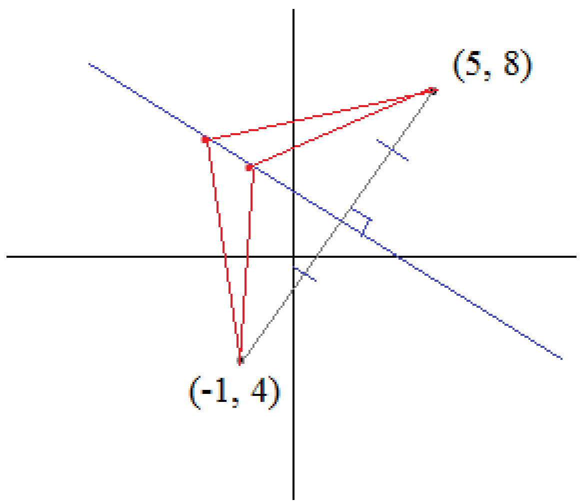
Geometry Review 002 Questions

(With solutions)



Topics include Pythagorean Theorem, sector area, perimeter, polygons, circles, similarity, and more.

Mathplane.com

Geometry Review Test 2

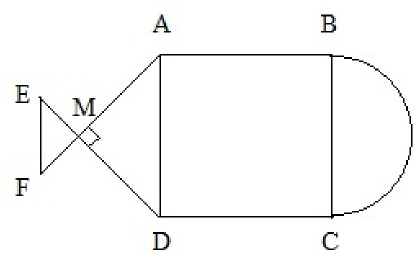
1) The sector area of a circle is $3cm^2$. And, the perimeter of the sector is 7cm. What is the (possible) length(s) of the radius?

2) Given: Area of square ABCD = 49 sq. feet

$\overline{AF} \perp \overline{DE}$

$\overline{EF} \parallel \overline{AD}$

$\overline{EM} \cong \overline{FM}$



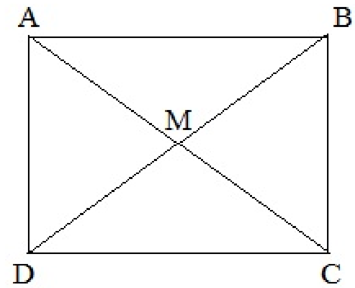
a) What is the area of semi-circle $\widehat{BC}$?

b) What is the length of $\overline{AM}$?

3) Given $\overline{AM} = \overline{MD} = \overline{DA} = 10$

a) What is the perimeter of rectangle ABCD?

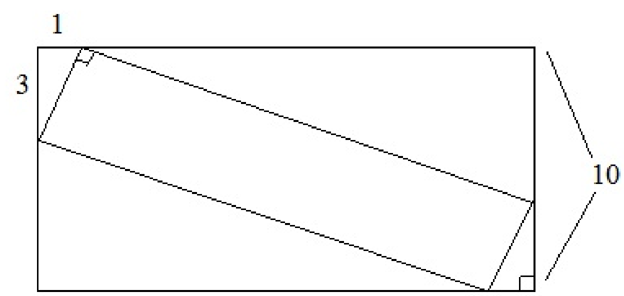
b) What is the area of $\triangle$DMC?



4) Given Figure A (rectangle inscribed in a rectangle):

a) What is the perimeter of the outer rectangle?
(Hint: similar triangles & proportions)

Figure A

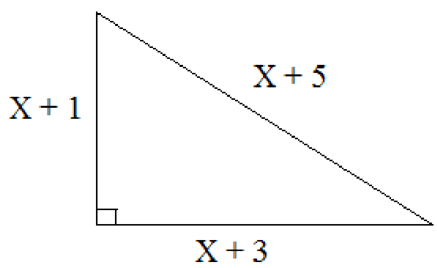


b) What is the area of the inner rectangle?
(Hint: "encasement" or pythagorean theorem)

5) In a kite, the ratio of the diagonals is 3:4
If the area is 150 square feet, what is the length of the larger diagonal?

6) A picture frame is shaped as a heptagon. The measure of the top interior angle is 126°. The remaining interior angles are congruent to each other. What is the measure of each remaining interior angle?

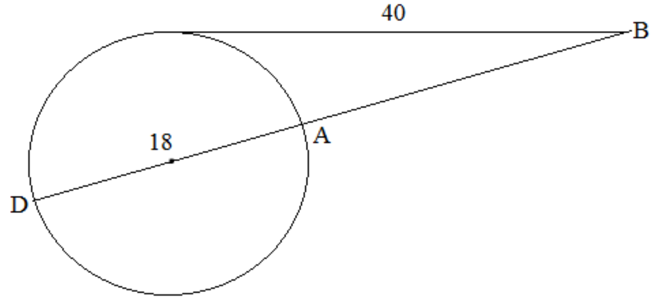
7) What is the perimeter of the triangle?



Geometry Review Test 2

8) Given: Diameter $\overline{AD} = 18$
Tangent segment is 40

Find: the length of $\overline{AB}$



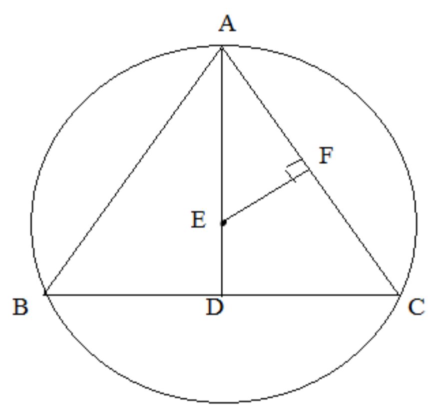
9) Given: ABC is an isosceles triangle inscribed in the circle where $\overline{AB} \cong \overline{AC}$

