( \text{D and E are midpoints} \)

   Prove: \( \angle B \cong \angle C \)

5) Prove the diagonals of an isosceles trapezoid are congruent.
6) Given: \( \overline{AE} \perp \overline{EC} \)
\( \overline{BE} \perp \overline{ED} \)

Prove: \( \angle 1 = \angle 3 \)

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7) Given: \( \overline{AB} \parallel \overline{CD} \)
\( \overline{AB} \cong \overline{CD} \)

Prove: \( \overline{AD} \parallel \overline{BC} \)

(Hint: Use an auxiliary line segment)

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</table>
"Noah's Arc"

"Perhaps you misunderstood the command to 'build an ark'?"

"I suppose I did...."

"Hey, at least it floats...."

"40 day-40 night cruise?!?!"

"Where are the rooms for two?"

Eventually, Noah realizes that this assignment was NOT a geometry construction.
1) Why is $\angle AOB \cong \angle COD$?

Since $AOC = BOD$, and $BOC = BOC$ (reflexive property), $AOB = COD$ (subtraction property)

(If congruent angles are subtracted from congruent angles, then the differences are congruent)

2) Why are $\overline{LQ}$ and $\overline{PN}$ congruent segments?

Since $LP = QN$, and $PQ = PQ$ (reflexive property), then $\overline{LQ}$ and $\overline{PN}$ are congruent (addition property)

(If congruent segments are added to equal segments, then the sums are the same!)

3) Angles 1 and 2 are congruent.

Why are $\angle KPM$ and $\angle LQJ$ congruent?

If angles are supplementary to congruent angles, then they are congruent...

4) $DV \perp VF$ and $EV \perp VG$

Why is angle 1 congruent to angle 2?

Complements of the same angle are congruent.
OR, subtraction property (right angles are congruent... then, subtract $\angle EVF$ from each, leaving 1 and 2)

5) If $TR = RI$, and $AB$ and $CD$ are trisectors, why are $CR$ and $BD$ congruent segments?

Division Property (If equal segments are divided by the same amount, the divided amounts are congruent)

6) $KM$ and $LN$ bisect each other.

Is $KM$ congruent to $NL$?

Yes... Since segments bisect each other, $KP = PM$ and $NP = PL$. Also, from diagram, $KP = PL$...

So, using substitution and addition properties, $KM = NL$

Or, use the multiplication property... (Since $KP = PL$ and both segments are doubled, the products are equal)
1) Why is the triangle isosceles?

\[ \overline{PR} \text{ and } \overline{PQ} \text{ are radii of the circle. Therefore, they have the same length.} \]

A triangle with 2 sides of the same length is isosceles.

2) Why is \( \overline{AB} \) an altitude?

\[ \overline{AB} = \overline{AB} \text{ (reflexive)} \]

\[ \triangle CAB = \triangle DAB \text{ (side-angle-side)} \]

If triangles are same, then

\[ \angle ABC = \angle ABD \text{ (CPCTC)} \]

Since angles are same and must add up to 180, each is 90°

Therefore, \( \overline{AB} \) is altitude.

3) Why are the triangles congruent?

Vertical angles are congruent therefore, triangles are congruent (angle-side-angle)

4) Why is \( \overline{NM} \) a median?

Since CMA is right angle, AMB is right angle.

\[ \overline{AM} = \overline{AM} \text{ (reflexive)} \]

\[ \triangle AMB = \triangle AMC \text{ (Angle-Side-Angle)} \]

\[ \overline{BM} = \overline{CM} \text{ (CPCTC)} \]

\( \overline{NM} \) is a median (segment from vertex to midpoint)

5) If \( \overline{ABCD} \) is a parallelogram, why are \( \angle A \) and \( \angle C \) congruent?

\[ \overline{BC} \parallel \overline{AD} \text{ (definition of a parallelogram)} \]

\[ \angle C = \angle D \text{ (alternate interior angles)} \]

\[ \overline{CD} \parallel \overline{AB} \text{ (def. of parallelogram)} \]

\[ \angle A = \angle D \text{ (corresponding angles)} \]

Since \( C = D \) and \( D = A \), then \( A = C \).

