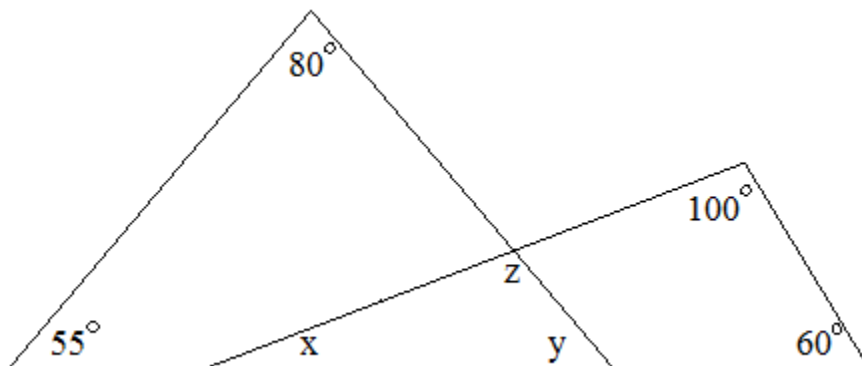


# Geometry Review 003

(With solutions)



*Topics include triangle properties, vertical angles, quadrilaterals, right triangles, parallel lines, restrictions, and more.*

Triangle Properties: Restrictions

Example: What are the restrictions of x?

$$m\angle A > m\angle B$$

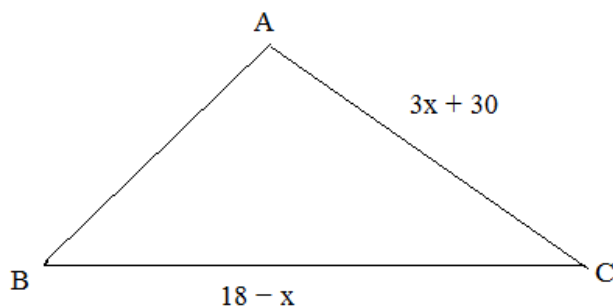
Since  $\angle A > \angle B$ ,

$$\overline{BC} > \overline{AC}$$

$$(18 - x) > (3x + 30)$$

$$-12 > 4x$$

$$x < -3$$



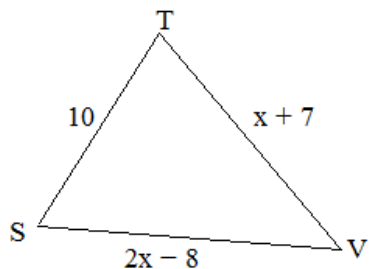
Also, since a side cannot be less than or equal to zero,

$$\overline{BC} \quad 18 - x > 0 \quad x < 18$$

$$\overline{AC} \quad 3x + 30 > 0 \quad x > -10$$

Therefore, the restrictions for x are  $-10 < x < -3$

Example: If the perimeter is less than 45, which side is the base?



$\triangle STV$  is an *isosceles* triangle

If 10 is the base:  $x + 7 = 2x - 8$   
 $x = 15$

therefore, the legs are 22  
 (If the legs are 22, then the perimeter exceeds 45)



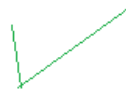
If  $2x - 8$  is the base:  $x + 7 = 10$   
 $x = 3$

Therefore, the legs are 10 and  
 the base is -2  
 (a segment cannot be negative!)