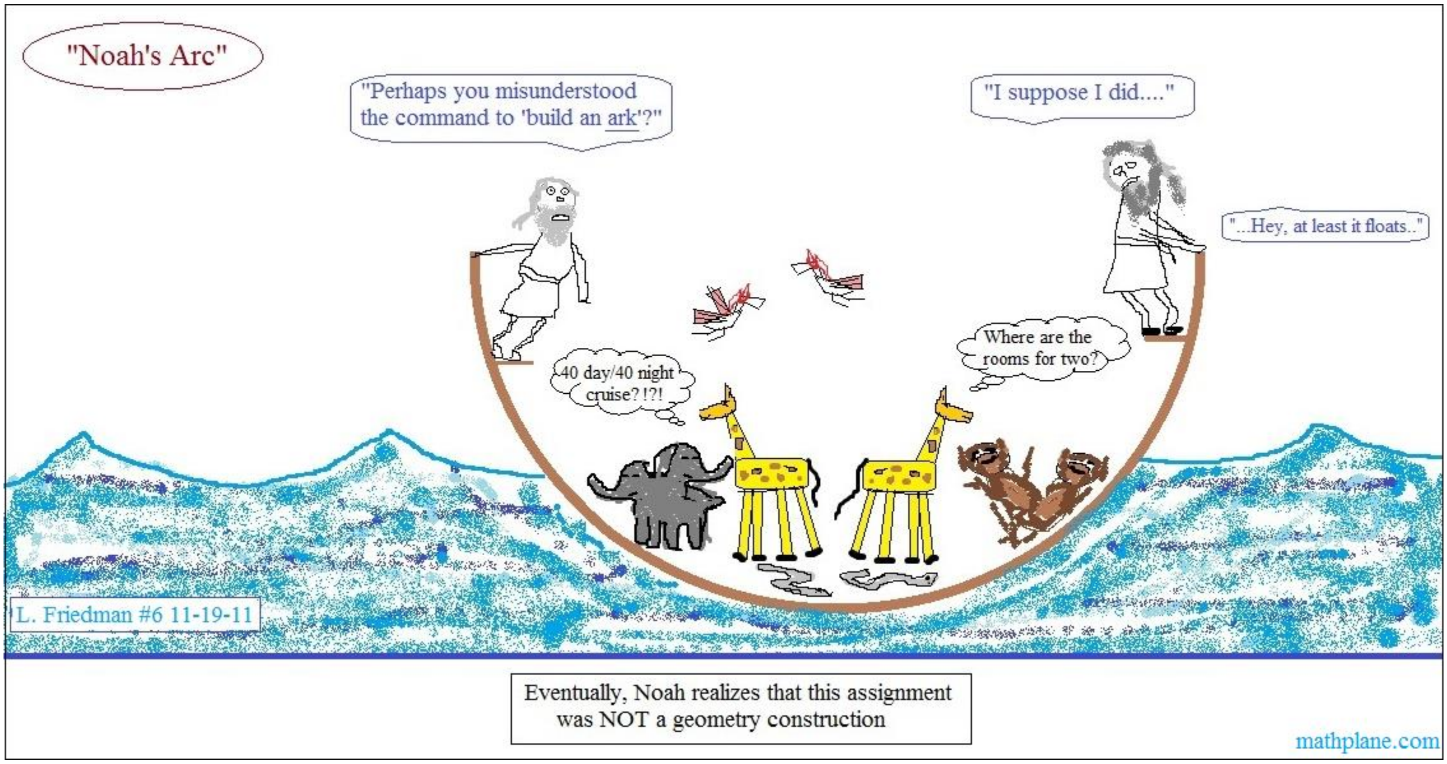
$\overline{AF} = 6$

$\overline{ED} = 1$

Find: a) the radius of the circle

b) the perimeter of the triangle

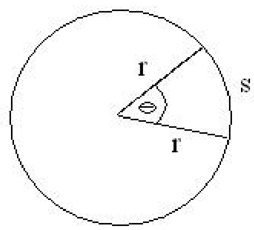



"Noah's Arc"
"Perhaps you misunderstood the command to 'build an ark'?"
"I suppose I did...."
"...Hey, at least it floats.."
40 day/40 night cruise? !?!
Where are the rooms for two?
L. Friedman #6 11-19-11
Eventually, Noah realizes that this assignment was NOT a geometry construction
mathplane.com

SOLUTIONS -→

1) The sector area of a circle is $3cm^2$. And, the perimeter of the sector is 7cm. What is the (possible) length(s) of the radius?

Step 1: Draw the figure; label the parts



Step 2: List measurements and formulas

Sector Area = $\frac{\theta}{360}\pi r^2 = 3cm^2$

Arc Length = $\frac{\theta}{360} 2\pi r = s$

Perimeter of sector = r + r + s

= 2r + s = 7

Step 3: Combine formulas and use algebra to find missing variables.

$\frac{\theta}{360} = \frac{3cm^2}{\pi r^2}$ (from sector area)

$\frac{\theta}{360} = \frac{s}{2\pi r}$ (from arc length)

$\frac{3cm^2}{\pi r^2} = \frac{s}{2\pi r}$ (substitution)

$\frac{3cm^2}{r} = \frac{s}{2}$ (multiply both by πr)

(multiply both by 2)

$\frac{6cm^2}{r} = s$

Step 4: Place s into perimeter formula to find r

$2r + s = 7$

$2r + \frac{6cm^2}{r} = 7$ (substitution)

$2r^2 + 6cm^2 = 7r$ (multiply entire equation by r)

$2r^2 - 7r + 6cm^2 = 0$ (Factor and solve)

$(2r - 3cm)(r - 2cm) = 0$

radius = 1.5 cm or 2 cm

Step 5: Check your answer

If r = 2 cm

Area of circle = 4π

Circumference = 4π

s = 3 (because we were given 2r + s = 7)