6) Why are the triangles congruent?

Since they are radii of the circle, the 4 marked sides are congruent.

Vertical angles

Triangles congruent by side-angle-side

NOTE: CPCTC is "Corresponding Parts of Congruent Triangles are Congruent"
SOLUTIONS

A) Given: \( \frac{AB}{CD} = \frac{EG}{EG} \)
Prove: \( AC \cong BD \)

1) \( AB = EG \)  
   1) Given
2) \( CD = EG \)  
   2) Given
3) \( AB = CD \)  
   3) Transitive Property (Segments that are congruent to the same segment are congruent)
4) \( BC = BC \)  
   4) Reflexive Property
5) \( AC = BD \)  
   5) Addition Property (If a segment is added to congruent segments, the sums are congruent)

OR,

1) \( AB = EG \)  
   1) Given
2) \( CD = EG \)  
   2) Given
3) \( AB = CD \)  
   3) Transitive Property (Segments that are congruent to the same segment are congruent)
4) \( AC = BD \)  
   4) Addition Property (If segment (BC) is added to congruent segments, then the sums are congruent)

B) Given: \( \angle M \) is the complement to \( \angle MQN \)
   \( \angle P \) is the complement to \( \angle PQN \)
   \( NQ \) bisects \( \angle MQP \)
Prove: \( \angle M \cong \angle P \)

1) \( NQ \) bisects \( \angle MQN \)  
   1) Given
2) \( \angle MQN \cong \angle PQN \)  
   2) Definition of (Angle) Bisector (If ray bisects an angle, the angle halves are congruent)
3) \( \text{Angles } M \text{ and } MQN \text{ are complementary} \)  
   3) Given
4) \( \text{Angles } P \text{ and } PQN \text{ are complementary} \)  
   4) Given
5) \( \angle M = \angle P \)  
   5) Substitution (If angles are complementary to congruent angles, then they are congruent) 
   "Congruent Complements"

C) Given: \( \overline{BE} \) bisects \( \angle FMD \)
Prove: \( \angle 1 \cong \angle 3 \)

1) \( \overline{BE} \) bisects \( \angle FMD \)  
   1) Given
2) \( \angle 2 = \angle 3 \)  
   2) Definition of Bisector (If segment bisects an angle, the angle halves are congruent)
3) \( \angle 1 = \angle 2 \)  
   3) Vertical angles are congruent
4) \( \angle 1 = \angle 3 \)  
   4) Transitive Property (Angles congruent to the same angle are congruent)
### Geometry Proofs

1) **Given:** $\odot O$; $\overline{DR} \perp \overline{OK}$  
   **Prove:** $\overline{DK} \cong \overline{KR}$

![Diagram 1](image1)

**Solutions**

<table>
<thead>
<tr>
<th>Statements</th>
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<tbody>
<tr>
<td>1. Circle $O$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\overline{DR} \perp \overline{OK}$</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. $\angle OKD$ and $\angle OKR$ are right angles</td>
<td>3. Definition of perpendicular</td>
</tr>
<tr>
<td>4. $\angle OKD \cong \angle OKR$</td>
<td>4. All right angles are congruent</td>
</tr>
<tr>
<td>5. Auxiliary line segments $\overline{OR}$ and $\overline{OD}$</td>
<td>5. A segment joins 2 points</td>
</tr>
<tr>
<td>6. $\overline{OR} \cong \overline{OD}$</td>
<td>6. All radii are congruent</td>
</tr>
<tr>
<td>7. $\overline{OK} \cong \overline{OK}$</td>
<td>7. Reflexive property</td>
</tr>
<tr>
<td>8. $\triangle DOK \cong \triangle ROK$</td>
<td>8. Hypotenuse Leg Theorem (HL)</td>
</tr>
<tr>
<td>9. $\overline{DK} \cong \overline{KR}$</td>
<td>(4, 6, 7)</td>
</tr>
</tbody>
</table>

