Detour and Diagramless Proofs

Topics include right angle theorem, indirect proofs, isosceles triangles, quadrilaterals, and more.

NOTE: A detour is taken to grab information needed to solve the main proof!

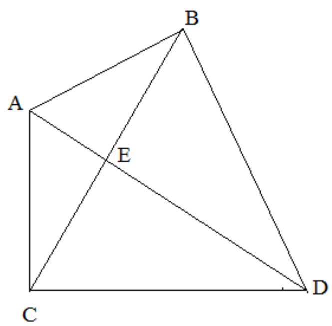
Detour Proofs

Example: Given: $\overline{AB} \perp \overline{BD}$

$\overline{AC} \perp \overline{CD}$

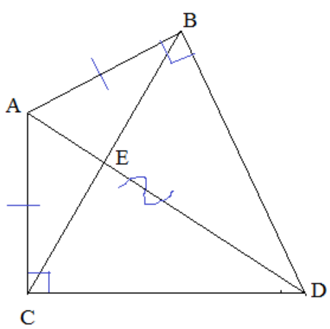
$\overline{AB} \cong \overline{AC}$

Prove: $\angle BED$ is a right angle

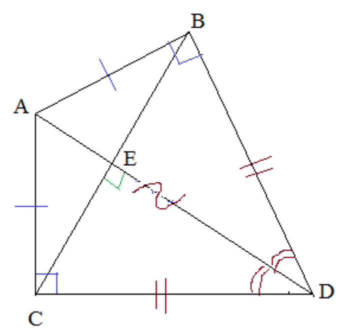


Detour (steps 1–7)

Statements	Reasons
1. $\overline{AB} \perp \overline{BD}$; $\overline{AC} \perp \overline{CD}$	1. Given
2. $\angle ACD$ and $\angle ABD$ are right angles	2. Definition of perpendicular (Perpendicular segments form right angles)
3. $\overline{AB} \cong \overline{AC}$	3. Given
4. $\overline{AD} \cong \overline{AD}$	4. Reflexive property
5. $\triangle ABD \cong \triangle ACD$	5. RHL (Right Angle-Hypotenuse-Leg) 2, 4, 3
6. $\overline{CD} \cong \overline{BD}$	6. CPCTC (Corresponding Parts of Congruent Triangles are Congruent)
7. $\angle BDE \cong \angle CDE$	7. CPCTC
8. $\overline{ED} \cong \overline{ED}$	8. Reflexive property
9. $\triangle BED \cong \triangle CED$	9. SAS (Side-Angle- Side) 6, 7, 8
10. $\angle CED = \angle BED$	10. CPCTC
11. $\angle BED$ is right angle	11. Right Angle Theorem (If angles are both congruent and supplementary, then each angle is a right angle)



NOTE: A detour was taken, using the "big triangles", to get information needed for the "medium triangles"



Example: Prove: "If 2 altitudes of a triangle are congruent, then the triangle is isosceles."

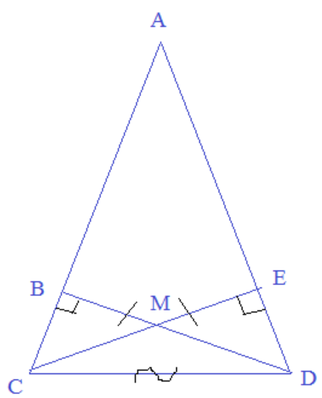
Diagramless Proofs

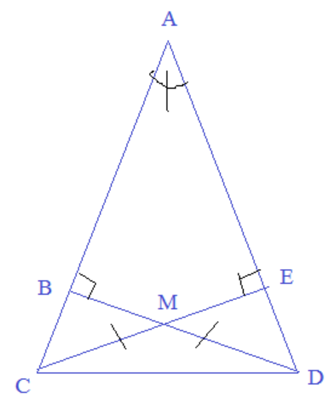
Draw a diagram... Devise a proof (using "if... , then....")