If  $x + 7$  is the base:  $2x - 8 = 10$   
 $x = 9$

Therefore, the legs are 10 and  
 the base is 16



The base is  $\overline{TV} = 16$

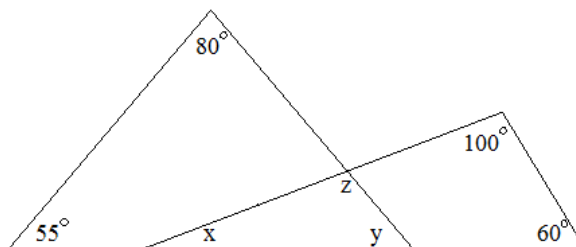
Geometry Angle and Triangle Exercises

1) Find  $x$ ,  $y$ , and  $z$

$m\angle x =$

$m\angle y =$

$m\angle z =$

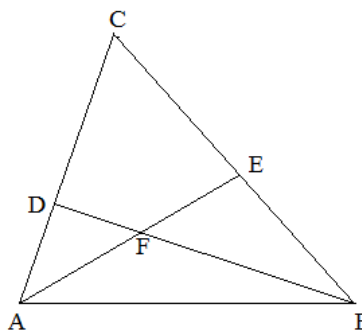


2) Given:  $\angle CAB = 80^\circ$

$\angle CBA = 60^\circ$

$\overline{AE}$  and  $\overline{BD}$  are altitudes

Find:  $m\angle C$  and  $m\angle AFB$



3) Given: Right triangle ABC inscribed in a circle

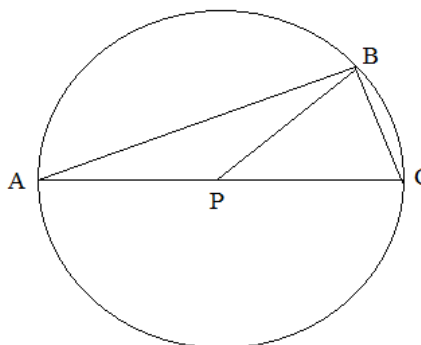
$\angle ABP = 4x + y$

$\angle PBC = 6x + 8$

$\angle APB = 18y + 100$

$\angle BPC = 5x + 4$

Find:  $\angle ABP$

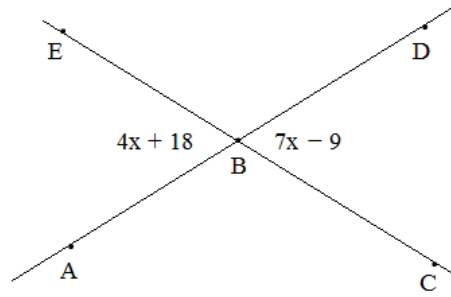


Geometry Angle and Triangle Exercises

4) Find  $x$  and  $\angle ABC$

$$x =$$

$$\angle ABC =$$

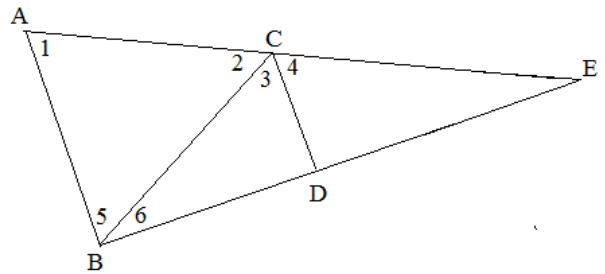


5) Given:  $\angle 1 = \angle 5$

$$\overline{AB} \parallel \overline{CD}$$

$$\angle 2 = 38^\circ$$

Find:  $\angle 3$

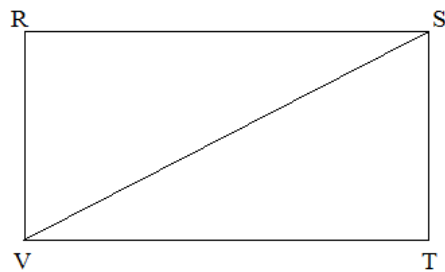


6) Given: Rectangle  $RSTV$

$$\angle RVS = x^2 + 30^\circ$$

$$\angle RSV = 6x - 12^\circ$$

Find the measure of  $\angle VST$

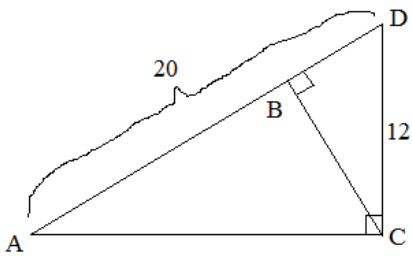


Geometry Angle and Triangle Exercises

7) Given: 45-45-90 Right Triangle  
Hypotenuse length: 25

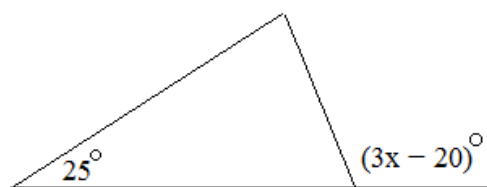
Find the perimeter.

8) Find the perimeter of  $\triangle ABC$

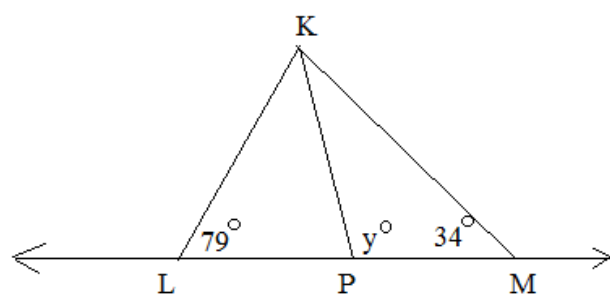


Geometry Angle and Triangle Exercises

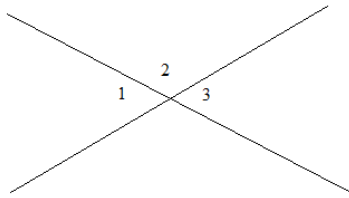
9) What are the restrictions of  $x$ ?



10) What are the restrictions of  $y$ ?

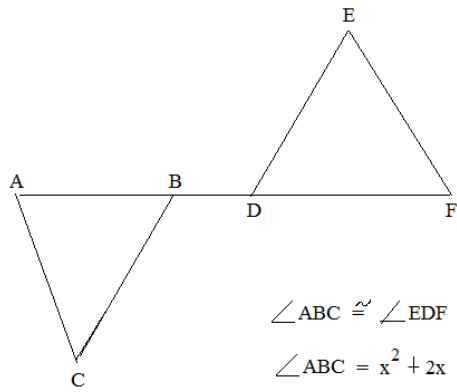


11)



If angles 1 and 3 are supplementary, what is the measure of angle 2?