2) **Given:** $\overline{AD}$ and $\overline{BC}$ bisect each other  
   **Prove:** $\overline{AC} \cong \overline{BD}$

![Diagram 2](image2)

**Solutions**

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<td>4. Side-Angle-Side (SAS)</td>
</tr>
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<td>5. $\overline{AC} \cong \overline{BD}$</td>
<td>(2, 3, 2)</td>
</tr>
<tr>
<td>6. $\overline{AC} \cong \overline{BD}$</td>
<td>5. CPCTC (Corresponding Parts of Congruent Triangles are Congruent)</td>
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</tbody>
</table>

3) **Given:** $\overline{RS} \cong \overline{RT}$  
   **Prove:** $\overline{AT}$ and $\overline{CS}$ are congruent

![Diagram 3](image3)

**Solutions**

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<tbody>
<tr>
<td>1. $\overline{RS} \cong \overline{RT}$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\triangle RST \cong \triangle RTS$</td>
<td>2. Sides-Angles Theorem (Isosceles Triangle)</td>
</tr>
<tr>
<td>3. $\overline{AT}$ and $\overline{CS}$ are medians</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. $A$ and $C$ are midpoints</td>
<td>4. Definition of median</td>
</tr>
<tr>
<td>5. $\overline{RA} \cong \overline{SA}$; $\overline{RC} \cong \overline{TC}$</td>
<td>5. Definition of midpoint</td>
</tr>
<tr>
<td>6. $\overline{SA} \cong \overline{CT}$</td>
<td>6. Division property (like division of congruent segments)</td>
</tr>
<tr>
<td>7. $\overline{ST} \cong \overline{ST}$</td>
<td>7. Reflexive property</td>
</tr>
<tr>
<td>8. $\triangle SAT \cong \triangle TCS$</td>
<td>8. Side-Angle-Side (SAS)</td>
</tr>
<tr>
<td>9. $\overline{AT} \cong \overline{CS}$</td>
<td>(6, 2, 7)</td>
</tr>
<tr>
<td>10. $\overline{AT} \cong \overline{CS}$</td>
<td>9. CPCTC</td>
</tr>
</tbody>
</table>
4) Given: \( AC \cong AB \)
D and E are midpoints

Prove: \( \angle B \cong \angle C \)

**Statements** | **Reasons**
--- | ---
1. \( AC \cong AB \) | 1. Given
2. \( AE \cong EC \) (AE is 1/2 of AC) | 2. Definition of midpoint
3. \( AD \cong DB \) (AD is 1/2 of AB) | 3. Definition of midpoint
4. \( AD \cong AE \) | 4. Division property ("like division" of congruent segments)
5. \( \angle A \cong \angle A \) | 5. Reflexive property
6. \( \triangle DAC \cong \triangle EAB \) | 6. Side-Angle-Side (SAS) (4, 5, 1)
7. \( \angle B \cong \angle C \) | 7. CPCTC

5) Prove the diagonals of an isosceles trapezoid are congruent.

**Definition of Isosceles Trapezoid:** A trapezoid in which the base angles and non-parallel sides are congruent

**Statements** | **Reasons**
--- | ---
1. RNPS is isosceles trapezoid with base RS | 1. Given (diagram)
2. \( NR \cong PS \) | 2. Definition of Isosceles Trapezoid
3. \( \angle NRS \cong \angle PSR \) | 3. Definition of Isosceles Trapezoid
4. \( RS \cong RS \) | 4. Reflexive property
5. \( \triangle NRS \cong \triangle PSR \) | 5. Side-Angle-Side (SAS) (2, 3, 4)
6. (diagonals) \( FR \cong NS \) | 6. CPCTC
6) Given: \( \overline{AE} \perp \overline{BC} \)
\( \overline{BE} \perp \overline{ED} \)