Given: $\overline{BD}$ and $\overline{CE}$ are altitudes

$\overline{BD} = \overline{CE}$

Prove: $\triangle$ACD is isosceles





Method 1:

Statements	Reasons
1) $\overline{BD} = \overline{CE}$	1) Given
2) $\overline{BD}$ and $\overline{CE}$ are altitudes	2) Given
3) $\angle$CBD and $\angle$DEC are right angles	3) Definition of Altitude
4) $\angle$CBD = $\angle$DEC	4) All right angles are congruent
5) $\overline{CD} = \overline{CD}$	5) Reflexive Property
6) $\triangle$BCD = $\triangle$EDC	6) RHL (4, 5, 1) (Right Angle-Hypotenuse-Leg)
7) $\angle$BCD = $\angle$EDC	7) CPCTC (Corresponding Parts of Congruent Triangles are Congruent)
8) $\triangle$ACD is isosceles	8) If base angles of triangle are congruent, then triangle is isosceles

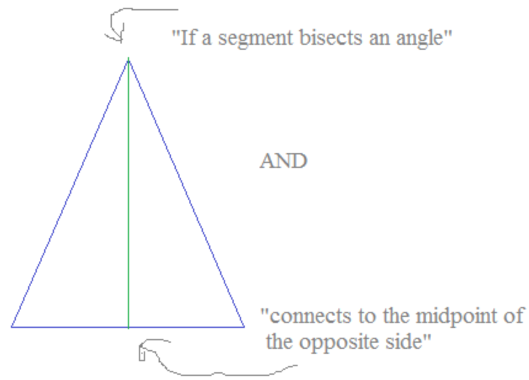
Method 2:

Statements	Reasons
1) $\overline{BD} = \overline{CE}$	1) Given
2) $\overline{BD}$ and $\overline{CE}$ are altitudes	2) Given
3) $\angle$ABD and $\angle$AEC are right angles	3) Definition of Altitude
4) $\angle$ABD = $\angle$AEC	4) All right angles are congruent
5) $\angle$A = $\angle$A	5) Reflexive property
6) $\triangle$AEC = $\triangle$ABD	6) AAS (5, 4, 1) (Angle-Angle-Side)
7) $\overline{AD} = \overline{AC}$	7) CPCTC
8) $\triangle$ACD is isosceles	8) Definition of Isosceles (If 2 or more sides congruent)

Example: If a segment bisects an angle of a triangle and connects to the midpoint of the opposite side, then the triangle is isosceles.

Diagramless Proofs

Step 1: Draw a diagram (go phrase by phrase)



Step 2: Devise a proof....

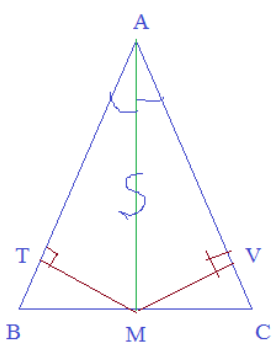
"IF" -----> the 'givens'

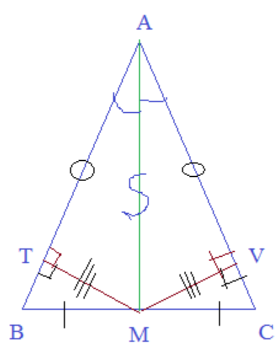
Given: $\overline{AM}$ bisects ∠BAC

M is the midpoint of $\overline{BC}$

"THEN" -----> what you're proving

Prove: △ABC is isosceles





Statements	Reasons
1. $\overline{AM}$ bisects ∠BAC	1. Given
2. M is the midpoint of $\overline{BC}$	2. Given
3. TM ⊥ AB MV ⊥ AC	3. Auxilary lines join 2 points
4. AVM and ATM rt angles	4. Definition of perpendicular
5. ATM and AVM congruent	5. All right angles are congruent
6. ∠MAC = ∠MAB	6. Definition of angle bisector
7. $\overline{AM}$ = $\overline{AM}$	7. Reflexive property
8. △MAT = △MAV	8. AAS (angle-angle-side) 5, 6, 7
9. MV = MT	9. CPCTC
10. BTM and CVM are congruent right angles	10. Def. of perpendicular, all right angles are congruent
11. BM = CM	11. Definition of midpoint
12. △CVM = △BTM	12. RHL (right angle-hypotenuse-leg) 10, 11, 9
13. angles B and C are congruent	13. CPCTC
14. AB = AC	14. If congruent angles, then congruent sides
15. ABC is isosceles	15. Def. of isosceles (at least 2 congruent sides)

Diagramless Proofs

1) If a triangle is isosceles, then the triangle formed by its base and the angle bisectors of its base angles is also isosceles.