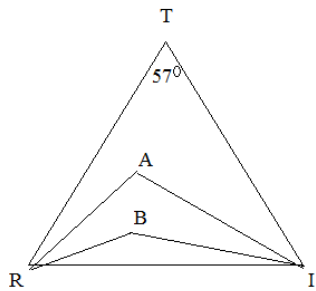
12)



$$\begin{aligned} \angle ABC &\cong \angle EDF \\ \angle ABC &= x^2 + 2x \\ \angle BDE &= 2x + 40 \end{aligned}$$

What is the measure of BDE?

13)



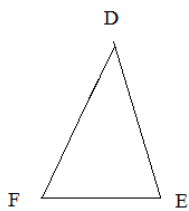
AR, BR, AI, and BI are angle trisectors..

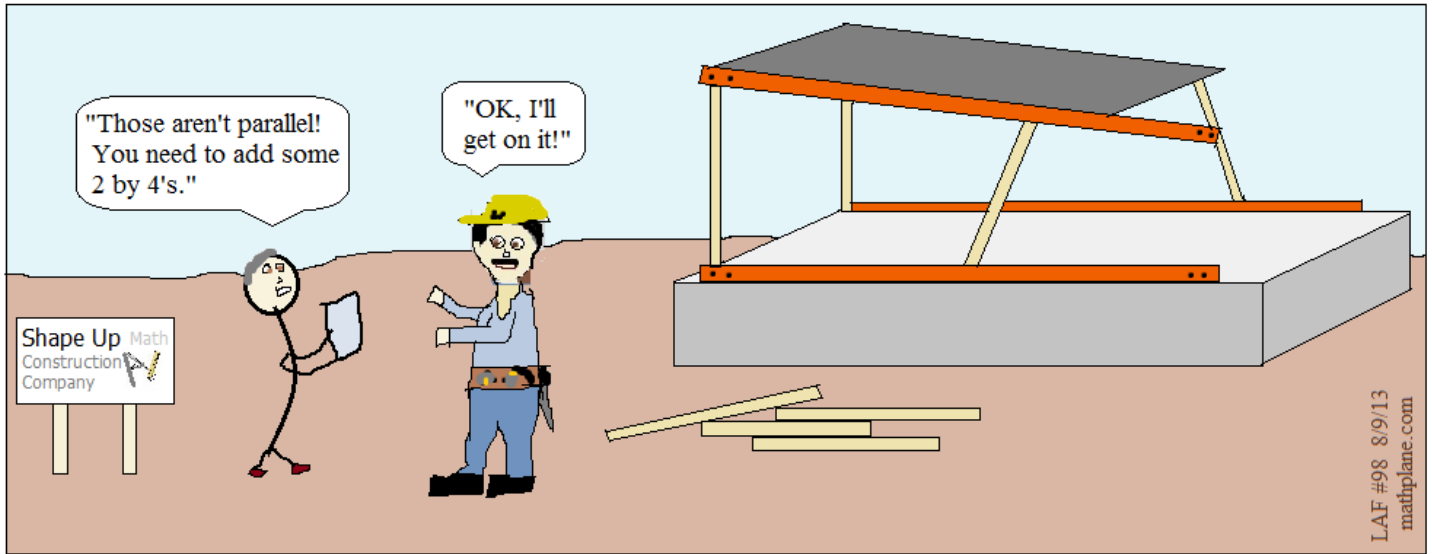
angle A = \_\_\_\_\_

angle B = \_\_\_\_\_

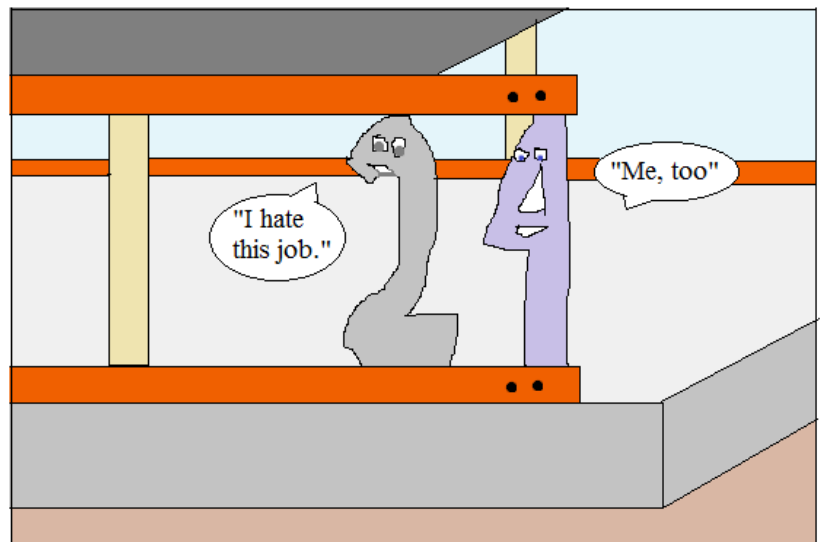
14) In  $\triangle DEF$ , the sum of  $\angle D$  and  $\angle E$  is  $110^\circ$  and the sum of  $\angle E$  and  $\angle F$  is  $150^\circ$

What is the sum of  $\angle D$  and  $\angle F$  ?