Arc Length = $\frac{\theta}{360} 2\pi r = s$

$\frac{\theta}{360} 2\pi(2cm) = 3$

$\frac{\theta}{360} 4cm = \frac{3}{\pi}$

$\theta = \frac{270}{\pi}$

Sector Area = $\frac{\theta}{360}\pi r^2 = 3cm^2$

$\frac{\frac{270}{\pi}}{360}\pi(2)^2 = 3cm^2$

$\frac{270}{360}(4) = 3cm$

$\frac{270}{360} = \frac{3}{4}$ ✓

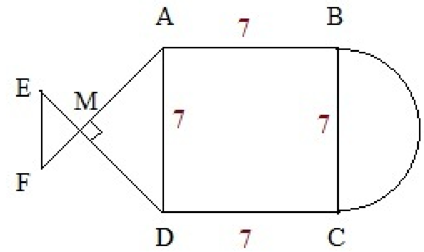
(**Then, check r = 1.5)

2) Given: Area of square ABCD = 49 sq. feet

$\overline{AF} \perp \overline{DE}$

$\overline{EF} \parallel \overline{AD}$

$\overline{EM} \cong \overline{FM}$



a) What is the area of semi-circle $\widehat{BC}$?

Since area of ABCD = 49,
AB = BC = CD = AD = 7 feet

Diameter BC = 7 feet
Radius of semi-circle = 3.5 feet

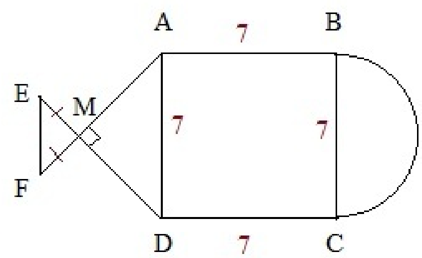
If $\overline{BC}$ is 7, then the radius of the semi-circle is 3.5

Area of a circle = πr^2

Area of a semi-circle = $(1/2)\pi r^2$

Area of semi-circle $\widehat{BC}$ = $(1/2)\pi\, 12.25 \cong$ **19.23 square feet**

b) What is the length of $\overline{AM}$?



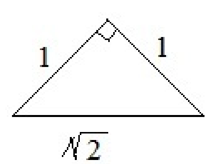
Since $\overline{EM} = \overline{FM}$
and $\angle M = 90^\circ$,

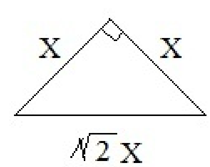
$\angle E = \angle F = 45^\circ$

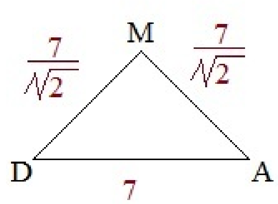
Since E and F are 45°
and
$\overline{EF}$ is parallel to $\overline{AD}$,
then
$\angle A = \angle D = 45^\circ$

(If parallel lines cut by a transversal, then alternate interior angles are congruent.)

$\triangle$ AMD is a 45-45-90 triangle!



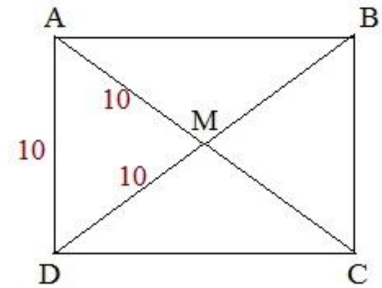




$\overline{AM} = \frac{7}{\sqrt{2}} = \boxed{\frac{7\sqrt{2}}{2} \text{ Feet}}$

3) Given $\overline{AM} = \overline{MD} = \overline{DA} = 10$

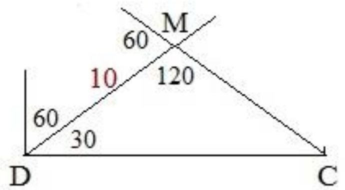
$\triangle$AMD is an equilateral triangle

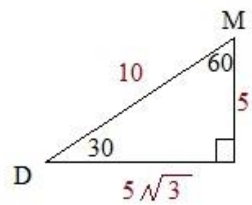


a) What is the perimeter of rectangle ABCD?

We know $\overline{AD} = \overline{BC} = 10$

Consider $\triangle$DMC, to find the other 2 sides.





Using properties of special right triangles, we find that $\overline{DC} = 10\sqrt{3}$

So, the perimeter of ABCD is

$10 + 10\sqrt{3} + 10 + 10\sqrt{3} \cong 54.64$

b) What is the area of $\triangle$DMC?

Area of a triangle = 1/2 bh

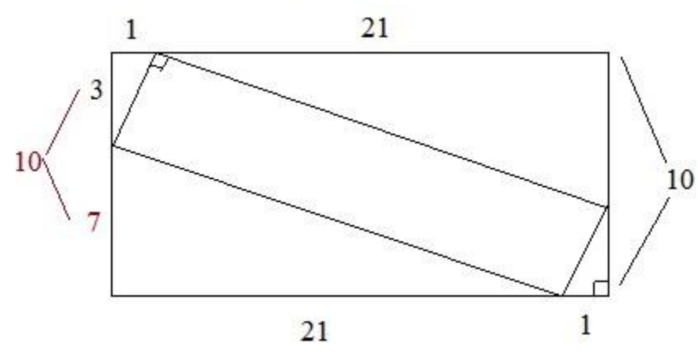
Area of $\triangle$DMC = $(1/2)\ 10\sqrt{3}\ (5) = 25\sqrt{3}$ square units

$\cong 43.3$ sq. units

4) Given Figure A (rectangle inscribed in a rectangle):

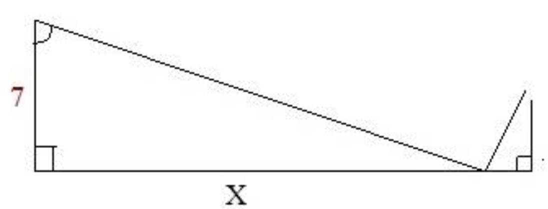
a) What is the perimeter of the outer rectangle? (Hint: similar triangles & proportions)

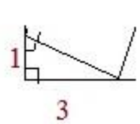
Figure A



Opposite sides of rectangle are same..

then, we observe that the small triangles are similar to the large triangles!