Prove: \( \angle 1 = \angle 3 \)

**Proof 1: Using the Subtraction Property**

<table>
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<td>1) ( \overline{AE} \perp \overline{BC} )</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) ( \overline{BE} \perp \overline{ED} )</td>
<td>2) Given</td>
</tr>
<tr>
<td>3) ( \angle AEC ) is right angle</td>
<td>3) Definition of Perpendicular (Perpendicular lines form right angles)</td>
</tr>
<tr>
<td>4) ( \angle BED ) is right angle</td>
<td>4) Definition of Perpendicular</td>
</tr>
<tr>
<td>5) ( \angle AEC \cong \angle BED )</td>
<td>5) All right angles are congruent</td>
</tr>
<tr>
<td>6) ( \angle 2 \cong \angle 2 )</td>
<td>6) Reflexive property</td>
</tr>
<tr>
<td>7) ( \angle 1 \cong \angle 3 )</td>
<td>7) Subtraction property (If equal angles are subtracted from congruent angles, then the differences are congruent)</td>
</tr>
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**Proof 2: Using the Congruent Complements**

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<td>1) ( \overline{AE} \perp \overline{BC} )</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) ( \overline{BE} \perp \overline{ED} )</td>
<td>2) Given</td>
</tr>
<tr>
<td>3) ( \angle AEC ) is right angle</td>
<td>3) Definition of Perpendicular (Perpendicular lines form right angles)</td>
</tr>
<tr>
<td>4) ( \angle BED ) is right angle</td>
<td>4) Definition of Perpendicular</td>
</tr>
<tr>
<td>5) Angles 1 and 2 are complementary</td>
<td>5) Definition of complementary (Two angles that add up to 90 degrees i.e. form a right angle)</td>
</tr>
<tr>
<td>6) Angles 3 and 2 are complementary</td>
<td>6) Definition of Complementary</td>
</tr>
<tr>
<td>7) ( \angle 1 \cong \angle 3 )</td>
<td>7) Congruent Complements (Complements of the same angle are congruent)</td>
</tr>
</tbody>
</table>

7) Given: \( \overline{AB} \parallel \overline{CD} \)
\( \overline{AB} \cong \overline{CD} \)

Prove: \( \overline{AD} \parallel \overline{BC} \)

(Hint: Use an auxiliary line segment)

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<td>1) ( \overline{AB} = \overline{CD} )</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) ( \overline{AC} ) is a line segment</td>
<td>2) Auxiliary line (2 points make a line)</td>
</tr>
<tr>
<td>3) ( \overline{AB} \parallel \overline{CD} )</td>
<td>3) Given</td>
</tr>
<tr>
<td>4) ( \angle BAC = \angle DCA )</td>
<td>4) If ( \parallel ) lines cut by transversal, then alternate interior angles are congruent</td>
</tr>
<tr>
<td>5) ( \overline{AC} \cong \overline{AC} )</td>
<td>5) Reflexive Property</td>
</tr>
<tr>
<td>6) ( \triangle BAC \cong \triangle DCA )</td>
<td>6) Side-Angle-Side (SAS) (1, 4, 5)</td>
</tr>
<tr>
<td>7) ( \angle ACB \cong \angle CAD )</td>
<td>7) Corresponding Parts of Congruent Triangles are Congruent (CPCTC)</td>
</tr>
<tr>
<td>8) ( \overline{AD} \parallel \overline{BC} )</td>
<td>8) If alternate interior angles are congruent, then lines are parallel</td>
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Thanks for visiting. (Hope it helped!)

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Let My People Know

"If I put my staff here, I can part the Red Sea into equal segments..."

"I understand that. Now, can he explain how that plant is always burning?"

Moses teaches the wonders of Geometry