The Math Guy misunderstood the Architect's suggestion...



Building Materials

# SOLUTIONS



Geometry Angle and Triangle Exercises

SOLUTIONS

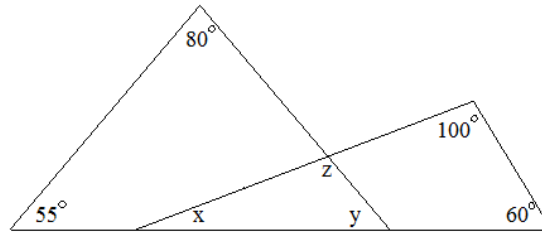
1) Find  $x$ ,  $y$ , and  $z$

$$m\angle x = 20^\circ$$

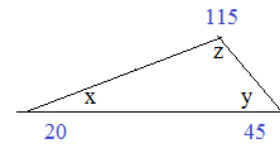
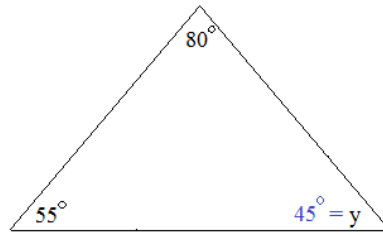
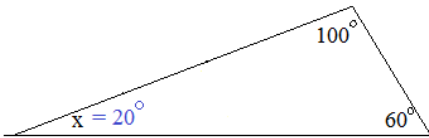
$$m\angle y = 45^\circ$$

$$m\angle z = 115^\circ$$

Examine each triangle separately



(Interior angles of a triangle add up to 180)



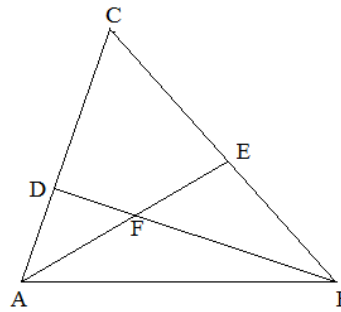
2) Given:  $\angle CAB = 80^\circ$

$$\angle CBA = 60^\circ$$

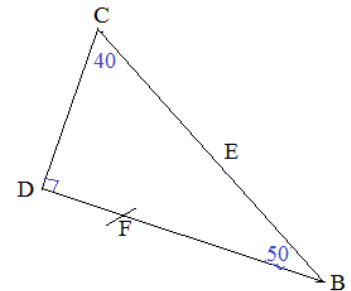
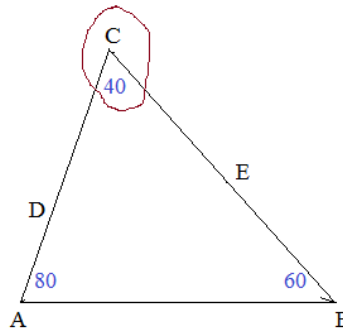
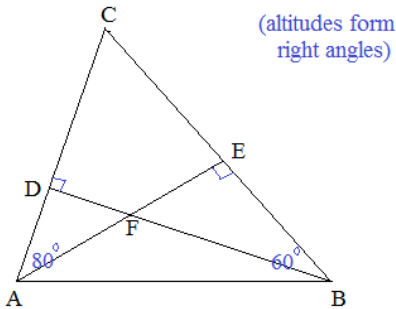
$\overline{AE}$  and  $\overline{BD}$  are altitudes

Find:  $m\angle C$  and  $m\angle AFB$

$$40^\circ \quad 140^\circ$$

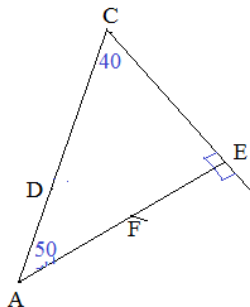


(Label the given angles)



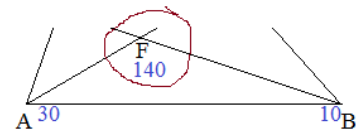
Since  $\angle C = 40$  and  $\angle D = 90$ ,  
 $\angle CBD = 50$

If  $\angle CBD = 50$  and  $\angle CBA = 60$ ,  
 $\angle DBA = 10$



Since  $\angle C = 40$  and  $\angle E = 90$ ,  
 $\angle CAE = 50$

And, if  $\angle CAE = 50$  and  $\angle CAB = 80$ ,  
then  $\angle EAB = 30$



Finally, looking at  $\angle AFB$ ,  $A = 30$ ,  $B = 10$ ;  
therefore,  $\angle AFB = 140^\circ$

## SOLUTIONS

3) Given: Right triangle ABC inscribed in a circle

$$\begin{aligned}\angle ABP &= 4x + y \\ \angle PBC &= 6x + 8 \\ \angle APB &= 18y + 100 \\ \angle BPC &= 5x + 4\end{aligned}$$