Statements	Reasons

2) Prove a trapezoid inscribed in a circle is isosceles.

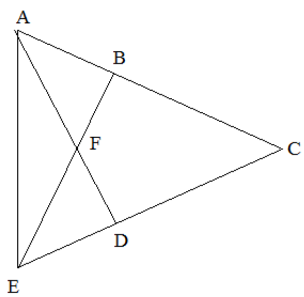
Statements	Reasons

Detour Proofs

3) Given: $\overline{AB} = \overline{DE}$

$\overline{BC} = \overline{CD}$

Prove: $\triangle$ AFE is isosceles



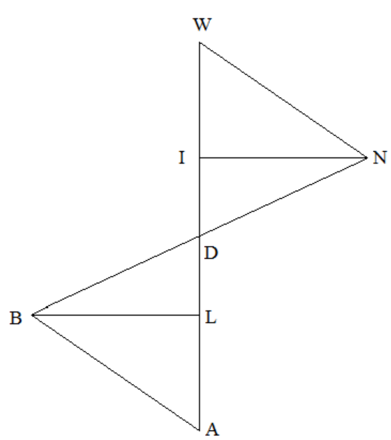
Statements	Reasons

4) Given: $\angle$WNI = $\angle$ABL

D is the midpoint of $\overline{BN}$

$\overline{ID} = \overline{LD}$

Prove: $\overline{WN} = \overline{AB}$



Statements	Reasons

Diagramless and Detour Proofs

5) Prove the diagonals of a rectangle create isosceles triangles.

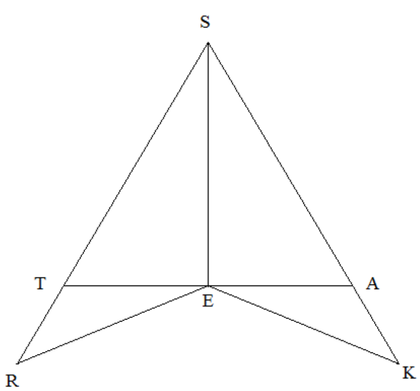
Statements	Reasons

6) Given: $\overline{SR} = \overline{SK}$

$\overline{RE} = \overline{KE}$

$\overline{TR} = \overline{AK}$

Prove: E is the midpoint of $\overline{TA}$



Statements	Reasons

Diagramless Indirect Proofs

7) "If a radius is NOT perpendicular to a chord, then the radius does NOT bisect chord."

Statements	Reasons

8) "In an isosceles triangle, if a point on the base is NOT the midpoint, then the segment from the vertex to that point does NOT bisect the vertex angle."

Statements	Reasons

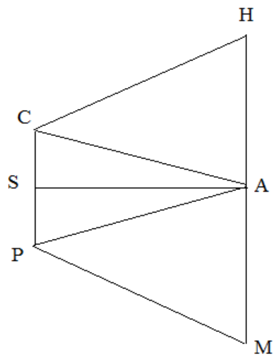
9) Given: S is midpoint of $\overline{CP}$

$\overline{AC} = \overline{AP}$

$\angle PAM = \angle CAH$

Prove: $SA \perp HM$

Statements	Reasons

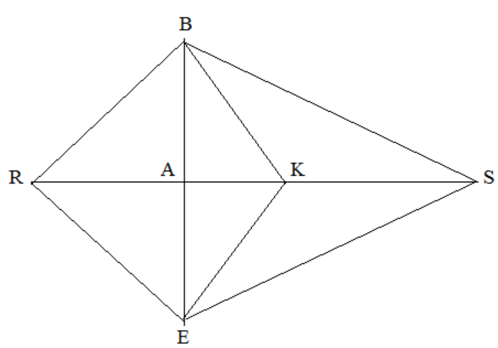


10) Given: $\overline{KS}$ bisects $\angle BSE$

$\overline{AK}$ bisects $\angle BKE$

Prove: $\overline{KR}$ bisects $\angle BRE$

Statements	Reasons



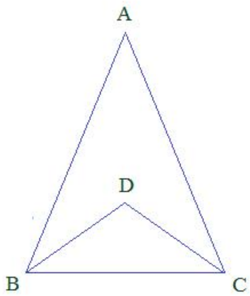
1) If a triangle is isosceles, then the triangle formed by its base and the angle bisectors of its base angles is also isosceles.