$\frac{7}{1} = \frac{x}{3}$

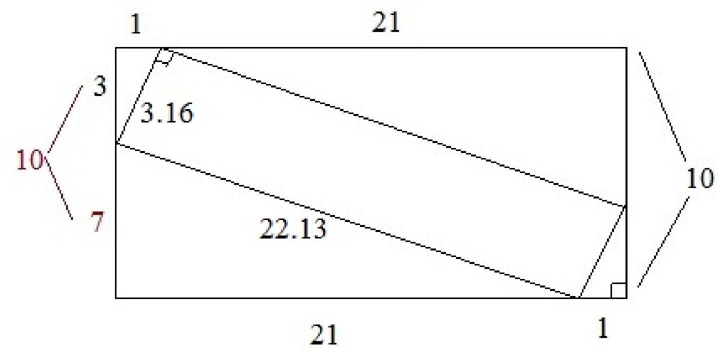
X = 21

Perimeter = 10 + 22 + 10 + 22

= 62 units

b) What is the area of the inner rectangle?
(Hint: "encasement" or pythagorean theorem)

Figure A



Once we get all the side measurements, we can use pythagorean theorem to get remaining sides..

$7^2 + 21^2 = C^2$

$49 + 441 = 490$

$C_1 = 7\sqrt{10}$

$1^2 + 3^2 = C$

$1 + 9 = 10$

$C_2 = \sqrt{10}$

Area of inner rectangle = $C_1 \times C_2$

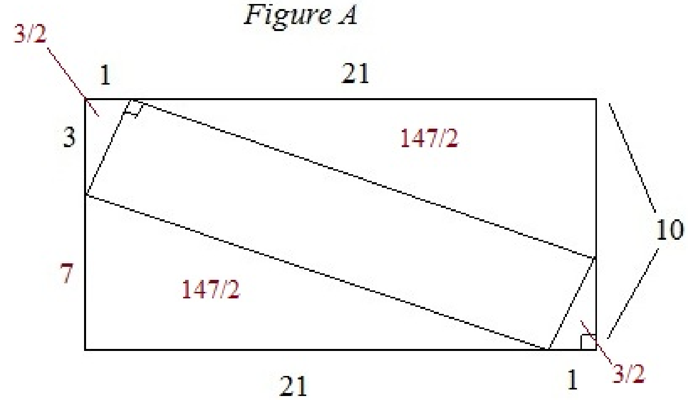
$= 7\sqrt{10} \times \sqrt{10}$

= 70 square units

Alternate Method: "Encasement"

Find area of outer rectangle..
then, subtract the area of the 4 triangles..
The remainder is the inner rectangle..

Figure A



Area of outer rectangle: 10 x 22 = 220

Area of small triangles: 1/2(1)(3) = 3/2 (each)

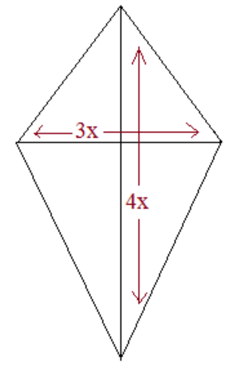
Area of large triangles: 1/2(7)(21) = 147/2 (each)

Area of outer: 220
Area of triangles: 3/2 + 3/2 + 147/2 + 147/2

Middle rectangle: 220 - (150) = **70 square units**

5) In a kite, the ratio of the diagonals is 3:4
If the area is 150 square feet, what is the length of the larger diagonal?

step 1: draw the figure and label



area = 150 sq feet

step 2: express the relevant equations

area of a kite = $\frac{d_1 d_2}{2}$

where d_1 and d_2 are the diagonals

step 3: solve

$$150 \text{ sq feet} = \frac{3x\,(\text{feet}) \cdot 4x(\text{feet})}{2}$$

$$150 = \frac{12x^2}{2}$$

$$300 = 12x^2$$

$$x^2 = 25$$

$x = 5$ or ~~-5~~

(distance cannot be negative)

step 4: answer question and check

What is the length of the larger diagonal?

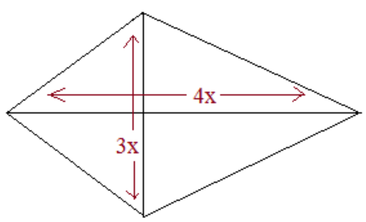
larger diagonal = 4x ----> **20 feet**

The smaller diagonal will be 3x ---> 15 feet

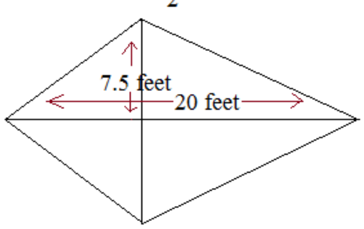
ratio is 15:20 or 3:4 ✓
and the area is

$\frac{15 \times 20}{2}$ = 150 square feet ✓

Additonal note:



Since a kite is symmetric, it has 2 congruent triangles. Therefore, to find the area, simply use area of a triangle: $\frac{1}{2}$ bh



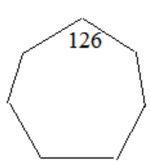
area of each triangle:

$\frac{1}{2}$ 20 feet (7.5 feet) = 75 sq. feet

area of kite (i.e. area of both triangles) = 150 sq. feet

6) A picture frame is shaped as a heptagon. The measure of the top interior angle is 126°. The remaining interior angles are congruent to each other. What is the measure of each remaining interior angle?