Find:  $\angle ABP$

ABP and PBC are complementary angles:

$$\begin{aligned}(4x + y) + (6x + 8) &= 90 \\ 10x + y &= 82\end{aligned}$$

APB and BPC are supplementary angles:

$$\begin{aligned}(18y + 100) + (5x + 4) &= 180 \\ 5x + 18y &= 76\end{aligned}$$

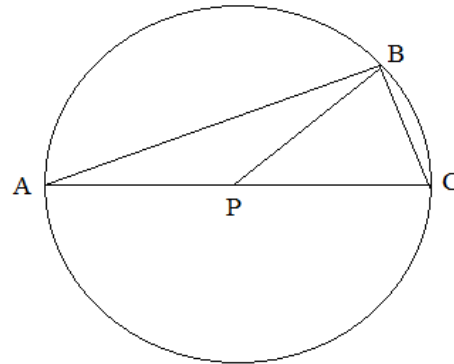
Since we have 2 equations and 2 unknowns, we can find  $x$  and  $y$  (using elimination method)

$$\begin{aligned}10x + y &= 82 \\ -10x - 36y &= -152\end{aligned}$$

$$\begin{aligned}-35y &= -70 \\ y &= 2\end{aligned}$$

$$\begin{aligned}10x + (2) &= 82 \\ x &= 8\end{aligned}$$

Since  $x = 8$  and  $y = 2$ , the measure of  $\angle ABP = 4(8) + (2) = 34^\circ$



$$\begin{aligned}\text{also, } \angle PBC &= 6x + 8 = 56^\circ \\ \angle APB &= 18y + 100 = 136^\circ \\ \angle BPC &= 5x + 4 = 44^\circ\end{aligned}$$

4) Find  $x$  and  $\angle ABC$

$$\begin{aligned}x &= 9 \\ \angle ABC &= 126^\circ\end{aligned}$$

Since vertical angles are congruent,

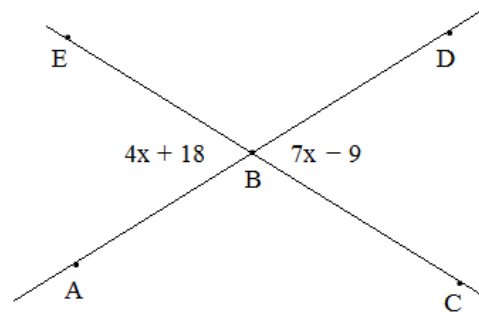
$$\begin{aligned}4x + 18 &= 7x - 9 \\ 27 &= 3x \\ x &= 9\end{aligned}$$

Since  $x = 9$ , then

$$\angle EBA = \angle DBC = 54^\circ$$

And, since  $\angle ABC$  is supplementary to either angle, then

$$\angle ABC = 126^\circ$$



SOLUTIONS

5) Given:  $\angle 1 = \angle 5$

$$\overline{AB} \parallel \overline{CD}$$

$$\angle 2 = 38^\circ$$

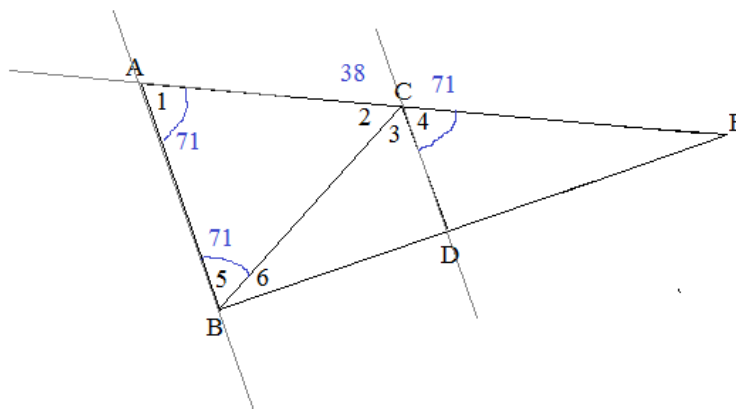
Find:  $\angle 3$

Interior angles of triangle add up to 180...  
 Since  $\angle 2 = 38$ , then  $\angle 1 + \angle 5 = 142$ ...  
 And, since they are congruent, each is  $71^\circ$

Since  $AB \parallel CD$ ,  $\angle 1 \cong \angle 4$   
 because of corresponding angles  
 Therefore,  $\angle 4 = 71^\circ$

$\overline{AE}$  is a straight angle, so the sum of  
 angles 2, 3, and 4 is 180..  
 Therefore,  $38 + \angle 3 + 71 = 180$

$$\angle 3 = 71^\circ$$



6) Given: Rectangle RSTV

$$\angle RVS = x^2 + 30^\circ$$

$$\angle RSV = 6x - 12^\circ$$

Find the measure of  $\angle VST$

Since RSTV is a rectangle,  $\angle VRS = 90^\circ$

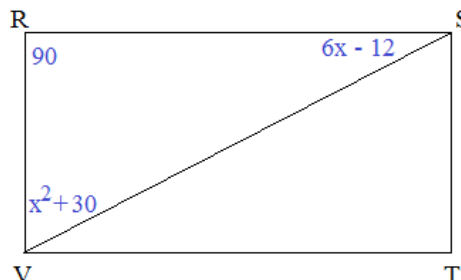
then,  $\angle RSV + \angle RVS = 90^\circ$

$$6x - 12 + x^2 + 30 = 90$$

$$x^2 + 6x - 72 = 0$$

$$(x + 12)(x - 6) = 0$$

$$x = -12, 6$$



If  $x = -12$ , then  $RVS = 174$   $RSV = -84$   
 EXTRANEOUS!!