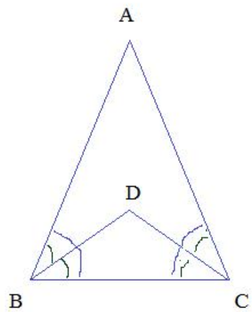
Diagramless Proofs

Given: Triangle ABC is isosceles
BD and CD are angle bisectors

SOLUTIONS

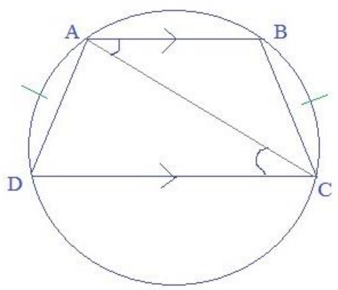
Prove: Triangle BDC is isosceles

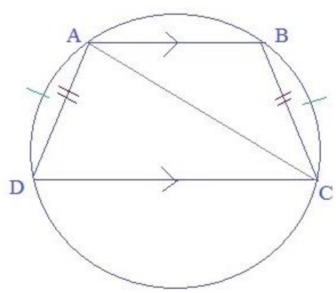




Statements	Reasons
1) △ABC is isosceles	1) Given
2) $\overline{AB} \cong \overline{AC}$	2) Definition of Isosceles
3) ∠ABC ≅ ∠ACB	3) If congruent sides, then congruent angles (or, base angles of isos. are congruent)
4) $\overline{BD}$ and $\overline{CD}$ are angle bisectors	4) Given
5) ∠DBC ≅ ∠DCB	5) "Like Division Property" If congruent angles are bisected, then the halves are congruent
6) BD = CD	6) If congruent angles (in a triangle), then opposite sides congruent
7) △ BDC is isosceles	7) Definition of isosceles (A triangle with 2 or more congruent sides)

2) Prove a trapezoid inscribed in a circle is isosceles.





Statements	Reasons
1) Trapezoid ABCD	1) Given
2) AB ∥ CD	2) Definition of Trapezoid (bases are parallel)
3) Draw Diagonal AC	3) Given (definition of trapezoid)
4) ∠ACD ≅ ∠BAC	4) If parallel lines are cut by transversal, then alternate interior angles are congruent.
5) $\overset{\frown}{AD} = \overset{\frown}{BC}$	5) If inscribed angles are congruent, then the arcs are congruent
6) AD = BC	6) If arcs in a circle are congruent, then their chords are congruent
7) ABCD is isosceles	7) If legs/sides of a trapezoid are congruent, then the trapezoid is isosceles.

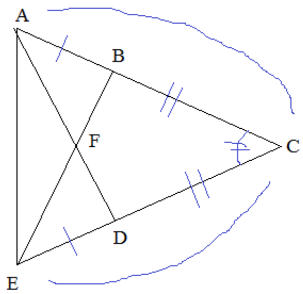
3) Given: $\overline{AB} = \overline{DE}$