Sketch the image and label:



(heptagon has 7 sides/7 interior angles)

Write equations:

The sum of the interior angles =

$(7 \text{ (sides)} - 2) \cdot 180^\circ = 900^\circ$

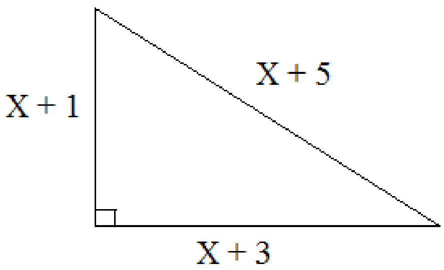
Solve:

Since the top angle is 126, the remaining 6 angles are 900 + 126 = 774°

Since the remaining 6 angles are congruent, each angle is 774 ÷ 6 = 129°

Each remaining angle is 129°

7) What is the perimeter of the triangle?



Use Pythagorean Theorem and algebra to find X:

$$a^2 + b^2 = c^2$$

$$(X + 1)^2 + (X + 3)^2 = (X + 5)^2$$

$$X^2 + 2X + 1 + X^2 + 6X + 9 = X^2 + 10X + 25$$

$$X^2 - 2X - 15 = 0$$

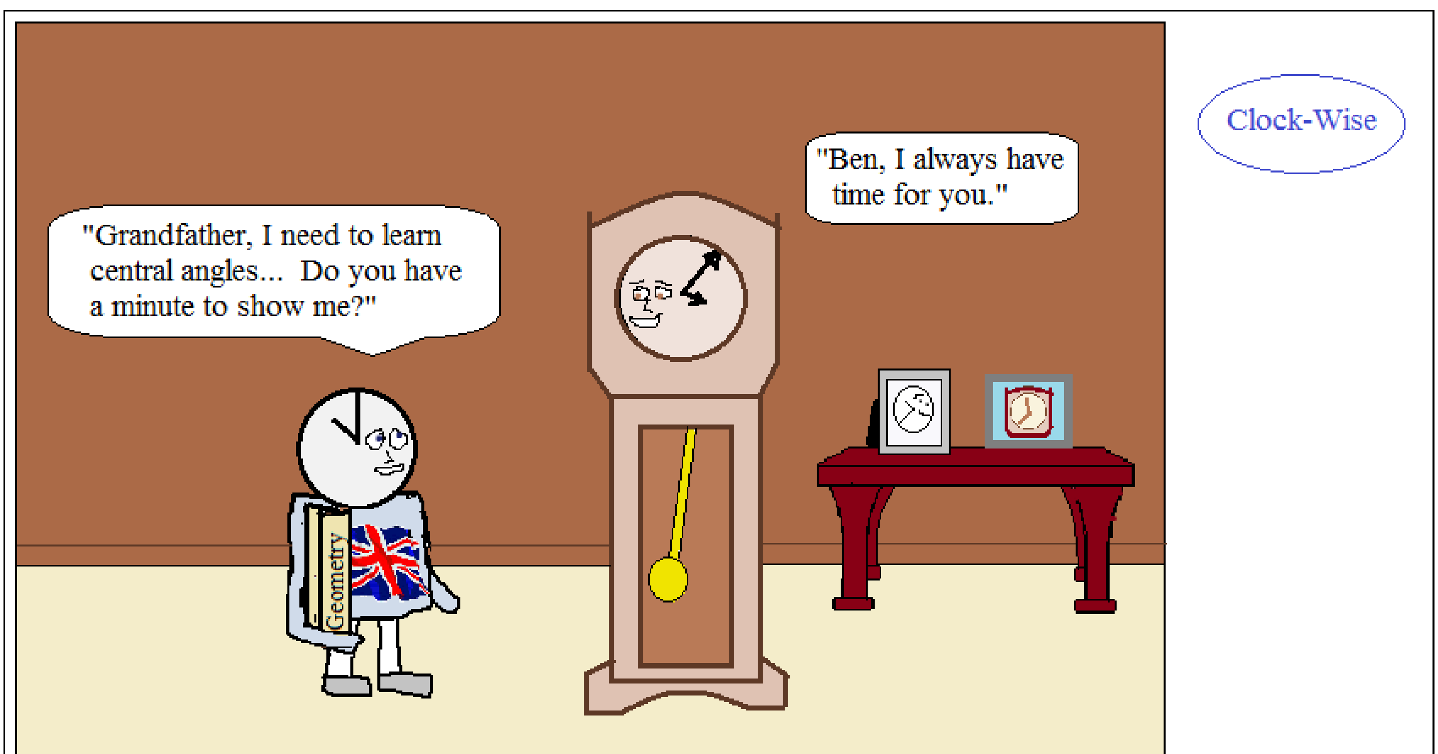
$$(X - 5)(X + 3) = 0$$

$X = -3$ or 5

Since length cannot be negative, we can eliminate $X = -3$

$X = 5$, so the lengths of the triangle are 6, 8, 10.

The perimeter is 24



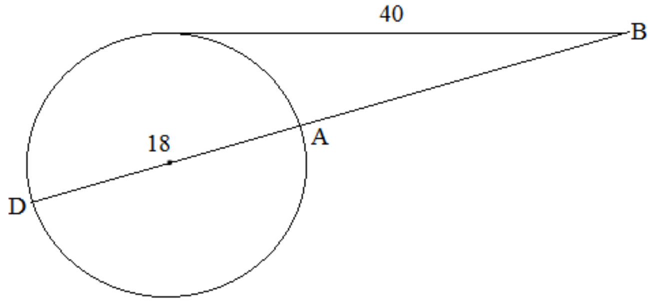
The old-timer demonstrates acute angles --- without being obtuse.