If  $x = 6$ , then  $RVS = 66$   $RSV = 24$

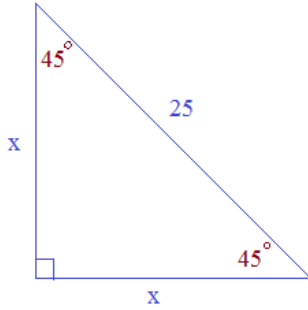
Since  $\angle RSV = 24$ , then  $\angle VST = 66$

Geometry Angle and Triangle Exercises

SOLUTIONS

- 7) Given: 45-45-90 Right Triangle  
Hypotenuse length: 25

Find the perimeter.



Ratio of sides of 45-45-90 right triangle:

$$1:1:\sqrt{2}$$

$$\frac{25}{x} = \frac{\sqrt{2}}{1}$$

$$\sqrt{2} x = 25$$

$$x = \frac{25}{\sqrt{2}} = \frac{25\sqrt{2}}{2}$$

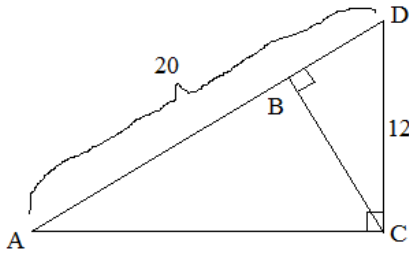
$$\text{Perimeter} = 25 + \frac{25\sqrt{2}}{2} + \frac{25\sqrt{2}}{2}$$

$$= 25 + \frac{50\sqrt{2}}{2}$$

$$= 25 + 25\sqrt{2}$$

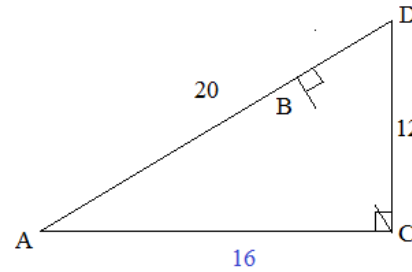
- 8) Find the perimeter of  $\triangle ABC$

Use geometry theorems and concepts to find sides until we know  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{AC}$ .



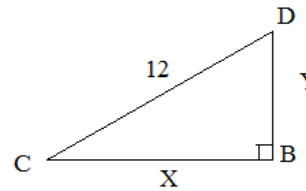
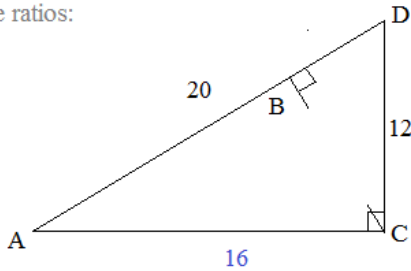
Pythagorean theorem...  $12^2 + 16^2 = 20^2$

(also, special 3-4-5 right triangle x 4  $\rightarrow$  12-16-20)



$$\overline{AC} = 16$$

Triangle ratios:



$$\frac{20}{12} = \frac{16}{X}$$

$$20X = 12(16)$$

$$X = 9.6$$

$$\overline{BC} = 9.6$$

$$\frac{20}{12} = \frac{12}{Y}$$

$$20Y = 12(12)$$

$$Y = 7.2$$

Since  $\overline{BD} = 7.2$  and  $\overline{AD} = 20$ ,

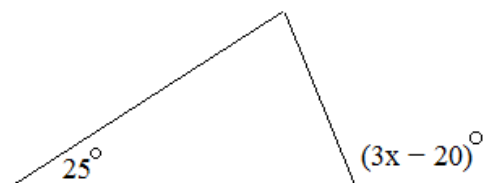
$$\overline{AB} = 12.8$$

Therefore, the perimeter of  $\triangle ABC$

$$16 + 9.6 + 12.8 = 38.4$$

Geometry Angle and Triangle Exercises

9) What are the restrictions of  $x$ ?



$$15 < x < 66.\bar{6}$$

SOLUTIONS

Exterior Angle = Sum of remote interior angles

Then,

Exterior Angle Theorem: The measure of the exterior angle is greater than either of the measures of the remote interior angles.

$$3x - 20 > 25$$

$$3x > 45$$

$$x > 15^\circ$$

But, it can't be 100!

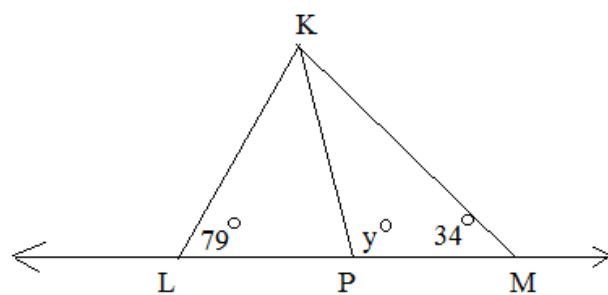
So, there must be a maximum value, too...

$$3x - 20 < 180$$

$$3x < 200$$

$$x < 200/3^\circ$$

10) What are the restrictions of  $y$ ?