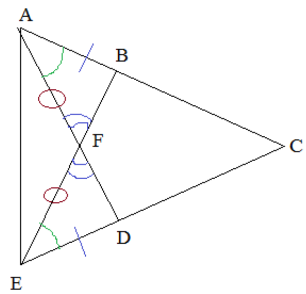
$\overline{BC} = \overline{CD}$

Prove: $\triangle$ AFE is isosceles

Detour Proofs

SOLUTIONS





Detour

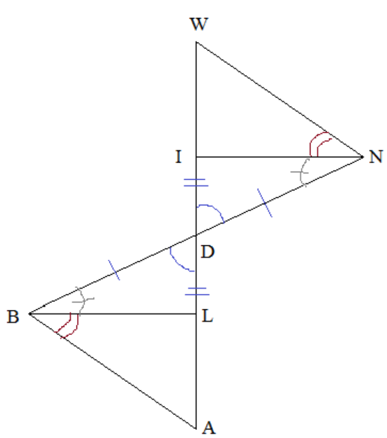
Statements	Reasons
1) $\overline{AB} = \overline{DE}$	1) Given
2) $\overline{BC} = \overline{CD}$	2) Given
3) $\overline{AC} = \overline{EC}$	3) Addition Property (If congruent sides are added to congruent sides, then the sums are equal)
4) $\angle$ACD = $\angle$ECB	4) Reflexive Property
5) $\triangle$ACD = $\triangle$ECB	5) Side-Angle-Side (2, 4, 3)
6) $\angle$BEC = $\angle$DAC	6) CPCTC (Corresponding Parts of Congruent Triangles Congruent)
7) $\angle$BFA = $\angle$DFE	7) Vertical angles congruent
8) $\triangle$FAB = $\triangle$FED	8) Angle-Angle-Side (6, 7, 1)
9) AF = EF	9) CPCTC
10) $\triangle$AFE is isosceles	10) Definition of Isosceles (At least 2 congruent sides)

4) Given: $\angle$WNI = $\angle$ABL

D is the midpoint of $\overline{BN}$

$\overline{ID} = \overline{LD}$

Prove: $\overline{WN} = \overline{AB}$



Detour

Statements	Reasons
1) D is the midpoint of $\overline{BN}$	1) Given
2) $\overline{DN} = \overline{DB}$	2) Definition of midpoint (midpoint divides segment into = parts)
3) $\angle$NDI = $\angle$LDB	3) Vertical angles congruent
4) $\overline{ID} = \overline{LD}$	4) Given
5) $\triangle$NDI = $\triangle$BDL	5) Side-Angle-Side (2, 3, 4)
6) $\angle$DBL = $\angle$DNI	6) CPCTC (corresponding parts of congruent triangles are congruent)
7) $\angle$DBA = $\angle$DNW	7) Additional property (If 2 congruent angles are added to congruent angles, the sums are the same)
8) $\triangle$DBA = $\triangle$DNW	8) Angle-Side-Angle (3, 2, 7)
9) $\overline{WN} = \overline{AB}$	9) CPCTC

NOTE: there are other methods that could prove..
(i.e. proving $\triangle$ABL = $\triangle$WNI would lead to answer)

5) Prove the diagonals of a rectangle create isosceles triangles.

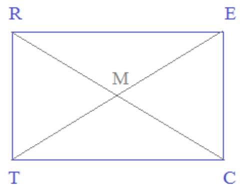
SOLUTIONS

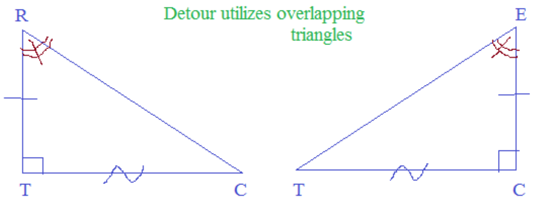
Diagramless and Detour Proofs

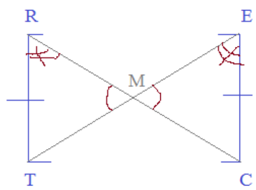
Given: Rectangle RECT

Prove: $\overline{RC}$ and $\overline{ET}$ bisect each other
(creating isosceles triangles)

"If rectangle, then diagonals create an isosceles triangle..."

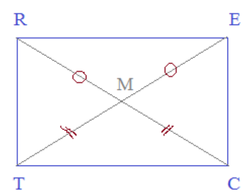






Detour (steps 2–7)

Statements	Reasons
1. Rectangle RECT	1. Given
2. ∠T and ∠C are right angles	2. Definition of rectangle
3. ∠T ≅ ∠C	3. All right angles are congruent
4. $\overline{RT} \cong \overline{EC}$	4. Definition of rectangle (opposite sides congruent)
5. $\overline{TC} = \overline{TC}$	5. Reflexive property
6. △RTC ≅ △ECT	6. SAS (Side-Angle-Side) 4, 3, 5
7. ∠TRM ≅ ∠CEM	7. CPCTC (Corresponding Parts of Congruent Triangles are Congruent)
8. ∠RMT ≅ ∠EMC	8. Vertical angles congruent
9. △RMT = △EMC	9. AAS (Angle-Angle-Side) 7, 8, 4
10. $\overline{EM} \cong \overline{RM}$; $\overline{TM} \cong \overline{CM}$	10. CPCTC
11. △RME and △TMC are isosceles triangles	11. Definition of isosceles triangle (2 or more congruent sides of a triangle are congruent)