LanceAF #85 5-10-13
www.mathplane.com

Geometry Review Test 2 SOLUTIONS

8) Given: Diameter $\overline{AD} = 18$
Tangent segment is 40

Find: the length of $\overline{AB}$



Method 1: Utilize the secant - tangent theorem

$40^2 = (AB)(AB + 18)$

$AB^2 + 18B - 1600 = 0$

$(AB + 50)(AB - 32) = 0$

$AB = 32$ (length AB must be positive)

"Secant-Tangent Theorem": If a tangent and a secant of a circle meet at a point outside the circle, then the product of the external secant and the entire secant equals the tangent squared.

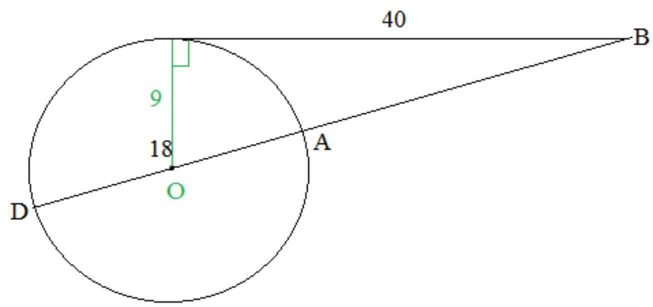
Method 2: Utilize chord and perpendicular tangent

Since diameter is 18, radius is 9

Tangent segment form right angle...

OB = 41 (9-40-41 right triangle)

Therefore, AB = 41 - 9 = 32



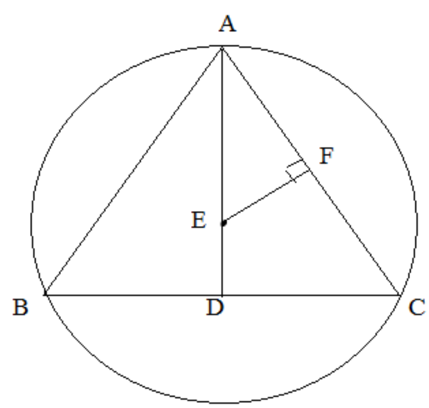
9) Given: ABC is an isosceles triangle inscribed in the circle where $\overline{AB} \cong \overline{AC}$

$\overline{AF} = 6$

$\overline{ED} = 1$

Find: a) the radius of the circle

b) the perimeter of the triangle



a) recognize that $\triangle AFE \sim \triangle ADC$

Set up the proportion:

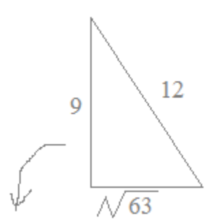
$$\frac{6}{(x+1)} = \frac{x}{12}$$

$x^2 + x = 72$

$(x - 8)(x + 9) = 72$

$x = 8$ or -9

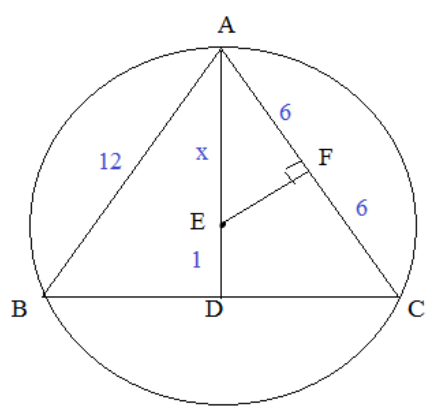
(-9 is extraneous)



b) Since x = 8, the length of AD is 9

Using Pythagorean Theorem, DC = $\sqrt{63}$

So, perimeter of ABC is $12 + 12 + 2\sqrt{63} = 24 + 6\sqrt{7}$

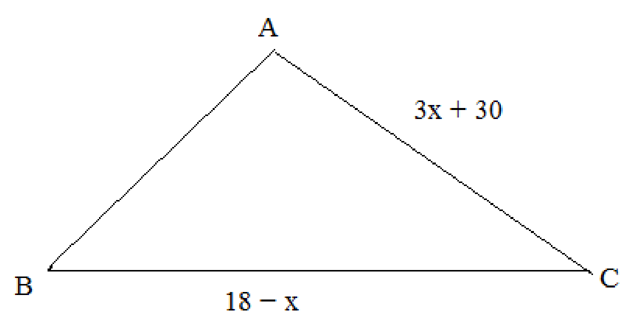


More Topics….

Triangle Characteristics

1) What are the restrictions of x?

$m\angle A > m\angle B$



Since $\angle A > \angle B$,

$\overline{BC} > \overline{AC}$

$(18 - x) > (3x + 30)$

$-12 > 4x$

$x < -3$

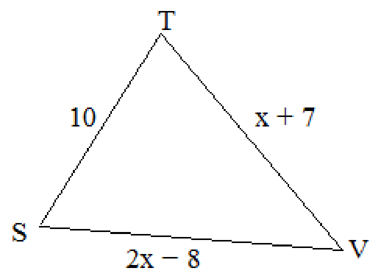
Also, since a side cannot be less than or equal to zero,

$\overline{BC}$	$18 - x > 0$	$x < 18$
$\overline{AC}$	$3x + 30 > 0$	$x > -10$

Therefore, the restrictions for x are $-10 < x < -3$

---

2) If the perimeter is less than 45, which side is the base?



If 10 is the base: $x + 7 = 2x - 8$
$x = 15$
therefore, the legs are 22
(If the legs are 22, then the perimeter exceeds 45)

If 2x - 8 is the base: $x + 7 = 10$
$x = 3$
Therefore, the legs are 10 and the base is -6
(a segment cannot be negative!)