$y$  is exterior angle of  $\triangle KLP$ .

So,  $y > 79^\circ$

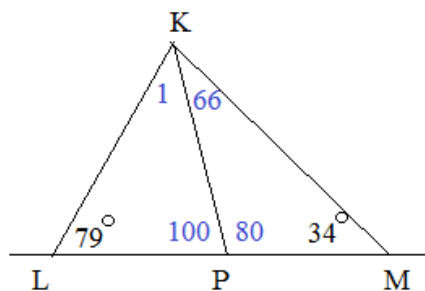
Then,  $y$  is interior angle of  $\triangle KPM$ .

So,  $y + 34 < 180$

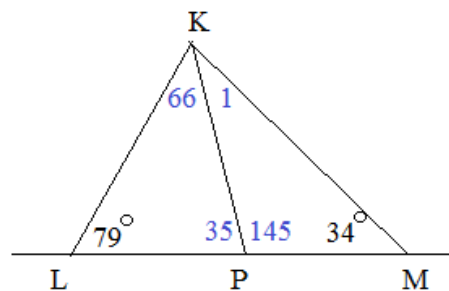
$$y < 146^\circ$$

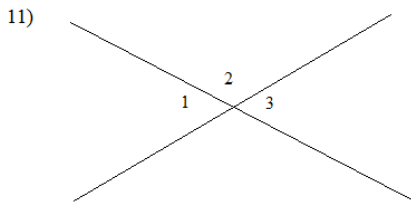
$$79 < y < 146$$

Random Check:



and,





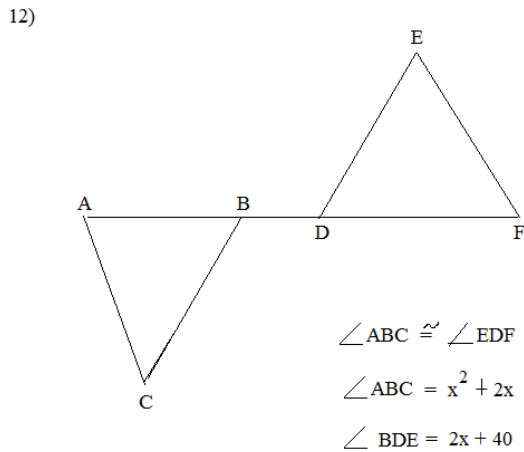
If angles 1 and 3 are supplementary,  
what is the measure of angle 2?

Since 1 and 3 are supp AND  
1 and 3 are congruent (vertical angles),  
they must be right angles (right angle theorem)

therefore, angle 2 is 90 degrees

**SOLUTIONS**

Note: the diagram is not drawn  
to scale...



What is the measure of BDE?

$$BDE + EDF = 180$$

$$EDF = 180 - (BDE)$$

$$EDF = 180 - (2x + 40) = 140 - 2x$$

$$ABC = EDF$$

$$x^2 + 2x = 140 - 2x$$

$$x^2 + 4x - 140 = 0$$

$$(x + 14)(x - 10) = 0$$

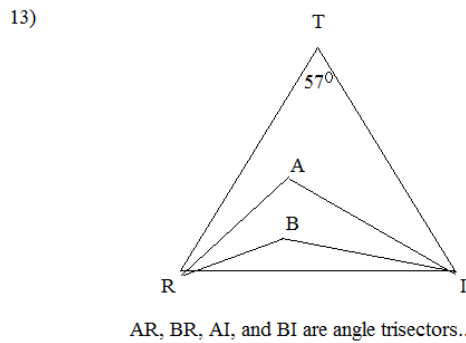
$$x = 10, -14$$

If  $x = 10$ , then

$$\begin{aligned} ABC &= 120 \\ BDE &= 60 \end{aligned}$$

If  $x = -14$ , then

$$\begin{aligned} ABC &= 168 \\ BDE &= 12 \end{aligned}$$



angle A = 98

angle B = 139

$$3x + 3y + 57 = 180$$

$$3x + 3y = 123$$

$$x + y = 41$$

$$A + 2x + 2y = 180$$

$$B + x + y = 180$$

$$A + 82 = 180$$

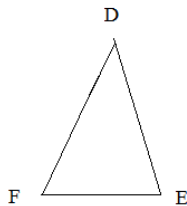
$$B + 41 = 180$$

$$A = 98$$

$$B = 139$$

14) In  $\triangle DEF$ , the sum of  $\angle D$  and  $\angle E$  is  $110^\circ$   
and the sum of  $\angle E$  and  $\angle F$  is  $150^\circ$

What is the sum of  $\angle D$  and  $\angle F$  ?



The sum of the interior angles of a triangle is 180 degrees.