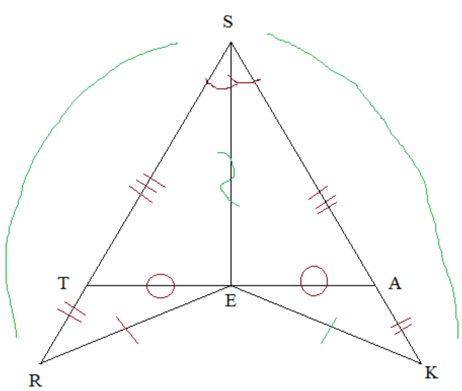


6) Given: $\overline{SR} = \overline{SK}$

$\overline{RE} = \overline{KE}$

$\overline{TR} = \overline{AK}$

Prove: E is the midpoint of $\overline{TA}$



Detour (steps 1–5)

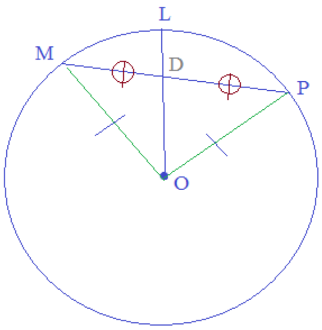
Statements	Reasons
1) $\overline{SR} = \overline{SK}$	1) Given
2) $\overline{RE} = \overline{EK}$	2) Given
3) $\overline{SE} = \overline{SE}$	3) Reflexive Property
4) △SER = △SEK	4) Side-Side-Side (1, 2, 3)
5) ∠TSE = ∠ASE	5) CPCTC
6) $\overline{TR} = \overline{AK}$	6) Given
7) $\overline{ST} = \overline{SA}$	7) Subtraction Property (If congruent segments are subtracted from congruent segments, then the differences are the same)
8) △TSE = △ASE	8) Side-Angle-Side (7, 5, 3)
9) $\overline{TE} = \overline{AE}$	9) CPCTC
10) E is midpoint of $\overline{TA}$	10) Definition of Midpoint (If point divides segment into congruent halves, then it is midpoint of segment)

Diagramless Indirect Proofs

7) "If a radius is NOT perpendicular to a chord, then the radius does NOT bisect chord."

Uses Equidistance Theorem
Auxilary Lines
Indirect Proof

Step 1: Sketch a diagram



Step 2: Design the proof

Given: Circle O

$\overline{LO}$ is NOT perpendicular to $\overline{MP}$

Prove: $\overline{LO}$ does NOT bisect $\overline{MP}$

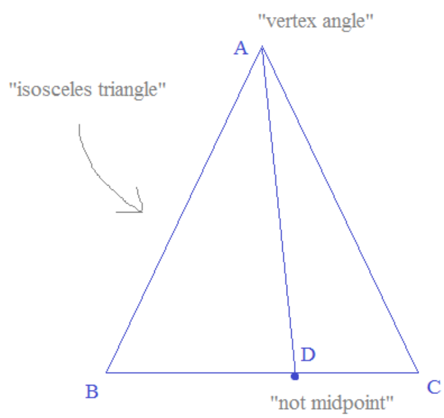
Step 3: Use indirect proof to solve

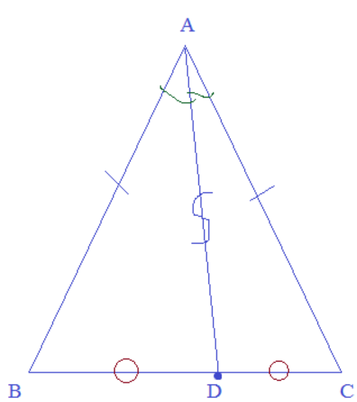
Statements	Reasons
1) Circle O	1) Given
2) $\overline{LO}$ NOT perpendicular to $\overline{MP}$	2) Given
3) Draw radii $\overline{OP}$ and $\overline{OM}$	3) Auxilary lines (line joins 2 points)
4) $\overline{OP} = \overline{OM}$	4) All radii congruent
5) $\overline{LO}$ bisects $\overline{MP}$	5) Assume for contradiction
6) $\overline{MD} = \overline{PD}$	6) Definition of bisector (Bisector divides segment into congruent halves)
7) $\overline{LO}$ is perpendicular bisector of $\overline{MP}$	7) Equidistance Theorem

However, statements 2) and 7) contradict each other!