If x + 7 is the base: $2x - 8 = 10$
$x = 9$
Therefore, the legs are 10 and the base is 16

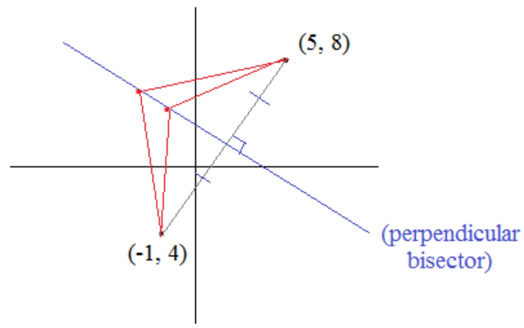
The base is $\overline{TV} = 16$

Write an equation that describes the set of points equidistant from both (-1, 4) and (5, 8).

Solution

Step 1: Graph and apply Geometric theorem

Perpendicular Bisector Theorem: The perpendicular bisector of a line segment is the locus of all points that equidistant from the endpoints



any point on the perpendicular bisector is equidistant from both points!

Step 2: Establish strategy and lists formulas or variables

To find the equation of a line, we need the *slope* and a *point*.

The bisector is the midpoint of (-1, 4) and (5, 8)

The slope of a perpendicular segment is the opposite reciprocal.

Step 3: Solve

The midpoint of (5, 8) and (-1, 4)

midpoint formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$\left(\frac{-1 + 5}{2}, \frac{4 + 8}{2}\right) = (2, 6)$$

The slope of segment joining (5, 8) and (-1, 4)

slope = $\frac{y_1 - y_2}{x_1 - x_2}$

$$\frac{8 - 4}{5 - (-1)} = \frac{4}{6} = \frac{2}{3}$$

Therefore, the slope of the perpendicular line is $\frac{-3}{2}$

Equation of a line: $y - y_1 = m(x - x_1)$
(pt. slope form)

slope m = $\frac{-3}{2}$ through point (2, 6)

$$y - 6 = \frac{-3}{2}(x - 2)$$

Step 4: Quick check

Pick a random point on the line. then, see if it is equidistant from (-1, 4) and (5, 8)

If x = 8, $y - 6 = \frac{-3}{2}(8 - 2)$

then y = -3

Let's test (8, -3)

distance formula: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

distance between (8, -3) and (-1, 4)

$$d = \sqrt{(8 - (-1))^2 + (-3 - 4)^2} = \sqrt{130}$$

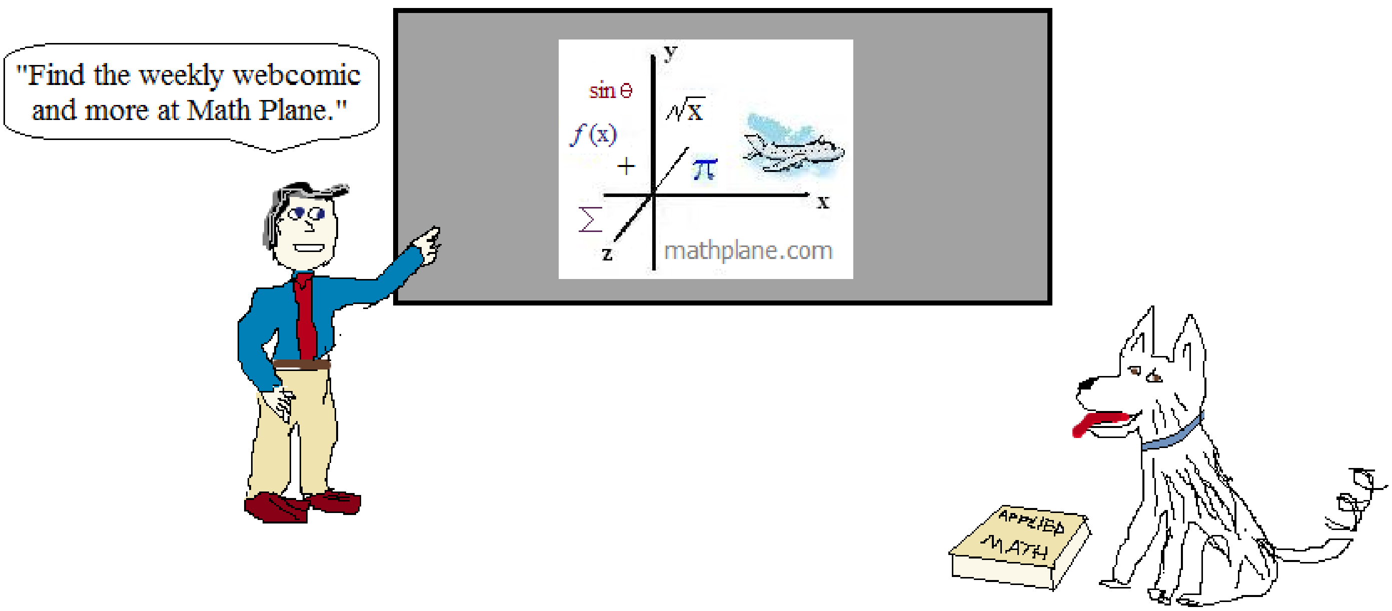
distance between (8, -3) and (5, 8)

$$d = \sqrt{(8 - 5)^2 + (-3 - 8)^2} = \sqrt{130}$$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, contact us.