If  $D + E = 110$ , then angle F must be 70

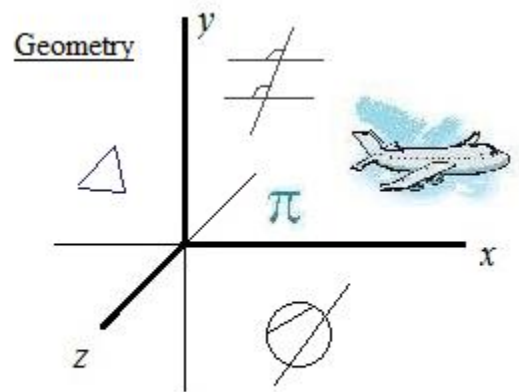
and, if  $E + F = 150$ , then angle D must be 30

therefore,  $D + F = 30 + 70 =$ 100

Thanks for visiting. (Hope it helped!)

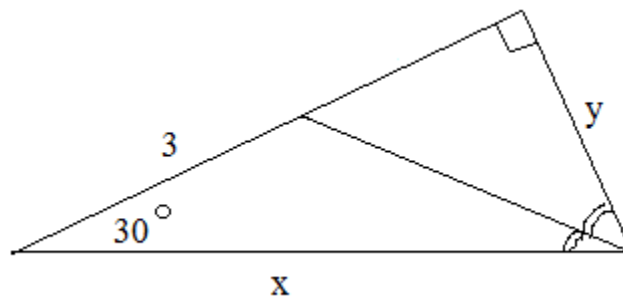
Find other Geometry exercises and more at [mathplane.com](http://mathplane.com)

And, follow comics and updates at Facebook or Google +



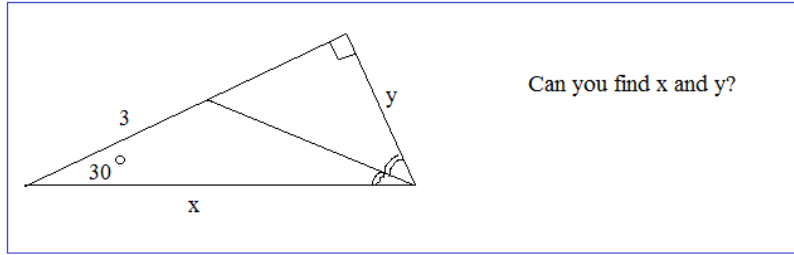
[www.mathplane.com](http://www.mathplane.com)

One more question....



Can you find  $x$  and  $y$ ? (Answer on next page)

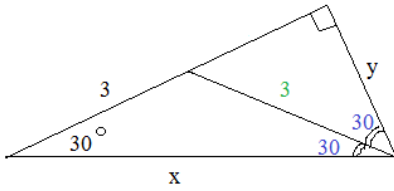
Angle Bisector, isosceles triangles, and 30-60-90....



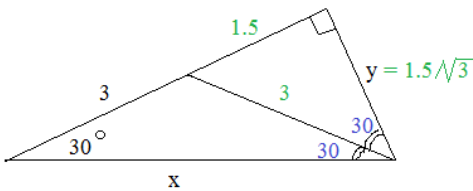
Can you find x and y?

Method 1: Using the bisectors...

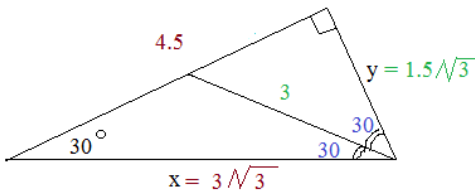
Since the main triangle is a right triangle, the bisected 60 degree angle produces two 30 degree angles...



since the left triangle is isosceles, the other side is 3....

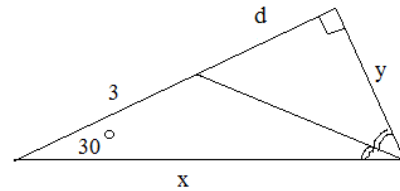


since the triangle on the right is 30-60-90, the small side is 1.5 and the medium side (y) is  $1.5\sqrt{3}$



since the large right triangle has side  $1.5\sqrt{3}$  across from the 30 degree angle, the hypotenuse is  $2x \rightarrow 3\sqrt{3}$

Method 2: Using the angle bisector theorem



$$\frac{y}{x} = \frac{d}{3} \quad \text{angle bisector theorem}$$

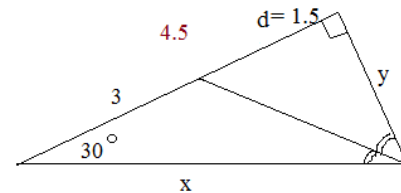
since large triangle is 30-60-90 right triangle, length of small side (y) is 1/2 the length of hypotenuse (x)..

$$2y = x$$

therefore,

$$\frac{y}{2y} = \frac{d}{3}$$

$$\text{so, } d = 1.5$$



If side opposite of 60 degree angle is 4.5, then small side is

$$\frac{4.5}{\sqrt{3}} = 1.5\sqrt{3} \quad y$$

$$\text{and hypotenuse is } \frac{4.5}{\sqrt{3}} \times 2 = 3\sqrt{3} \quad x$$