8) "In an isosceles triangle, if a point on the base is NOT the midpoint, then the segment from the vertex to that point does NOT bisect the vertex angle."

Step 1: Sketch diagram by picking out key phrases...





Step 2: Write out "givens" using IF statements...
And, write "prove" using THEN statements...

Given: Isosceles Triangle ABC

D is NOT a midpoint

Prove: $\overline{AD}$ is NOT an angle bisector

Statements	Reasons
1) $\triangle$ABC is isosceles	1) Given
2) $\overline{AB} = \overline{AC}$	2) Definition of Isosceles (2 or more congruent sides)
3) D is NOT a midpoint of $\overline{BC}$	3) Given
4) $\overline{AD}$ is angle bisector	4) Assume for contradiction
5) $\overline{AD} = \overline{AD}$	5) Reflexive Property
6) $\triangle$ ABD = $\triangle$ ACD	6) Side-Angle-Side (SAS) (2, 3, 4)
7) $\overline{BD} = \overline{CD}$	7) CPCTC (Corresponding parts of congruent triangles are congruent)
8) D is midpoint of BC	8) Definition of Midpoint (If point divides segment into congruent halves, then it is a midpoint)

However, statements 3) and 8) contradict each other!

9) Given: S is midpoint of $\overline{CP}$

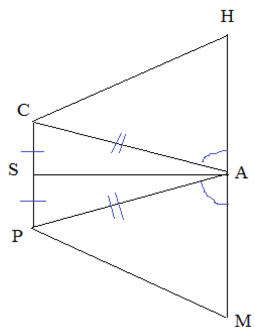
$\overline{AC} = \overline{AP}$

$\angle$PAM = $\angle$ CAH

Prove: SA $\perp$ HM

SOLUTIONS

Uses Detour and Right Angle Theorem



Detour ---->

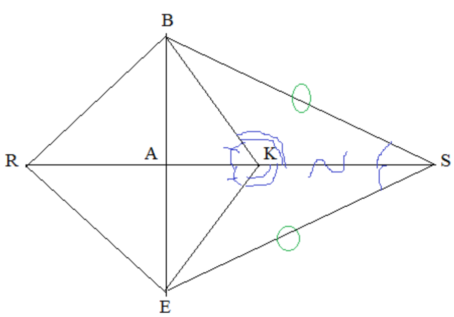
Statements	Reasons
1) S is midpoint of CP	1) Given
2) $\overline{CS} = \overline{PS}$	2) Definition of Midpoint
3) $\overline{AC} = \overline{AP}$	3) Given
4) $\overline{SA} = \overline{SA}$	4) Reflexive Property
5) $\triangle$CAS = $\triangle$ PAS	5) Side-Side-Side (2, 3, 4)
6) $\angle$CAS = $\angle$PAS	6) CPCTC
7) $\angle$PAM = $\angle$ CAH	7) Given
8) $\angle$SAM = $\angle$SAH	8) Addition Property
9) SAM and SAH are supplementary angles	9) Definition of Supplementary
10) SAM and SAH are right angles	10) Right Angle Theorem If angles are congruent and supplementary, then they are right angles..
11) SA $\perp$ HM	11) If right angles, then segments are perpendicular