Cheers



Also, at Facebook, Google+, Pinterest, and TeachersPayTeachers

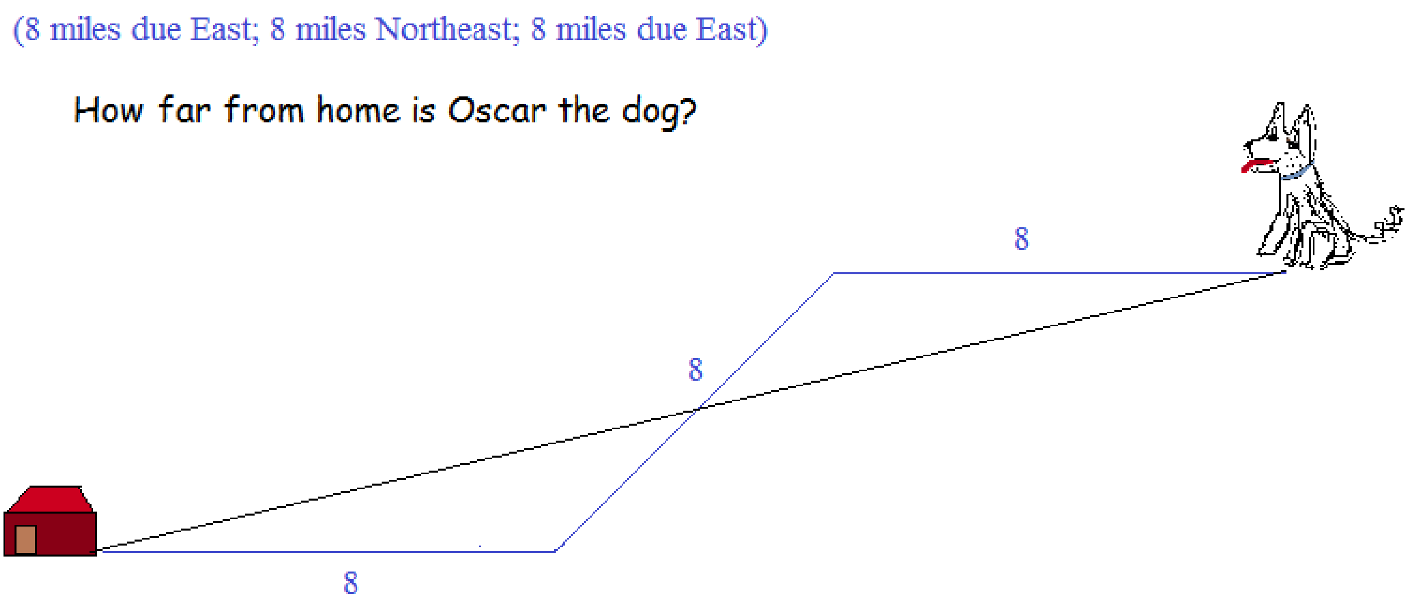
One more question….

Oscar the dog leaves home and walks 8 miles due East.

Then, he turns and walks another 8 miles Northeast.

And, then, he turns and walks due East 8 miles more.

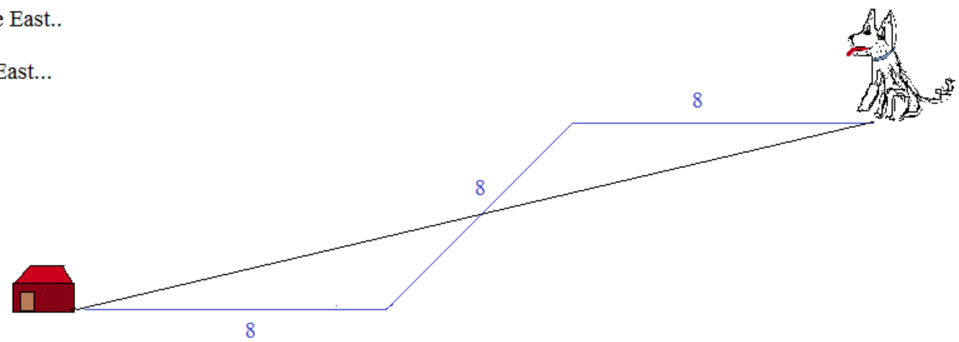
How far from home is Oscar the dog?



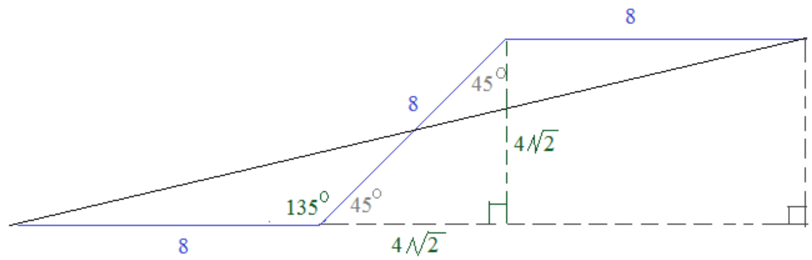
SOLUTION ON NEXT PAGE -→

Oscar the dog leaves home and walks 8 miles due East..
then, he turns and continues 8 miles NorthEast..
And, then, he turns and goes 8 miles further due East...

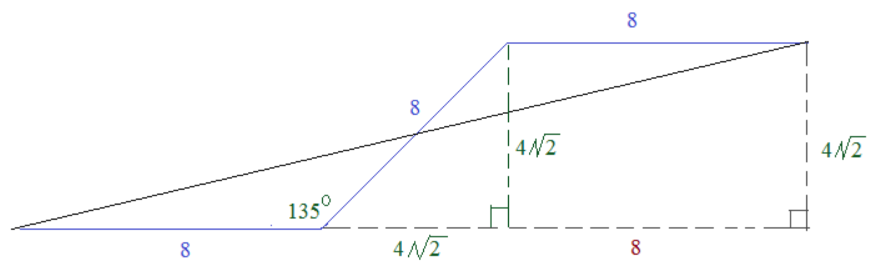
How far from home is Oscar?



recognize 45-45-90 right triangle,
properties of rectangles.....



Use Pythagorean Theorem
to get full length....



$$(16 + 4\sqrt{2})^2 + (4\sqrt{2})^2 = (\text{distance})^2$$

$$256 + 128\sqrt{2} + 32 + 32 = (\text{distance})^2$$

$$320 + 128\sqrt{2} = (\text{distance})^2$$

distance ≈ 22.38 miles