10) Given: $\overline{KS}$ bisects $\angle$BSE

$\overline{AK}$ bisects $\angle$BKE

Prove: $\overline{KR}$ bisects $\angle$BRE

detour

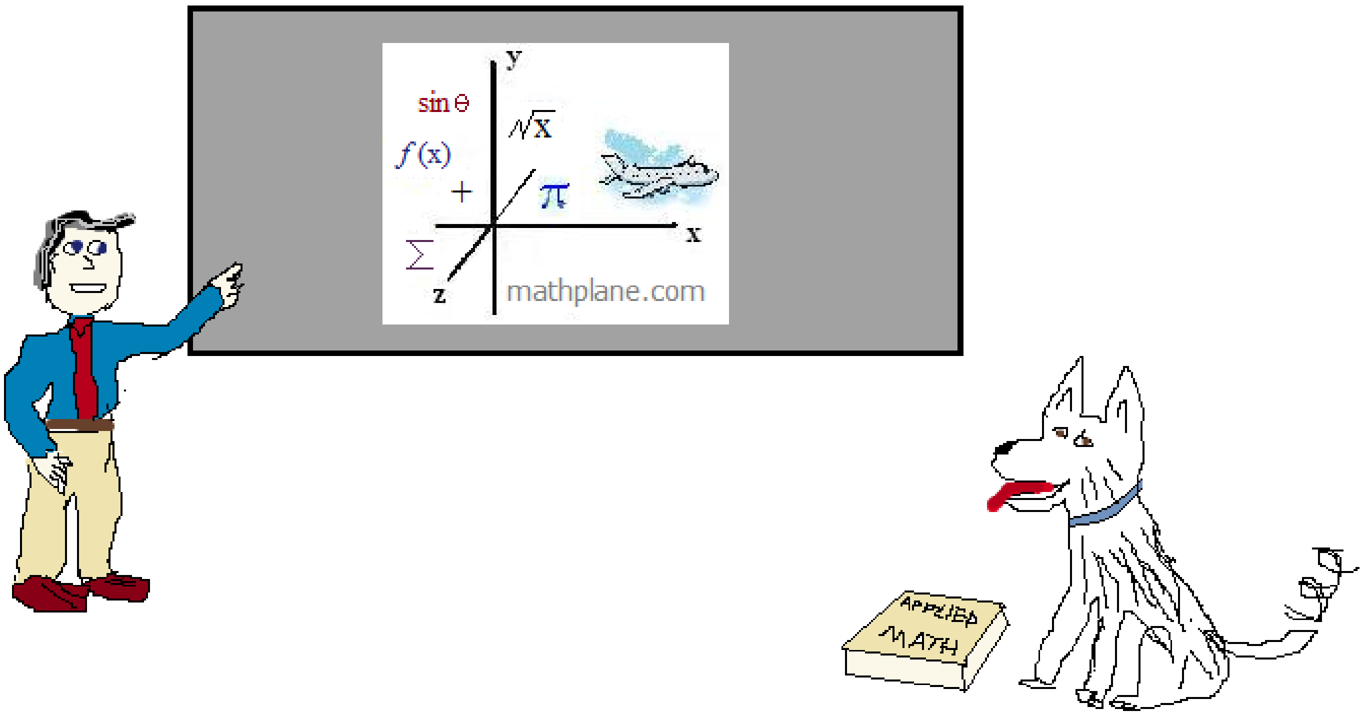


Statements	Reasons
1) $\overline{KS}$ bisects $\angle$BSE	1) Given
2) $\angle$BSK = $\angle$ ESK	2) Definition of angle bisector
3) $\overline{KS} = \overline{KS}$	3) Reflexive Property
4) $\overline{AK}$ bisects $\angle$BKE	4) Given
5) $\angle$AKB = $\angle$AKE	5) Definition of angle bisector
6) $\angle$AKB is supplementary to $\angle$BKS $\angle$AKE is supplementary to $\angle$EKS	6) Definition of Supplementary (Adjacent angles that form a straight angle are supplementary)
7) $\angle$BKS = $\angle$EKS	7) If 2 angles are congruent, then their supplements are congruent
8) $\triangle$ BSK = $\triangle$ESK	8) Angle-Side-Angle (2, 3, 7)
9) BS = SE	9) CPCTC
10) RS = RS	10) Reflexive Property
11) $\triangle$ SBR = $\triangle$ SER	11) Side-Angle-Side (9, 2, 10)
12) $\angle$BRK = $\angle$ ERK	12) CPCTC
13) KR bisects BRE	13) Definition of Angle Bisector

Thanks for visiting. Hope it helps!

If you have questions, suggestions, or requests, let us know.

Cheers



Also, at Mathplane *Express* for mobile and tablets at Mathplane.ORG